CALCULATING FLOW RATE FROM VELOCITY PROFILE

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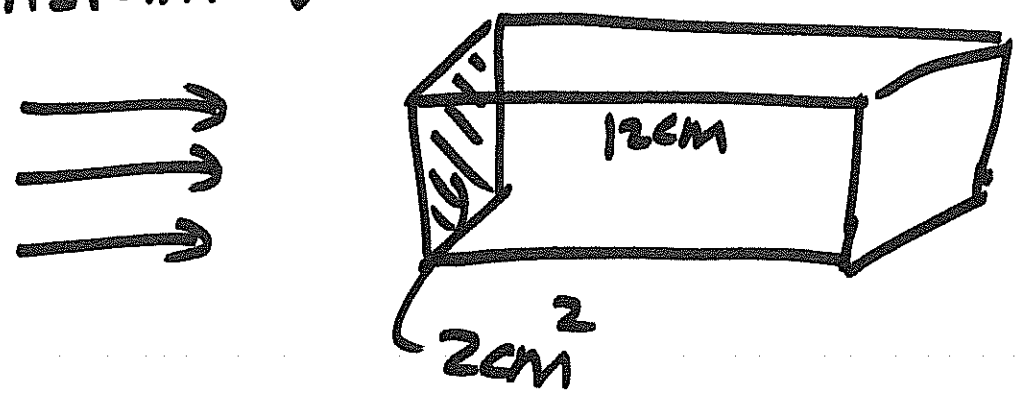
constant $V = 4\,cm/s$



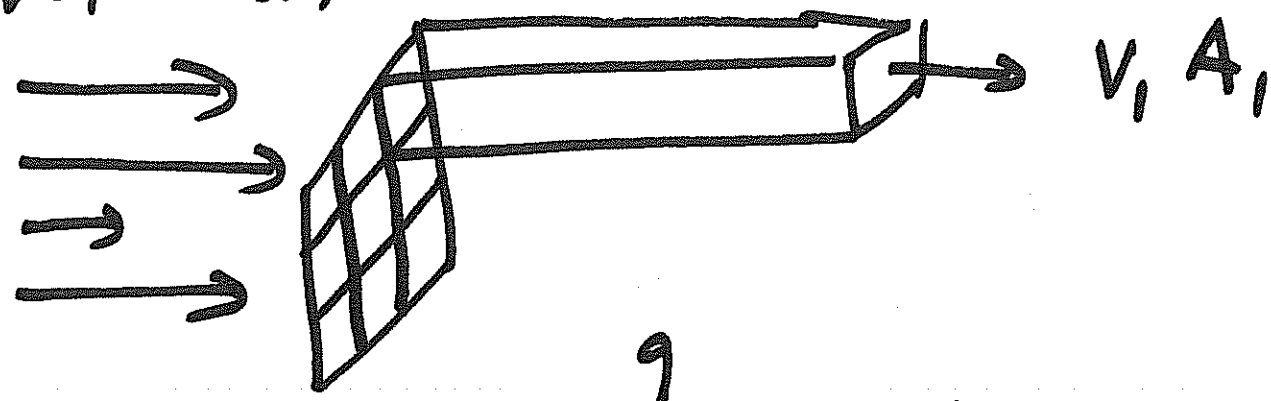
Choose time = 3 s

$$\frac{4\,cm}{s}\ (\text{velocity}) \times 3\,s\ (\text{time}) = 12\,cm \quad A$$

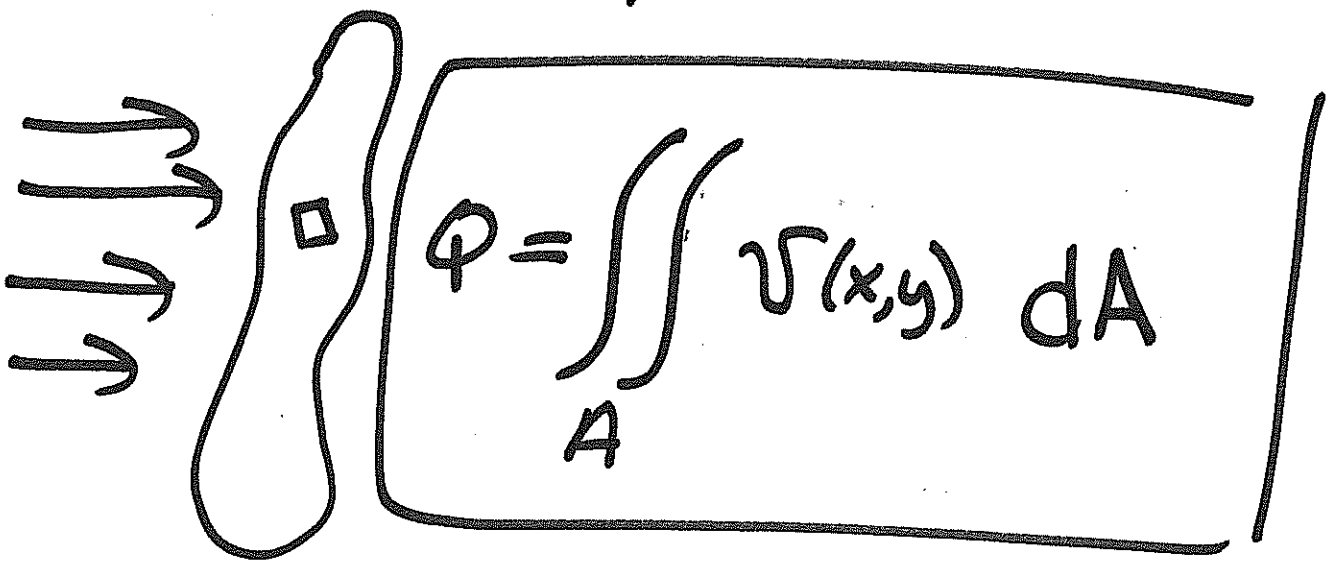
$$\frac{\text{Volume}}{\text{time}} = \frac{(12\,cm)(2\,cm^2)}{3\,s\ (\text{time})} = \boxed{\frac{8\,cm^3}{s}}$$

