Recommended Practice Problems with solutions

CM3110

Fluid Mechanics

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Ref: C. J. Geankoplis
Transport Processes + Separation Process Principles, Prentice Hall
(2003) 4th ED, Upper Saddle River NJ

FLUID STATICS

EXAMPLE 2.2-1 p37 — pressure in 2 fluid vessel

EXAMPLE 2.2-4 p40 classic manometer (part 2 is hard; omit)

EXAMPLE 2.2-5 p41 harder manometer/vessel prob.

AVERAGE VELOCITY

EXAMPLE 2.6-3 p59 — $\langle v \rangle$ for pipe flow

MEB

$$\frac{\Delta P}{\rho} + \frac{\Delta \langle v \rangle^2}{2\alpha} + g\Delta z + F = \frac{W_{s,on}}{\dot{m}}$$

Example 2.7-4 p68 pump + piping calc F

Example 2.7-5 p69 pump calc $W_{s,on}$

EXAMPLE 2.7-6 p72 flow thru contraction

EXAMPLE 2.7-7 p73 tank discharge

STEADY STATE

MACROSCOPIC MOMENTUM BAL

(MACRO mom)

$$0 = \sum_{i=1}^{N} \left[\frac{-\rho A \langle v \rangle^2 \hat{u} \cos\theta}{\beta} \right]_{i^{th} \text{ stream}} + \underbrace{\underline{F}_p}_{\text{pressure}} + \underbrace{\underline{F}_v}_{\text{wall (viscous + other)}} + \underbrace{\underline{F}_g}_{\text{gravity}}$$

($\underline{F}_p + \underline{F}_v + \underline{F}_g$: force "on" fluid)

EXAMPLE 2.8-2 p78 force on a nozzle (see attached p8)

EXAMPLE 2.8-3 p79 force on a bend

EXAMPLE 2.8-4 p80 sudden enlargement (harder)

NAVIER - STOKES - MICRO MOM

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}$$

(1) SECTION 2.9 B DISCUSSION - Poiseuille flow in a tube

(a) Calc the steady state velocity profile for an incompressible, Newtonian fluid in pressure-driven flow in a tube. The tube is of radius R and length L. The inlet pressure is P_0 and the exit pressure is P_L.

Answer: EQN 2.9-9 p85

(b) Calc $\langle v \rangle$ Answer - 2.9-11

(c) calc Q (Hagen Poiseuille) Answer: $Q = \langle v \rangle \pi R^2$

(d) calc z-directed force on wall

Answer: see attached (p7)

(2) SECTION 2.9C – falling film

... (ditto)..., Newtonian fluid flowing down a vertical wall under the influence of gravity. You may assume unidirectional flow.

answer: equation 2.9-25 p87

(b) calc ⟨v⟩; answer eqn 2.9-28

(c) calc Q; answer eqn 2.9-29

(d) calc force on wall; answer: see attached p7

(3) EXAMPLE 3.8-1 p194 laminar flow between plates
Answer: 3.8-9 p195

(4) EXAMPLE 3.8-2 vertical plates one moving

TURBULENT FLOW

Moody Chart f(Re) p94

Example 2.10-6 f(Re) problem p100

NONCIRCULAR CONDUITS

Calculate the pressure drop for steady turbulent flow of water at Re = 13,000 through a conduit that is a square of size 1" on a side. The length of the conduit is 250'.

DRAG

Example 3.1-1 p124 force on a sphere

Example 3.1-2 drag on a cylinder

PACKED BEDS

Example 3.1-4 $\Delta P(Q)$ in packed bed

NON NEWTONIAN FLUIDS

(1) Example 3.5-1 calc Δp, f

(2) Derive equation 3.5-12 from equation 3.5-21 (av velocity from velocity profile)

(3) What is the shear stress at the center of the tube in steady, pressure-driven flow of a non-Newtonian fluid?

Answer: see later

How about for a Newtonian fluid??

