Ch 2. Shell Momentum Balances
Velocity Distributions in Laminar Flow.

Shell Momentum Balance. Steady Flow

$$\underbrace{\text{rate in} - \text{rate out}}_{\text{convective flux}} + \underbrace{\text{rate in} - \text{rate out}}_{\text{molecular flux}} + \text{gravity force} = 0$$

- identify non-vanishing v components
- identify pressure dependence
- write momentum balance on fluid "shell"
- allow shell thickness $\to 0$, use derivative definition
- solve for τ distribution
- introduce Newton's Law
- solve for v distribution

Boundary Conditions,

- solid-liquid interface, $v_{fluid} = v_{solid}$,
- liquid-liquid, $v_{tangential}$ and $p + \tau_{ii}$ ~ continuous,
- gas-liquid, τ_{ii} in liquid $= 0$.

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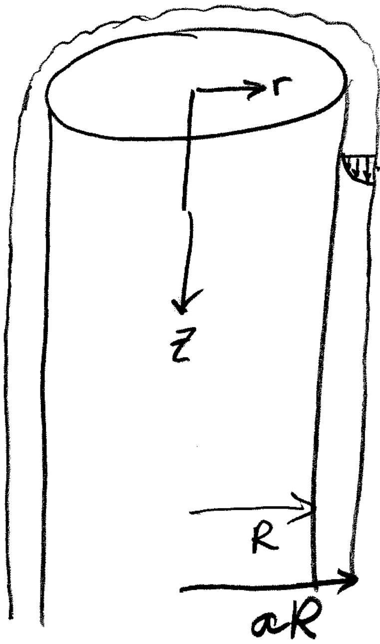
2B.6 Shell Balance Example



$$v_z = \frac{\rho g R^2}{4\mu}\left(1-\left(\frac{r}{R}\right)^2 + 2a^2 \ln\left(\frac{r}{R}\right)\right)$$

$$v_{z,max} = \frac{\rho g R^2}{4\mu}\left(1-\left(\frac{aR}{R}\right)^2 + 2a^2 \ln\left(\frac{aR}{R}\right)\right)$$

$$= \{ \quad (1-a^2+2a^2 \ln a)$$

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$$\underset{z}{\overset{*}{}} \quad \tau_{rz} = -\mu\left[\cancelto{0}{\frac{\partial v_r}{\partial z}} + \frac{\partial v_z}{\partial r}\right] = -\mu \frac{d v_z}{dr}$$

$$\tau_{rz}\Big|_R = -\mu \cdot \frac{d}{dr}\left[\frac{\rho g R^2}{4\mu}\left(1-\left(\frac{r}{R}\right)^2 + 2a^2 \ln\left(\frac{r}{R}\right)\right)\right]_R$$

$$= -\cancel{\mu} \cdot \frac{\rho g R^2}{4\cancel{\mu}}\left[-\frac{2}{R^2} r + 2a^2 \cdot \frac{1}{r}\right]_R$$

$$= -\frac{\rho g R^2}{4}\left[-\frac{2}{R} + 2a^2 \cdot \frac{1}{R}\right]$$

$$= -\frac{\rho g R}{4}\left[-2+2a^2\right] = \boxed{-\frac{\rho g R}{2}\left[a^2-1\right]}$$