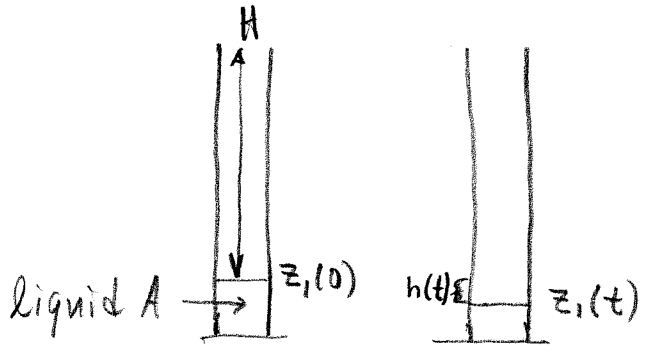
Example 18.2-1 Diffusion With a Moving Interface.



assume that the diffusion occurs at quasi-steady-state.

$$N_{Az}\big|_{z_1} = \frac{c\mathcal{D}_{AB}}{(z_2 - z_1(t))} \cdot \frac{x_{A1} - x_{A2}}{(x_B)_{ln}}$$

$$-S\frac{\rho^{(A)}}{M_A}\frac{dz_1}{dt} = N_{Az}\big|_{z_1} \cdot S = \frac{c\mathcal{D}_{AB}}{(z_2 - z_1(t))} \cdot \frac{x_{A1} - x_{A2}}{(x_B)_{ln}}$$

$$-(z_2 - z_1(t))\,dz_1 = \frac{c\mathcal{D}_{AB}}{(\rho^{(A)}/M_A)} \cdot \frac{(x_{A1} - x_{A2})}{(x_B)_{ln}} \cdot dt$$

but $H = z_2 - z_1(0)$ or $z_2 = H + z_1(0)$

$h(t) = z_1(0) - z_1(t)$ or $z_1(t) = z_1(0) - h(t)$

$$-(H + z_1(0) - z_1(0) + h(t))(-dh) = \frac{c\mathcal{D}_{AB}}{(\rho^{(A)}/M_A)} \cdot \frac{(x_{A1} - x_{A2})}{(x_B)_{ln}}\,dt$$

$$\int_0^h (H+h)\,dh = \frac{c\mathcal{D}_{AB}(x_{A1} - x_{A2})}{(\rho^{(A)}/M_A)(x_B)_{ln}} \int_0^t dt$$

$$Hh + \frac{1}{2}h^2 = \frac{c\mathcal{D}_{AB}(x_{A1} - x_{A2})}{(\rho^{(A)}/M_A)(x_B)_{ln}}\,t = \frac{1}{2}Ct$$

where $C = \dfrac{2c\,\mathcal{D}_{AB}(x_{A1}-x_{A2})}{(\rho^{(A)}/M_A)(x_B)_{ln}}$

quadratic eqn is

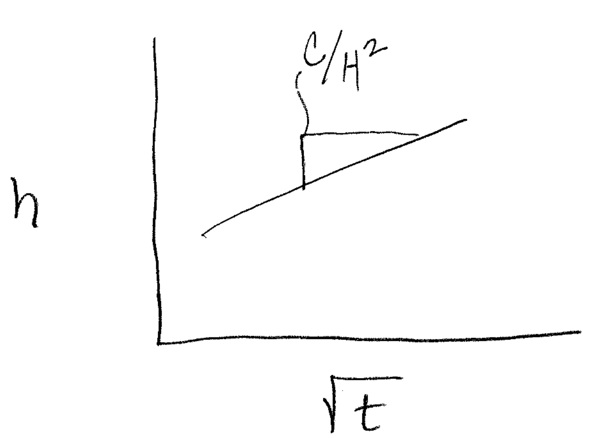
$$h^2 + 2Hh - Ct = 0$$

$$h = \frac{-2H \pm \sqrt{4H^2+4Ct}}{2(1)} = \frac{\pm \cancel{2}\sqrt{H^2+Ct} - \cancel{2}H}{\cancel{2}}$$

only + roots

$$\boxed{h = H\left(\sqrt{1+Ct/H^2} - 1\right)}$$

Measuring h vs t, $\mathcal{D}_{AB}$ can be obtained.