Answer: see later

ANSWERS

poiseville flow
(d) Answer
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$$F_z = \int_0^L \int_0^{2\pi} \tau_{rz}\Big|_{r=R} \, dA$$

$\tau_{rz}\big|_{r=R}$: EQN 2.9-6 p84

$dA = R\,d\theta\,dz$

$$F_z = (P_0 - P_L)\,\pi R^2$$

(vertical wall)
(d) Answer
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$$F_z = \int_0^L \int_0^W \tau_{xz}\Big|_{x=\delta} \, dA$$

$dA = dy\,dz$

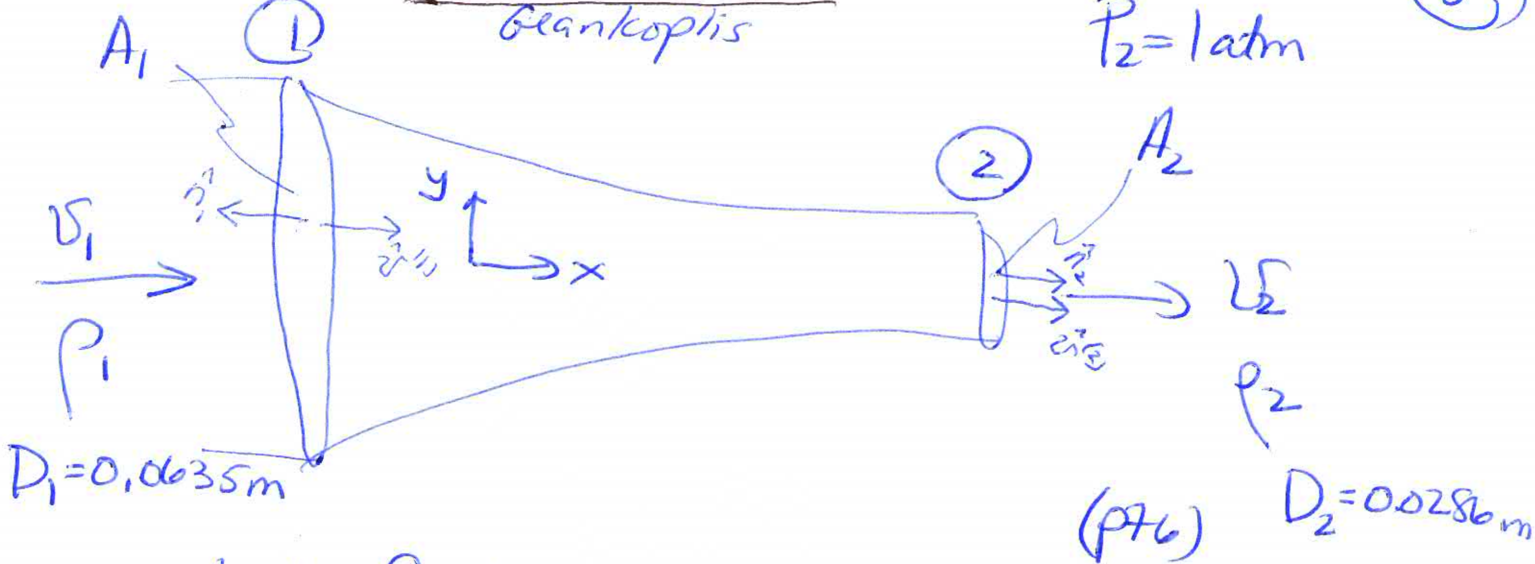
carry out.

p6,3 see 2.9-6 at $r=0$ $\tau=0$
for both Newtonian +
non Newtonian

Force on a Nozzle   Example 2.8-2 pg. 78

Geankoplis

(8)



neglect friction

$Q = 0.03154 \frac{m^3}{s}$

Calc FORCE ON NOZZLE.

