PART I and PART II

CALCULATING FORCE ON A SURFACE IN FLOW FROM VELOCITY FIELD

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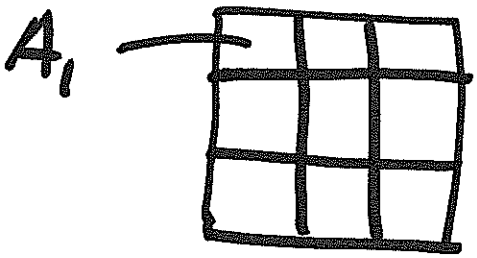
$$\text{Force} = A \cdot \text{stress} = \left(\frac{\text{force}}{\text{area}}\right)(\text{Area})$$

$$\boxed{\text{Force} = (\text{stress})(\text{Area})}$$

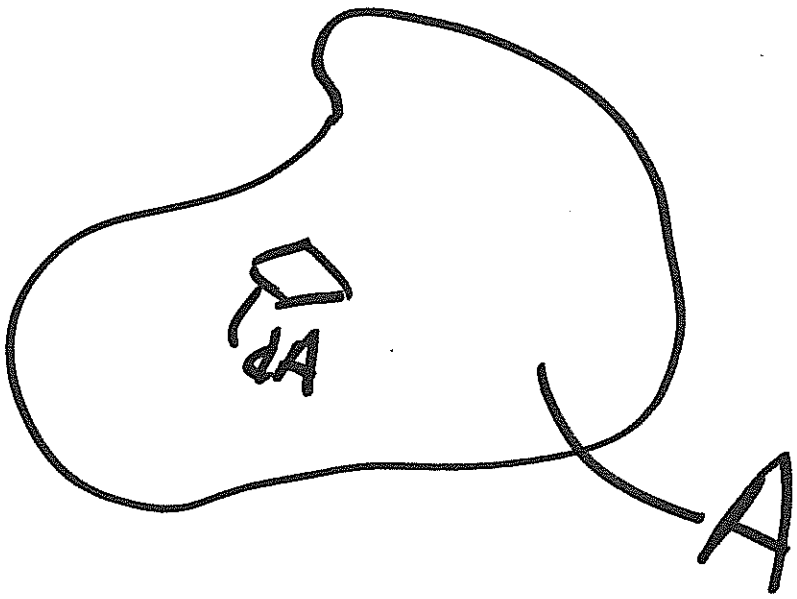
stress is uniform ←



total force

$$F = \text{stress}\big|_{A_1} \cdot A_1 + \text{stress}\big|_{A_2} A_2 \ldots$$

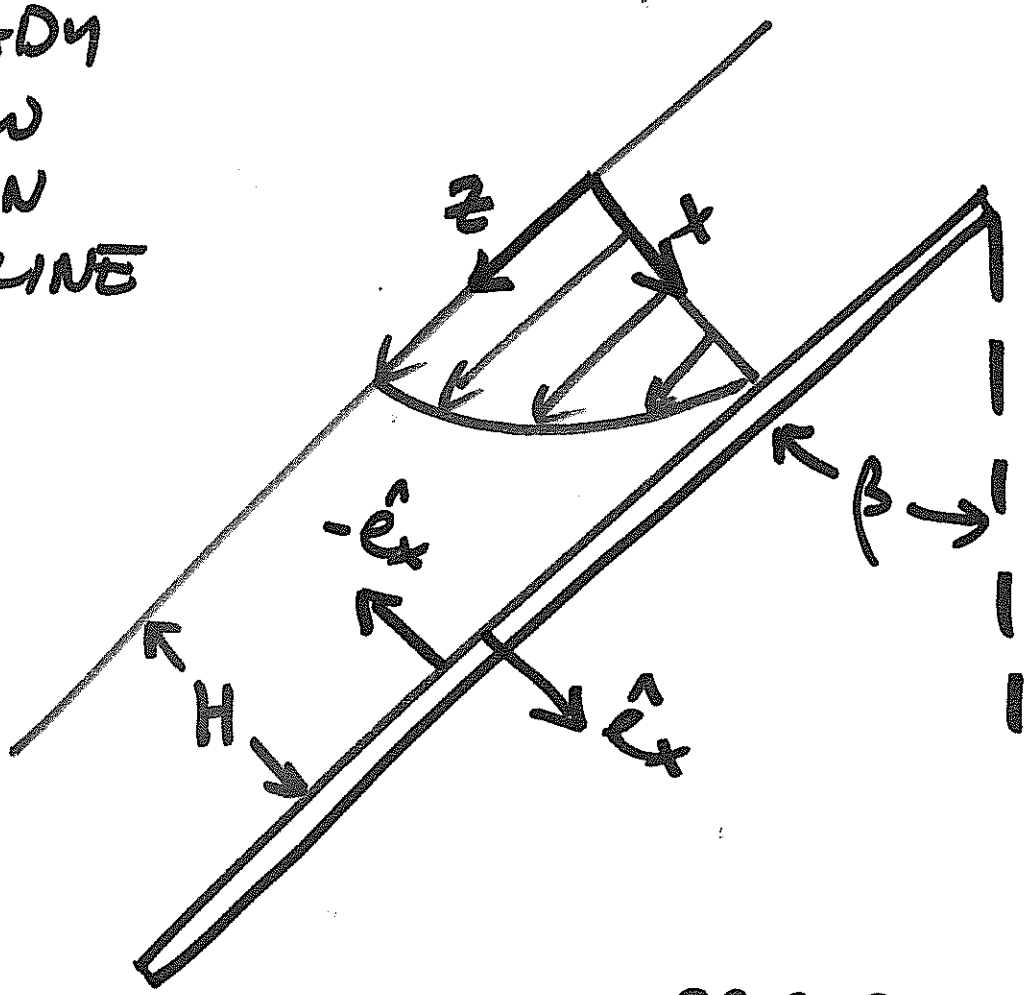
$$F = \sum_{i=1}^{9} (\text{stress})\big|_{A_i} A_i$$



