Recommended
Practice Problems
with solutions
CM3110
Fluid Mechanics

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Ref: C. J. Geankoplis
Transport Processes + Separation Process Principles, Prentice Hall
Fluid Statics

Example 2.2-1 p37 - pressure in fluid vessel

Example 2.2-4 p40 classic manometer (part 2 is hard; omit)

Example 2.2-5 p41 hand-in manometer vessel pm

Average Velocity
Example 2.6-3 - \( \langle v \rangle \) for pipe flow

MEB
\[ \frac{dP}{\rho} + \frac{\Delta v^2}{2\alpha} + s \Delta z + F = \frac{W_b \text{cm}}{m^3} \]

Example 2.7-4 p68 pump + piping calc F

Example 2.7-5 p69 pump calc \( W_b \text{cm} \)

Example 2.7-6 p72 flow thru contraction

Example 2.7-7 p73 tank discharge
STEADY STATE
MACROSCOPIC MOMENTUM BALANCE
(MACRO mom)

\[ D = \sum_{i=1}^{N} \left( -pA <v_i^2 > \hat{u} \cos \theta \right) \frac{\beta}{\text{\textbar{}}_{\text{\textit{th}} \text{stream}}} \]

- Force on fluid
- Force on a nozzle
- Force on a bend
- Sudden enlargement

EXAMPLE 2.8-2 p78
EXAMPLE 2.8-3 p79
EXAMPLE 2.8-4 p80

pressure
wall (viscous friction)
Section 2.9B Discussion - Poiseuille Flow in a Tube

1. Calculate the steady state velocity profile for an incompressible, Newtonian fluid in pressure-driven flow in a tube. The tube is of radius $R$ and length $L$. The inlet pressure is $P_0$ and the exit pressure is $P_e$. Answer: Equation 2.9-9 p. 85

2. Calculate $<v_3>$ Answer: 2.9-11

3. Calculate $Q$ Answer: $Q = <v_3> 4\pi R^2$

4. Calculate the directed force on wall
   Answer: see attached (p7)
SECTION 2.9C - Falling Film

... (ditto) ... Newtonian fluid flowing down a vertical wall under the influence of gravity. You may assume unidirectional flow.

Equation

Answer: 2.9-25 p87

b) calc L/V; answer eqn 2.9-28

c) calc Q; answer eqn -2.9-29

d) calc force on wall; answer: see attached PT

3) Example 3.8-1 Laminar flow between plates

Answer: 3.8-9 p95

4) Example 3.8-2 Vertical plates one moving
TURBULENT FLOW

M Moody chart \( f(Re) \) p94

Example 2.10-6 \( f(Re) \) problem p100

NONCIRCULAR CONDUITS

Calculate the pressure drop for steady turbulent flow of water at \( Re = 13,000 \) through a conduit that is a square of size 1" on a side. The length of the conduit is 250'.

DRAG

Example 3.1-1 force on a sphere p124

Example 3.1-2 drag on a cylinder

PACKED BEDS

Example 3.1-4 \( \Delta P(Q) \) in packed bed
1. Example 3.5-1: Calculate $A_p, f$

2. Derive equation 3.5-12 from equation 3.5-21 (for velocity profile)

3. What is the shear stress at the center of the tube in steady, pressure-driven flow of a non-Newtonian fluid?
   Answer: See later
   How about for a Newtonian fluid?
   Answer: See later
\[ F_2 = \int_0^L \int_0^{x_2} \frac{T}{r^2} \, dA \, dA \]

\[ (\text{vertical wall}) \]

\[ F_2 = \int_0^L \int_0^{x_2} \frac{T}{r^2} \, dA \, dA \]

\[ \text{carry out} \]

\[ F_2 = (P_0 - P_2) \pi R^2 \]

\[ \text{Answer:} \]

\[ \text{page 4} \]

\[ \text{Answer:} \]

\[ \text{page 3} \]

\[ \text{Poiseuille Flow} \]

\[ \text{P6.3 See 2.9-6 at r=0 C=0 for both Newtonian and non-Newtonian} \]
Force on a Nozzle

Example 2.8-2

\[ P_2 = 1 \text{ atm} \]

\[ D_1 = 0.0035 \text{ m} \]

\[ D_2 = 0.0286 \text{ m} \]

