Equations for Inside Front Cover

Unit conversions summary: www.chem.mtu.edu/˜fmorriso/cm310/convert.pdf

Mechanical Energy Balance

$$\frac{\Delta p}{\rho} + \frac{\Delta \langle v \rangle^2}{2\alpha} + g\Delta z + F_{friction} = -\frac{W_{s,by\ fluid}}{m} \qquad \begin{cases} \alpha_{laminar} = 0.5 \\ \alpha_{turbulent} \approx 1 \end{cases}$$

$$F_{friction} = \left[4f\frac{L}{D} + \sum_{fittings_i} n_i K_{f,i}\right]\frac{\langle v \rangle^2}{2}$$

Note error on inside front cover:

Fanning Friction Factor (pipe flow)

$$f = \frac{\mathcal{F}_{\text{drag}}}{\frac{1}{2}\rho\langle v\rangle^2 \cancel{\pi R^2}\ 2\pi RL} = \frac{\Delta p D}{2L\rho\langle v\rangle^2}$$

Drag Coefficient (sphere drop)

$$C_D = \frac{\mathcal{F}_{\text{drag}}}{\frac{1}{2}\rho v_\infty^2 \pi R^2} = \frac{4gD\left(\rho_{\text{body}} - \rho\right)}{3\rho v_\infty^2}$$

Momentum balance on a CV (Reynolds transport theorem)

$$\frac{d\mathbf{P}}{dt} + \iint_{CS} (\hat{n}\cdot \underline{v})\ \rho \underline{v}\ dS = \sum_{\substack{\text{on}\\ \text{CV}}} \underline{f}$$

Hydrostatic pressure $p_{bottom} = p_{top} + \rho g h$

Hagen-Poiseuille equation (steady, laminar tube flow, incompressible)

$$Q = \frac{\pi(p_0 - p_L)R^4}{8\mu L}$$

Stokes-Einstein-Sutherland equation (steady, slow flow around a sphere)

$$\mathcal{F}_{\text{drag}} = 6\pi R\mu v_\infty$$

Macroscopic Momentum Balance on a CV

$$\frac{d\mathbf{P}}{dt} + \sum_{i=1}^{\#streams}\left[\frac{\rho A\cos\theta\langle v\rangle^2}{\beta}\,\hat{v}\right]\Bigg|_{A_i} = \sum_{i=1}^{\#streams}\left[-pA\hat{n}\right]_{A_i} + \underline{R} + M_{CV}\underline{g} \qquad \begin{cases}\beta_{laminar} = 0.75\\ \beta_{turbulent}\approx 1\end{cases}$$

Navier-Stokes equation (microscopic momentum balance, incompressible, Newtonian fluids)

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}$$

Continuity equation (microscopic mass balance, incompressible fluids)

$$\nabla\cdot\underline{v} = 0$$

Total stress tensor

$$\underline{\underline{\tilde{\Pi}}} = -p\underline{\underline{I}} + \underline{\underline{\tilde{\tau}}}$$

$$\begin{pmatrix}\tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13}\\ \tilde{\Pi}_{21} & \tilde{\Pi}_{22} & \tilde{\Pi}_{23}\\ \tilde{\Pi}_{31} & \tilde{\Pi}_{32} & \tilde{\Pi}_{33}\end{pmatrix}_{123} = \begin{pmatrix}\tilde{\tau}_{11}-p & \tilde{\tau}_{12} & \tilde{\tau}_{13}\\ \tilde{\tau}_{21} & \tilde{\tau}_{22}-p & \tilde{\tau}_{23}\\ \tilde{\tau}_{31} & \tilde{\tau}_{32} & \tilde{\tau}_{33}-p\end{pmatrix}_{123}$$

Dynamic pressure $\mathcal{P} \equiv p + \rho g h$

Newtonian constitutive equation

$$\underline{\underline{\tilde{\tau}}} = \mu\left(\nabla\underline{v} + (\nabla\underline{v})^T\right)$$

$$= \mu\begin{pmatrix} 2\frac{\partial v_1}{\partial x_1} & \frac{\partial v_2}{\partial x_1}+\frac{\partial v_1}{\partial x_2} & \frac{\partial v_3}{\partial x_1}+\frac{\partial v_1}{\partial x_3}\\ \frac{\partial v_2}{\partial x_1}+\frac{\partial v_1}{\partial x_2} & 2\frac{\partial v_2}{\partial x_2} & \frac{\partial v_2}{\partial x_3}+\frac{\partial v_3}{\partial x_2}\\ \frac{\partial v_3}{\partial x_1}+\frac{\partial v_1}{\partial x_3} & \frac{\partial v_2}{\partial x_3}+\frac{\partial v_3}{\partial x_2} & 2\frac{\partial v_3}{\partial x_3}\end{pmatrix}_{123}$$