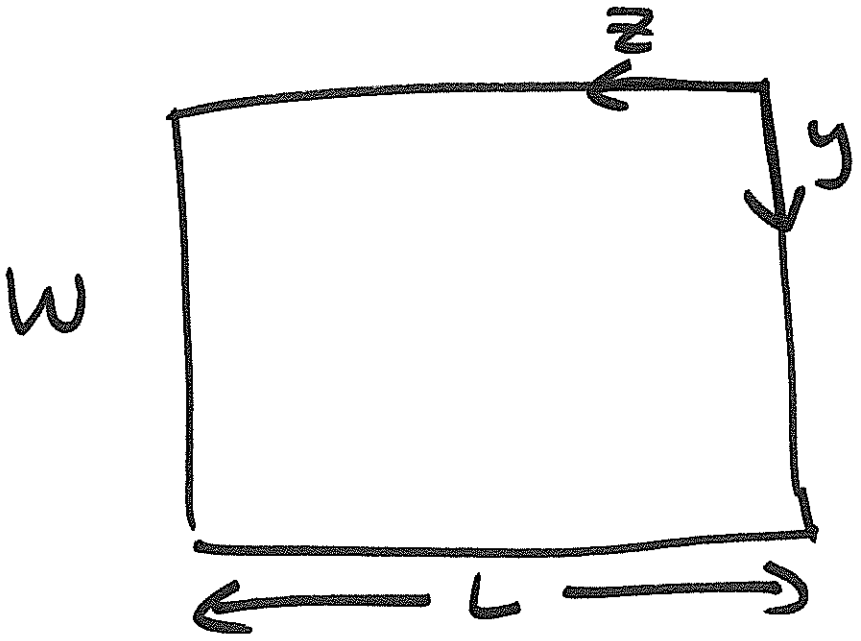
total force

$$F = \iint_A \text{stress}\Big|_{\text{at surface as a function of position}} dA$$

STEADY FLOW DOWN INCLINE



$$v_z(x) = \frac{\rho g \cos\beta}{2\mu}(H^2 - x^2)$$



$$F = \int_0^L \int_0^W \text{stress}\Big|_{\text{plate surface}} \, dy \, dz$$

Newton's Law

$$\tau_{xz} = -\mu \frac{dv_z}{dx}$$

- flux of z-momentum in the x-dir
- stress on x-surface in z-direction

x-surface = surface whose unit normal is $\hat{e}_x$

$$F = \int_0^L \int_0^W -\tau_{xz}\Big|_{\text{surface}} \, dy \, dz$$

$$F = \int_0^L \int_0^W -\tau_{xz}\Big|_{surface} \, dy\,dz$$

$$= \int_0^L \int_0^W -\tau_{xz}\Big|_{x=H} \, dy\,dz$$

Newtons Law of Viscosity

$$\tau_{xz} = -\mu \frac{dv_z}{dx}$$

$$v_z = \frac{\rho g \cos\beta}{2\mu}(H^2 - x^2)$$

$$\frac{dv_z}{dx} = \frac{\rho g \cos\beta}{2\mu}(-2x)$$

$$\tau_{xz} = \mu\left(-\frac{\rho g \cos\beta}{\mu} x\right)$$

$$\boxed{\tau_{xz} = \rho g \cos\beta \, x}$$

$$F = \int_0^L \int_0^W -\tau_{xz}\Big|_{x=H} \, dy\,dz$$

$$\tau_{xz} = \rho g \cos\beta \, x$$

$$\tau_{xz}\Big|_{x=H} = \rho g \cos\beta \, H$$

$$F = \int_0^L \int_0^W -\rho g \cos\beta H \, dy\,dz$$

$$= -\rho g \cos\beta H \int_0^L \int_0^W dy\,dz$$

$$\boxed{F = -\rho g \cos\beta \, HWL}$$

compressive Force

$-F$ = tensile force