$\rho_1 = \rho_2 = 1000\,kg/m^3$

MEB

$$\frac{\Delta P}{\rho} + \frac{\Delta \langle V \rangle^2}{2\alpha} + g\Delta z + F = \frac{W_{s,on}}{\dot{m}}$$

($g\Delta z$ → horizontal; F → neglect; $W_{s,on}/\dot{m}$ → no moving parts)

$\alpha = 1$ (turbulent)

$$\frac{P_2 - P_1}{\rho} + \frac{\langle V \rangle_2^2 - \langle V \rangle_1^2}{2} = 0$$

$$P_1 = \frac{\left(\langle V \rangle_2^2 - \langle V \rangle_1^2\right)\rho}{2} + P_2$$

$$A_1 = \frac{\pi D^2}{4} = \frac{\pi(.0635\,m)^2}{4}$$

$$= 3.669 \times 10^{-3}\,m^2$$

$$\langle V \rangle_1 = \frac{Q}{A_1} = \frac{0.03154\,m^3/s}{3.669 \times 10^{-3}\,m^2}$$

$$\boxed{\langle V \rangle_1 = 9.959\,m/s}$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(0.0286\,m)^2}{4}$$

$$= 6.424 \times 10^{-4}\,m^2$$

$$\langle V \rangle_2 = \frac{Q}{A_2} = \frac{0.03154\,m^3/s}{6.424 \times 10^{-4}\,m^2}$$

$$\boxed{\langle V \rangle_2 = 49.09\,m/s}$$

$$P_1 = \frac{\left[(49.09\,m/s)^2 - (9.959\,m/s)^2\right]}{2}\left(1000\,\frac{kg}{m^3}\right) + 1atm$$

$$= 1.155\times10^6 \;\frac{m^2}{s^2}\frac{kg}{m^3}\frac{N\,s^2}{kg\,m}\frac{Pa}{N/m^2}$$

$$+ 1.01325\times10^5\ Pa$$

$$\boxed{P_1 = 1.2566\times10^6\ Pa\text{-abs}}$$

MACRO MOMENTUM

$$0 = \left(\frac{-\rho A\langle v\rangle^2\cos\theta\,\hat{v}}{\beta}\right)_{inlet} + \left(\frac{-\rho A\langle v\rangle^2\cos\theta\,\hat{v}}{\beta}\right)_{outlet}$$

$$+ \underline{F}_p + \underline{F}_v + \underline{F}_g$$

$\underline{F}_p$ ← inlet + outlet

$\underline{F}_v = \underline{R}$ the force on walls

$\underline{F}_g$ → neglect

Recall: $\underline{F}_p = -pA\hat{n}$

$\hat{n}$ ← outwardly pointing unit normal to the cross-section

$\theta_1 = 180 \quad \cos 180 = -1$

$\theta_2 = 0 \quad \cos 0 = 1$

$\beta_1 = \beta_2 = 1$

$$\hat{v}^{(1)} = \hat{e}_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$\hat{v}^{(2)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$\dot{m} = \rho A \langle v \rangle$$

$$-\dot{m}_1 \langle v \rangle_1^2 (-1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz} + -\dot{m}_2 \langle v \rangle_2^2 (1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$+ (-P_1 A_1) \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz} + (-P_2 A_2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$+ \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = 0$$

z-component:

$$R_z = 0$$

y-component:

$$R_y = 0$$

x-component:

$$\dot{m}_1 \langle V \rangle_1 - \dot{m}_2 \langle V \rangle_2 + P_1 A_1 - P_2 A_2 + R_x = 0$$

$$m_2 = m_1 = Q\rho = \left(0.03154 \frac{m^3}{s}\right)\left(1000 \frac{kg}{m^3}\right)$$

$$= 31.54 \frac{kg}{s}$$

$$-R_x = 31.54 \frac{kg}{s}\left[\left(9.96 \frac{m}{s}\right) - \left(49.09 \frac{m}{s}\right)\right]\frac{s^2 N}{kg\, m}$$

$$+ (1.257 \times 10^6 \text{ Pa})(3.669 \times 10^{-3} m^2)$$

$$- (1.01325 \times 10^5 \text{ Pa})(6.424 \times 10^{-4} m^2)$$

$$= (-1234 + 4{,}612 - 65) \text{ N}$$

$$= 3313 \text{ N}$$