Total molecular fluid force on a finite surface $\mathcal{S}$

$$\underline{\mathcal{F}} = \iint_{\mathcal{S}}\left[\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right]_{\text{at surface}} dS$$

Stationary fluid $\left[\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right] = -p\hat{n}$

Moving fluid $\left[\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right] = -p\hat{n} + \hat{n}\cdot\underline{\underline{\tilde{\tau}}}$

Total fluid torque on a finite surface $\mathcal{S}$

$$\underline{\mathcal{T}} = \iint_{\mathcal{S}}\left[\underline{R}\times\left(\hat{n}\cdot\underline{\underline{\tilde{\Pi}}}\right)\right]_{\text{at surface}} dS$$

Total flow rate out through a finite surface $\mathcal{S}$

$$Q = \dot{V} = \iint_{\mathcal{S}}\left[\hat{n}\cdot\underline{v}\right]_{\text{at surface}} dS$$

Average velocity across a finite surface $\mathcal{S}$

$$\langle v\rangle = \frac{Q}{\mathcal{S}}$$

Coordinate system	surface differential dS
Cartesian (top, $\hat{n} = \hat{e}_z$)	$dS = dxdy$
Cartesian (side a, $\hat{n} = \hat{e}_y$)	$dS = dxdz$
Cartesian (side b, $\hat{n} = \hat{e}_x$)	$dS = dydz$
cylindrical (top, $\hat{n} = \hat{e}_z$)	$dS = rdrd\theta$
cylindrical (side, $\hat{n} = \hat{e}_r$)	$dS = Rd\theta dz$
spherical, ($\hat{n} = \hat{e}_r$)	$dS = R^2 \sin\theta d\theta d\phi$

Coordinate system	volume differential dV
Cartesian	$dV = dxdydz$
cylindrical	$dV = rdrd\theta dz$
spherical	$dV = r^2 \sin\theta dr d\theta d\phi$

Coordinate system	coordinates	basis vectors
spherical	$x = r\sin\theta\cos\phi$	$\hat{e}_r = (\sin\theta\cos\phi\hat{e}_x) + (\sin\theta\sin\phi\hat{e}_y) + \cos\theta\hat{e}_z$
	$y = r\sin\theta\sin\phi$	$\hat{e}_\theta = (\cos\theta\cos\phi)\hat{e}_x + (\cos\theta\sin\phi)\hat{e}_y + (-\sin\theta)\hat{e}_z$
	$z = r\cos\theta$	$\hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\hat{e}_y$
cylindrical	$x = r\cos\theta$	$\hat{e}_r = \cos\theta\hat{e}_x + \sin\theta\hat{e}_y$
	$y = r\sin\theta$	$\hat{e}_\theta = (-\sin\theta)\hat{e}_x + \cos\theta\hat{e}_y$
	$z = z$	$\hat{e}_z = \hat{e}_z$

$$\text{Divergence Theorem} \quad \iint_{\mathcal{S}} \hat{n}\cdot\underline{F}\; dS = \iiint_{\mathcal{V}} \nabla\cdot\underline{F}\; dV$$

$$\text{Stokes Theorem} \quad \oint_{\mathcal{C}} \hat{t}\cdot\underline{F}\; dl = \iint_{\mathcal{S}} \hat{n}\cdot(\nabla\times\underline{F})\; dS$$

Vector identities:

$$\begin{aligned}
\nabla\cdot\nabla\times\underline{F} &= 0 \quad \text{(Divergence of curl = 0)} \\
\nabla\times\nabla f &= 0 \quad \text{(Curl of gradient = 0)} \\
\nabla(fg) &= f\nabla g + g\nabla f \\
\underline{F}\cdot\nabla\underline{F} &= \frac{1}{2}\nabla\left(\underline{F}^2\right) - \underline{F}\times(\nabla\times\underline{F}) \\
\nabla\cdot(f\underline{F}) &= f\nabla\cdot\underline{F} + \underline{F}\cdot\nabla f \\
\nabla\times\nabla\times\underline{F} &= \nabla(\nabla\cdot\underline{F}) - \nabla^2\underline{F} \\
\nabla\cdot(\underline{F}\times\underline{G}) &= \underline{G}\cdot(\nabla\times\underline{F}) - \underline{F}\cdot(\nabla\times\underline{G})
\end{aligned}$$