**Neglect friction**

\[ Q = 0.03184 \text{ m}^3 \frac{m}{s} \]

\[ P_1 = P_2 = 1000 \text{ kg/m}^3 \]

**Calculate force on nozzle.**

\[ \frac{\Delta P}{\rho} + \frac{\Delta \left( U^2 \right)}{2\rho} + g\Delta z + F = \frac{\omega_s^2 \rho}{m} \]

\[ \alpha = 1 \text{ (turbulent)} \]

\[ P_2 - P_1 + \frac{\left( U^2 \right)_2 - \left( U^2 \right)_1}{\rho} = 0 \]

\[ P_1 = \left( \frac{\left( U^2 \right)_2 - \left( U^2 \right)_1}{\rho} \right) \rho + P_2 \]
\[ A_1 = \frac{\pi D_1^2}{4} = \frac{\pi (0.0635 \text{ m})^2}{4} \]
\[ = 3.669 \times 10^{-3} \text{ m}^2 \]

\[ \langle V \rangle_1 = \frac{Q}{A_1} = \frac{0.03154 \text{ m}^3/\text{s}}{3.669 \times 10^{-3} \text{ m}^2} \]
\[ \langle V \rangle_1 = 9.159 \text{ m/s} \]

\[ A_2 = \frac{\pi D_2^2}{4} = \frac{\pi (0.0286 \text{ m})^2}{4} \]
\[ = 6.424 \times 10^{-4} \text{ m}^2 \]

\[ \langle V \rangle_2 = \frac{Q}{A_2} = \frac{0.03154 \text{ m}^3/\text{s}}{6.424 \times 10^{-4} \text{ m}^2} \]
\[ \langle V \rangle_2 = 49.09 \text{ m/s} \]
\[
P_1 = \frac{\left[(49.09 \text{ m/s})^2 - (9.959 \text{ m/s})^2\right]}{2} \left(1000 \frac{\text{kg}}{\text{m}^3}\right) + 1 \text{ atm}
\]
\[
= 1.155 \times 10^6 \ \text{Pa} - \text{abs}
\]

**Macro Momentum**

\[
0 = \left( -\rho A \left( V_2 \cos \theta V^i_2 \right) \right)_{\text{inlet}} + \left( -\rho A \left( V_2 \cos \theta V^i_2 \right) \right)_{\text{outlet}} + F_p + F_{\text{outlet}} + F_{\text{inlet}}
\]

Recall: \( F_p = -\rho A \hat{n} \)

Outwardly pointing unit normal to the cross-section
\[ \theta_1 = 180 \quad \cos 180 = -1 \]
\[ \theta_2 = 0 \quad \cos 0 = 1 \]
\[ \beta_1 = \beta_2 = 1 \]
\[ \vec{U}^{(0)} = \vec{e}_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{x,y,z} \]
\[ \vec{U}^{(2)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{x,y,z} \]
\[ \dot{m} = m A \vec{V} \]

\[ -m_1 \langle v \rangle_{x}^2 (-1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + m_2 \langle v \rangle_{y}^2 (1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (-\gamma A_1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (\gamma A_2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = 0 \]
\( z \)-component:
\[ R_z = 0 \]

\( y \)-component:
\[ R_y = 0 \]

\( x \)-component:
\[ m_1 \langle V \rangle_1 - m_2 \langle V \rangle_2 + P_1 A_1 - P_2 A_2 + R_x = 0 \]

\[ m_2 = m_1 = \text{QP} = \left( 0.03154 \text{ m}^3 \right) \left( \frac{1000 \text{ kg}}{\text{m}^3} \right) \]
\[ = 31.54 \text{ kg} \]

\[ -R_x = 31.54 \text{ kg} \frac{\text{m}}{\text{s}} \left[ \left( \frac{9.96 \text{ m}}{8} \right) - \left( 49.09 \text{ m} \right) \right] \frac{\text{m}^2 \text{N}}{\text{kg} \text{m}} \]
\[ + (1.257 \times 10^6 \text{ Pa}) (3.669 \times 10^{-3} \text{ m}^2) \]
\[ - (1.01325 \times 10^5 \text{ Pa}) (6.424 \times 10^{-4} \text{ m}^2) \]

\[ = (-1234 + 4612 - 45) \text{ N} \]
\[ = 3313 \text{ N} \]