HEAT XFER

MACRO E-BAL:

\[ \Delta E_k + \Delta E_p + \Delta h = Q_{in} + W_{s,m} \]

- Change w/ T
- Phase change
- Rxn

MEB:

\[ \frac{\Delta P}{\rho} + \frac{\Delta v^2}{2\rho} + g \Delta z + F = W_{s,m} \]

\* MEB = a macro E-bal specialized for circumstances common in mechanical systems.
In Heat Exchanger Equipment:

\[ \Delta E_{lc} + \Delta E_p + \Delta h = Q_{in} + W_{em} \]

- Heat Exchanger
- Condenser
- Boil-er
- Reactor

\[ \Delta h = Q \]

*This is the Macro E-bal specialized for circumstances common in heat xfer equipment.*
\[ \Delta E_p + \Delta E_k + \Delta H = Q_{\text{in}} + \Delta E_{\text{can}} \]

\[ Q_{\text{in}} = \Delta H \]

\[ = \sum \text{in.} \hat{m}_i \hat{A}_i - \sum \text{in.} \hat{m}_j \hat{A}_j \]

\[ Q_{\text{in}} = m_2 \hat{A}_2 + m_3 \hat{A}_3 - m_1 \hat{A}_1 \]

*See Felder + Rousseau*
CV Bar in Heat Transfer:

Control Volume:

- Hot
- \( T_1 \)
- \( x \)
- \( B \)
- \( \Delta x \)
- Height \( H \)
- Width \( W \)

Choose control volume:

- Steady State: accumulation = 0
- No current
- No reactions

\[ \frac{9x}{A} \bigg|_{x}^{x+\Delta x} \]
\[
\frac{g_x}{A} \left|_{x+\Delta x} \right. = \frac{g_x}{A} \left|_x \right.
\]

\[
\frac{g_x}{A} \left|_{x+\Delta x} \right. - \frac{g_x}{A} \left|_x \right. = 0
\]

\[
\lim_{\Delta x \to 0} \frac{\Delta}{\Delta x} = 0
\]

\[
\frac{d}{dx} \left( \frac{g_x}{A} \right) = 0
\]

The general definition of a derivative
Integrate: \[ \sqrt{\frac{q_x}{A}} = c_1 \]

(Flux is constant)

Farin's Law: \[ \frac{q_x}{A} = -k \frac{\partial T}{\partial x} \]

\[ c_1 = -k \frac{\partial T}{\partial x} \]

\[ \frac{\partial T}{\partial x} = -\frac{\beta}{k} \]

Integrate: \[ T = \left( -\frac{\beta}{k} \right) x + c_2 \]
Boundary conditions:

\[ x = 0 \quad T = T_1, \]
\[ x = B \quad T = T_2 \]

Solve for \( c_1, c_2 \) (not shown; do the algebra)

\[ T = \left( \frac{T_2 - T_1}{B} \right) x + T_1 \]

What is the flux?
\[ \frac{d}{dx} T(x) = -k \frac{dT}{dx} \]

Fouria's Law!

\[ T(x) = \left( \frac{T_2 - T_1}{B} \right) x + T_1 \]

\[ \frac{dT}{dx} = \left( \frac{T_2 - T_1}{B} \right) \]

\[ \frac{d}{dx} T(x) = -k \left( \frac{T_2 - T_1}{B} \right) \]

 Flux depends on \( k \) and \( \frac{T_2 - T_1}{B} \) (flux profile does not)}
If I double the thermal conductivity $k$, how does the temperature profile change?

$T_1$  

$T_2$  

*(trick question: it does not change $T(x)$; it does change flux)*