3.6 Uses of Equations of Change.

Steady Flow in a Long Circular Tube. ρ, μ constant $v = \delta_z v_z(r,z)$

$v_r = v_\theta = 0$

Eqn. of Continuity: Table B.4

$$\cancel{\frac{\partial \rho}{\partial t}} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r \cancel{v_r}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho \cancel{v_\theta}) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\rho \frac{\partial v_z}{\partial z} + v_z \cancel{\frac{\partial \rho}{\partial z}} = 0 \quad \Rightarrow \quad \boxed{\frac{\partial v_z}{\partial z} = 0}$$

"$v = \delta_z v_z(r)$ only"

Eqn. of Motion: Table B.6

$\mathcal{P} = \mathcal{P}(z)$ alone

r-component $\Rightarrow$ $0 = -\frac{\partial p}{\partial r} + \rho g_r$ or $\boxed{0 = -\frac{\partial \mathcal{P}}{\partial r}}$

θ-component $\Rightarrow$ $0 = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$ or $0 = -\frac{\partial p}{\partial \theta} + \rho g_\theta r$

$$\boxed{0 = -\frac{\partial \mathcal{P}}{\partial \theta}}$$

z-component $\Rightarrow$ $0 = -\frac{\partial \mathcal{P}}{\partial z} + \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right)$

$$0 = -\frac{\partial \mathcal{P}}{\partial z} + \mu \frac{1}{r}\frac{d}{dr}\left(r \frac{dv_z}{dr}\right) \Rightarrow v_z = v_z(r)$$

$$\mu \frac{1}{r}\frac{d}{dr}\left(r \frac{dv_z}{dr}\right) = C_0 = \frac{d\mathcal{P}}{dz}$$

C_0 = constant.

$$\therefore \mathcal{P} = C_0 z + C_1$$

$$v_z = \frac{C_0}{4\mu} r^2 + C_2 \ln r + C_3$$

BC1 $\quad z=0 \quad \mathcal{P}=\mathcal{P}_0 \quad \rightarrow C_1=\mathcal{P}_0$

BC2 $\quad z=L \quad \mathcal{P}=\mathcal{P}_L \quad \rightarrow C_0=(\mathcal{P}_L-\mathcal{P}_0)/L$

BC3 $\quad r=R \quad v_z=0 \quad \rightarrow C_3=\dfrac{\mathcal{P}_0-\mathcal{P}_L}{4\mu L}R^2$

BC4 $\quad r=0 \quad v_z=\text{finite} \rightarrow C_2=0$

$$\mathcal{P}=\mathcal{P}_0-(\mathcal{P}_0-\mathcal{P}_L)(z/L)$$

$$v_z=\frac{\mathcal{P}_L-\mathcal{P}_0}{4L\mu}r^2+\frac{\mathcal{P}_0-\mathcal{P}_L}{4\mu L}R^2$$

$$\boxed{=\frac{(\mathcal{P}_0-\mathcal{P}_L)R^2}{4\mu L}\left(1-\left(\frac{r}{R}\right)^2\right)}$$

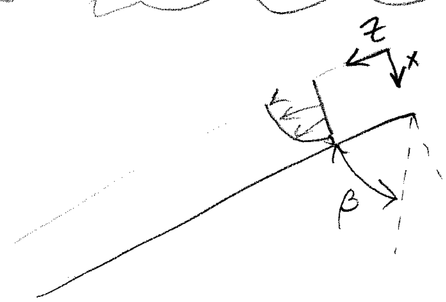
$$\tau_{rz}=-\mu\frac{\partial v_z}{\partial r}$$

$$=\frac{\cancel{\mu}(\mathcal{P}_0-\mathcal{P}_L)}{2\cancel{\mu}L}r$$

$$\tau_{rz,max}=\frac{(\mathcal{P}_0-\mathcal{P}_L)R}{2L}$$

F_z = shear force of fluid on wall

$=(2\pi RL)\,\tau_{rz,max}$



Falling Film Problem.

$v_z=v_z(x) \qquad \rho=\text{constant}$

$v_x=v_y=0 \qquad \longrightarrow$ Table B.1 $\quad \tau_{zx}=\tau_{xz}=-\mu\dfrac{\partial v_z}{\partial x}$

Eqn. of Continuity: $\Rightarrow$ all terms are zero

Eqns. of Motion: in Terms of τ Table B.5

x-component: $\quad 0=-\dfrac{\partial p}{\partial x}+\rho g\sin\beta$

y-component $\quad 0=-\dfrac{\partial p}{\partial y} \qquad p\neq p(y)$

z-component $\quad 0=-\dfrac{\partial p}{\partial z}-\dfrac{\partial}{\partial x}\tau_{xz}+\rho g\cos\beta$

integrating x-component eqn,

$$p = \rho g x \sin\beta + f(y,z)$$

but $p \neq p(y)$ as shown above, and we recognize that the pressure in gas above film is nearly constant at P_{atm} for short L, so at $x=0$, for all z, $p = P_{atm}$.

$$\boxed{p = \rho g x \sin\beta + P_{atm}.}$$

z-component eqn becomes.

$$\boxed{\frac{d}{dx}\tau_{xz} = \rho g \cos\beta} \qquad (3.6\text{-}19)$$

$$(2.2\text{-}10)$$

See example 2.2-2 for solution when $\mu = \mu_0 e^{-\alpha x/\delta}$

$$\boxed{v_z = \frac{\rho g \delta^2 \cos\beta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]}$$

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