$\underline{\mathcal{D}_{AB} = f(C)}$.

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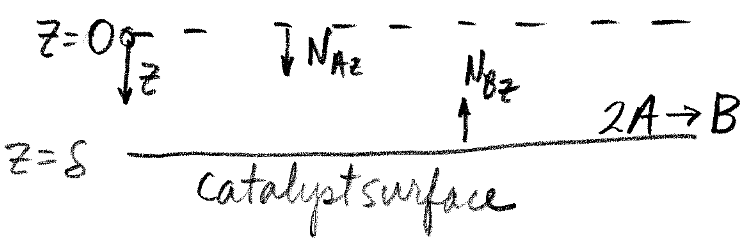
Binary Diffusion:

$$N_{Az} = -c\mathcal{D}_{AB}\frac{\partial x_A}{\partial z} + x_A(N_{Az}+N_{Bz}) \qquad (18.0\text{-}1)$$

Prior to solving a shell balance eqn. for x_A, we must determine $(N_{Az}+N_{Bz})$

Reaction at a Catalyst Surface,

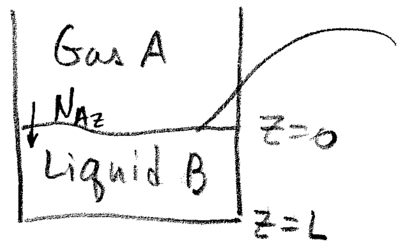


$$N_{Bz} = -\frac{1}{2}N_{Az}$$

$$N_{Az} = -c\mathcal{D}_{AB}\frac{\partial x_A}{\partial z} + x_A\left(N_{Az} - \frac{1}{2}N_{Az}\right)$$

$$\therefore \boxed{N_{Az} = -\frac{c\mathcal{D}_{AB}}{1-\frac{1}{2}x_A}\frac{dx_A}{dz}}$$

Diffusion With Homogenous Chemical Reaction



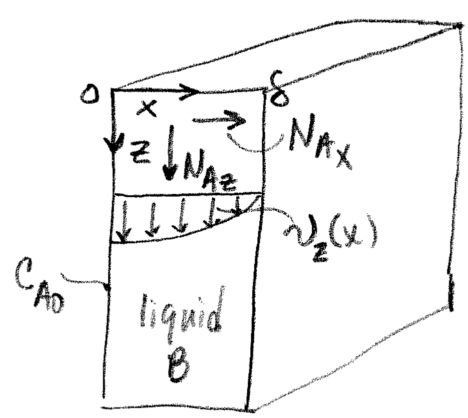
C_{A0} = solubility conc. of A in liquid B, ~0 (v. small)

for $C_{A0} \sim 0$, $x_A \sim 0$, $c \sim$ constant

$$\therefore N_{Az} = -c\mathcal{D}_{AB}\frac{\partial x_A}{\partial z} + \cancel{x_A}^{\,0}(N_{Az}+N_{Bz}) \sim \boxed{-c\mathcal{D}_{AB}\frac{\partial x_A}{\partial z}}$$

$$\boxed{= -\mathcal{D}_{AB}\frac{\partial c_A}{\partial z}} \quad \text{or}$$

Absorption of a Dilute Gas into a Falling Liquid Film



$$N_{Az} = \underbrace{\cancel{-c\mathcal{D}_{AB}\frac{\partial x_A}{\partial z}}^{\,0}}_{\text{diffusion is small relative to convection}} + \underbrace{x_A(N_{Az}+N_{Bz})}_{\text{convection}} \sim x_A c v^* \sim c_A v_z(x) \longrightarrow$$

see next pg.