$$\boxed{Q = VA}$$

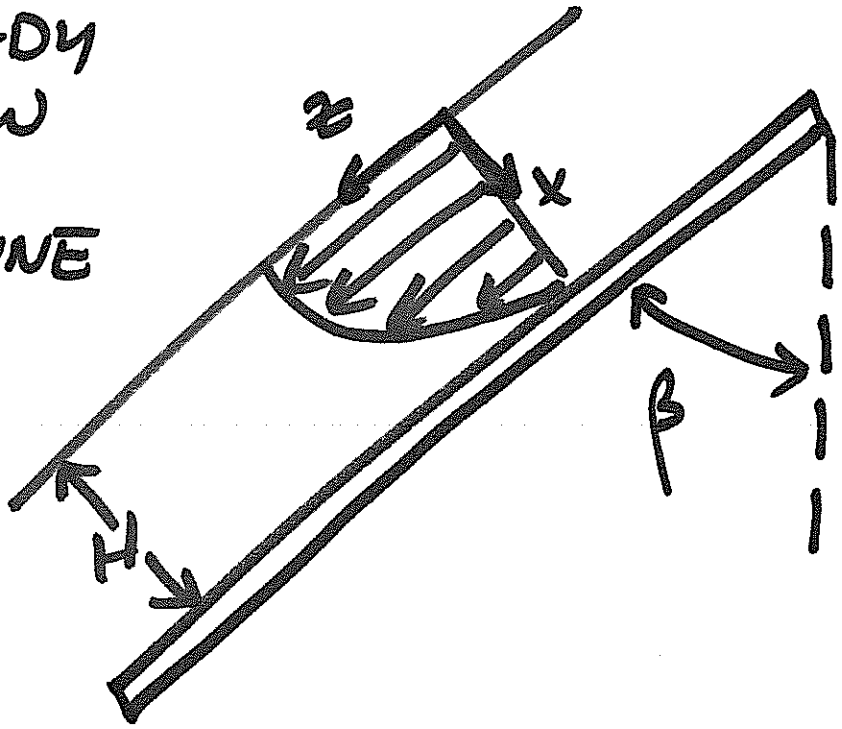
V(positions)



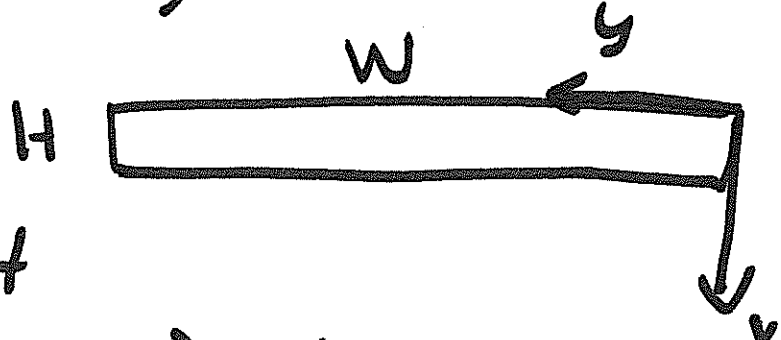
$$\Phi = \sum_{i=1}^{9} V_i A_i$$



STEADY FLOW DOWN INCLINE



$$v_z(x) = \frac{\rho g \cos\beta}{2\mu}(H^2 - x^2)$$



$$Q = \int_0^W \int_0^H v_z(x)\, dx\, dy$$

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$$= \int_0^W \int_0^H A_1 (H^2 - x^2)\, dx\, dy$$

$$= \int_0^H \underbrace{\left(A_1 (H^2 - x^2) y \Big|_0^W \right)}_{A_1 (H^2 - x^2)(W - 0)} dx$$

$$= \int_0^H A_1 W (H^2 - x^2)\, dx$$

$$= A_1 W \left(H^2 x - \frac{x^3}{3} \right) \Big|_0^H$$

$$Q = A_1 W \left(\frac{H^3}{1} - \frac{H^3}{3} - (0 - 0) \right)$$

$$Q = A_1 W \frac{2}{3} H^3 = \frac{\rho g \cos\beta}{2\mu} W \frac{2 H^3}{3} //$$