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FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb_m = 35.27392 oz 1 lb_m = 16 oz = 5 x 10^{-4} ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10^6 microns (μm) = 10^{10} angstroms (Å) = 39.37 in = 3.2808 ft = 1.0936 yd = 0.0006214 mile 1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m^3 = 1000 liters = 10^6 cm^3 = 10^6 ml = 35.3145 ft^3 = 220.83 imperial gallons = 264.17 gal = 1056.68 qt 1 ft^3 = 1728 in^3 = 7.4805 gal = 0.028317 m^3 = 28.317 liters = 28 317 cm^3
Force	1 N = 1 kg·m/s^2 = 10^5 dynes = 10^5 g·cm/s^2 = 0.22481 lb_f 1 lb_f = 32.174 lb_m·ft/s^2 = 4.4482 N = 4.4482 x 10^5 dynes
Pressure	1 atm = 1.01325 x 10^5 N/m^2 (Pa) = 101.325 kPa = 1.01325 bars = 1.01325 x 10^6 dynes/cm^2 = 760 mm Hg at 0˚ C (torr) = 10.333 m H_2O at 4˚ C = 14.696 lb_f/in^2 (psi) = 33.9 ft H_2O at 4˚C 100 kPa = 1 bar
Energy	1 J = 1 N·m = 10^7 ergs = 10^7 dyne·cm = 2.778 x 10^{-7} kW·h = 0.23901 cal = 0.7376 ft·lb_f = 9.47817 x 10^{-4} Btu
Power	1 W = 1 J/s = 0.23885 cal/s = 0.7376 ft·lb_f/s = 9.47817 x 10^{-4} Btu/s = 3.4121 Btu/h = 1.341 x 10^{-3} hp (horsepower)
Viscosity	1 Pa·s = 1 N·s/m^2 = 1 kg/m·s = 10 poise = 10 dynes·s/cm^2 = 10 g/cm·s = 10^3 cp (centipoise) = 0.67197 lb_m/ft·s = 2419.088 lb_m/ft·h
Density	1 kg/m^3 = 10^{-3} g/cm^3 = 0.06243 lb_m/ft^3 10^3 kg/m^3 = 1 g/cm^3 = 62.428 lb_m/ft^3
Volumetric Flow	1 m^3/s= 35.3145 ft^3/s=15,850.2 gal/min (gpm) 1 gpm = 6.30907 x 10^{-5} m^3/s=2.22802 x 10^{-3} ft^3/s=3.7854 liter/min 1 liter/min=0.26417 gpm

Ver. 21-Sep-2011

Temperature	$T(^oC) = \frac{5}{9}\left[T(^oF) - 32\right]$
	$T(^oF) = \frac{9}{5}T(^oC) + 32 = 1.8T(^oC) + 32$
Absolute Temperature	T(K) = T(˚C) + 273.15
	T(˚R) = T(˚F) + 459.67
Temperature Interval (ΔT)	1 C˚ = 1 K = 1.8 F˚ = 1.8 R˚
	1F˚ = 1 R˚ = (5/9) C˚ = (5/9) K

USEFUL QUANTITIES

SG = ρ(20˚C)/ρ_{water} (4˚C)

ρ_{water}(4˚C) = 1000 kg/m^3 = 62.43 lb$_m$/ft^3 = 1.000 g/cm^3

ρ_{water}(25˚C) = 997.08 kg/m^3 = 62.25 lb$_m$/ft^3 = 0.99709 g/cm^3

g = 9.8066 m/s^2 = 980.66 cm/s^2 = 32.174 ft/s^2

μ_{water} (25˚C) = 8.937 x 10^{-4} Pa·s = 8.937 x 10^{-4} kg/m·s
= 0.8937 cp =0.8937 x 10^{-2} g/cm·s = 6.005 x 10^{-4} lb$_m$/ft·s

Composition of air:

N_2	78.03%
O_2	20.99%
Ar	0.94%
CO_2	0.03%
H_2, He, Ne, Kr, Xe	0.01%
	100.00%

M_{air} = 29 g/mol = 29 kg/kmol = 29 lb$_m$/lbmole

$\hat{C}_{p,water}$ (25˚C) = 4.182 kJ/kg K = 0.9989 cal/g˚C = 0.9997 Btu/lb$_m$˚F

R = 8.314 m^3·Pa/mol·K = 0.08314 liter·bar/mol·K = 0.08206 liter·atm/mol·K
= 62.36 liter·mm Hg/mol·K = 0.7302 ft^3·atm/lbmole·˚R
= 10.73 ft^3·psia/lbmole·˚R
= 8.314 J/mol·K
= 1.987 cal/mol·K = 1.987 Btu/lbmole·˚R