for dilute A in B

$$v^* = \frac{\sum_{\alpha=1}^{N} c_\alpha v_\alpha}{\sum_{\alpha=1}^{N} c_\alpha} = \frac{\cancelto{0}{c_A v_A} + c_B v_B}{\cancelto{0}{c_A} + c_B} \sim v_B = v_z(x)$$

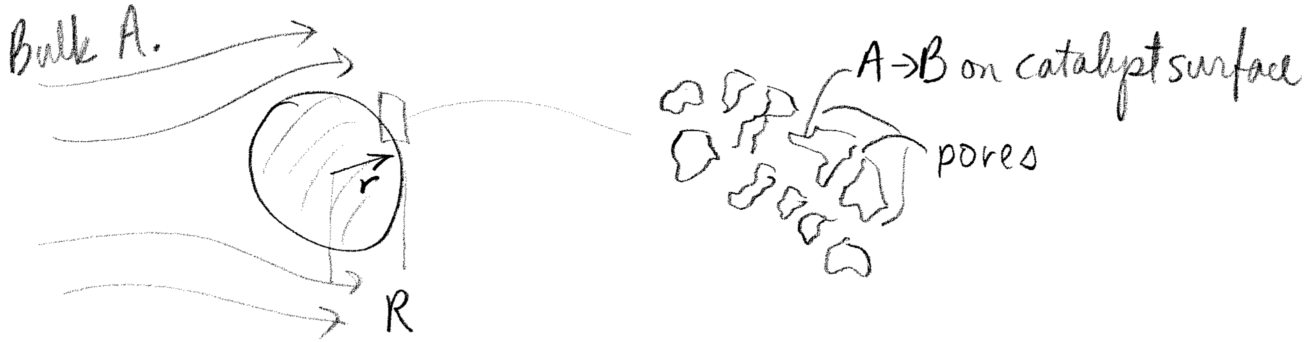
$$N_{Ax} = -c\mathcal{D}_{AB}\frac{\partial x_A}{\partial x} + \underbrace{x_A(\cancelto{0}{N_{Ax}+N_{Bx}})}_{\text{convection is negligible}}$$

$\sim -c\mathcal{D}_{AB}\frac{\partial x_A}{\partial x}$ -- no component of flow in x-direction!

$x_A \sim 0$

$\sim -\mathcal{D}_{AB}\frac{\partial c_A}{\partial x}$ ~ when $x_A \sim 0$ (v. small, dilute) → c is constant!

Diffusion and Chemical Reaction Inside a Porous Catalyst



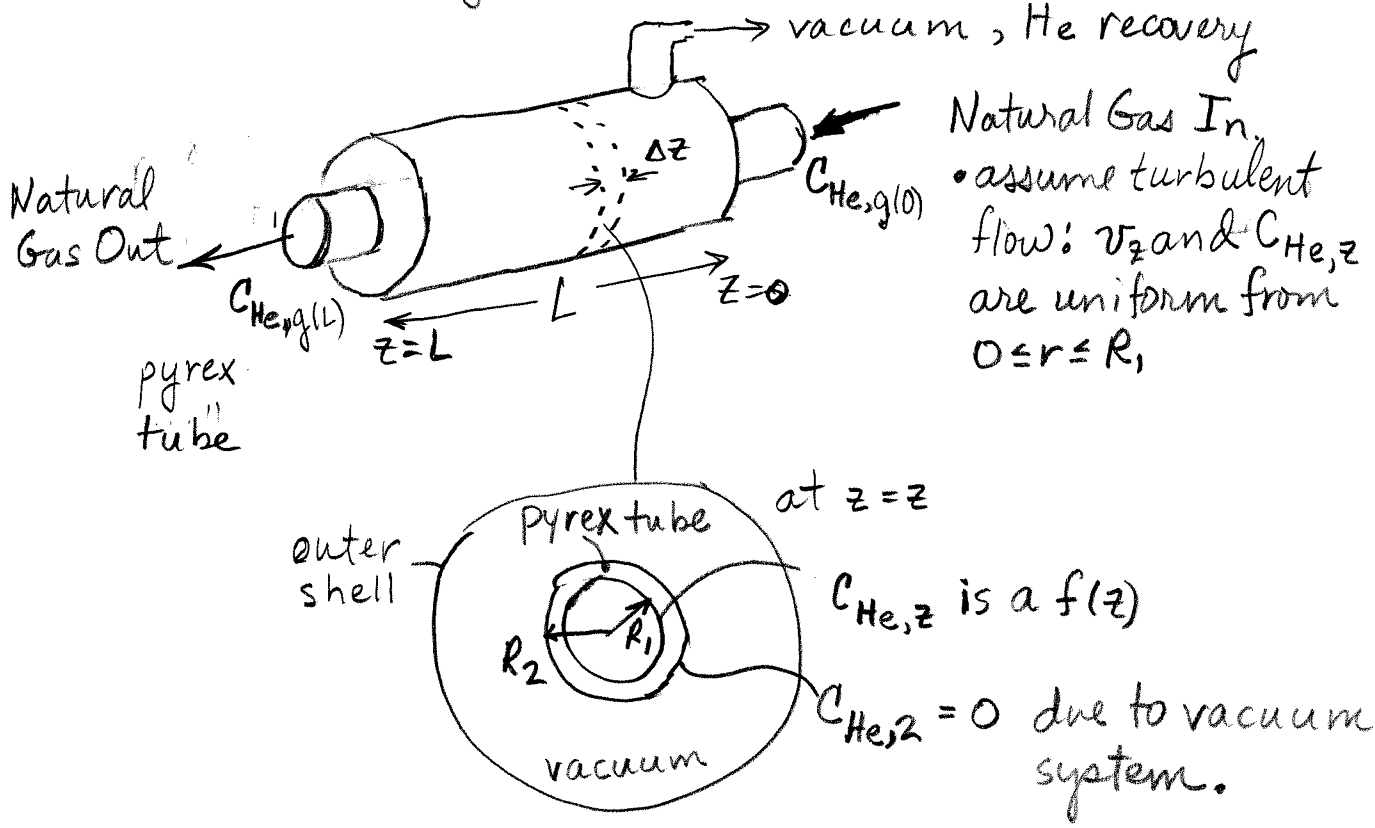
$$N_{Ar} = -c\mathcal{D}_A\frac{dx_A}{dr} + x_A(N_{Ar}+N_{Br}).$$

for this reaction stoichiometry $N_{Br} = -N_{Ar}$, c = constant

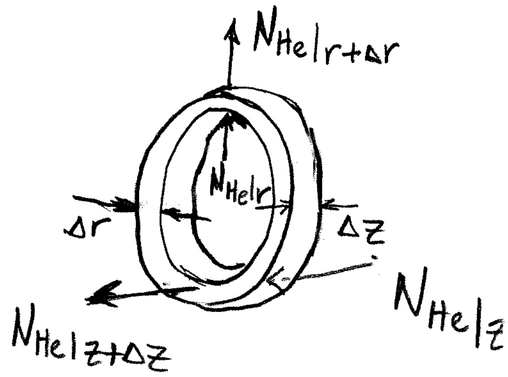
$$\therefore N_{Ar} = -c\mathcal{D}_A\frac{dx_A}{dr} = \boxed{-\mathcal{D}_A\frac{dc_A}{dr}}$$

where $\mathcal{D}_A \equiv$ effective diffusivity of A with the catalyst. - must be measured, but some correlations are available!

Prob. 18B.8 Separating He From Natural Gas.



Shell Balance in Pyrex Tube: steady-state



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- rate of He diffusion in at r — $N_{He}|_r \cdot 2\pi r \Delta z$
- " " " " out at $r+\Delta r$ — $N_{He}|_{r+\Delta r} \cdot 2\pi (r+\Delta r) \Delta z$
- " " " " in at z — $N_{He}|_z \cdot 2\pi r \Delta r$
- " " " " out at $z+\Delta z$ — $N_{He}|_{z+\Delta z} \cdot 2\pi r \Delta r$