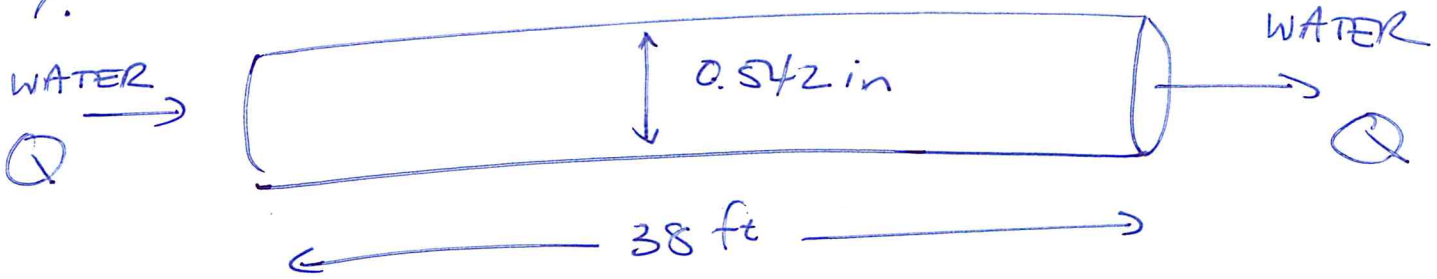
CM3110
2019
EXAM 1
SOLUTION

1.



$Q = 4.1$ gpm

$\langle v \rangle = ?$

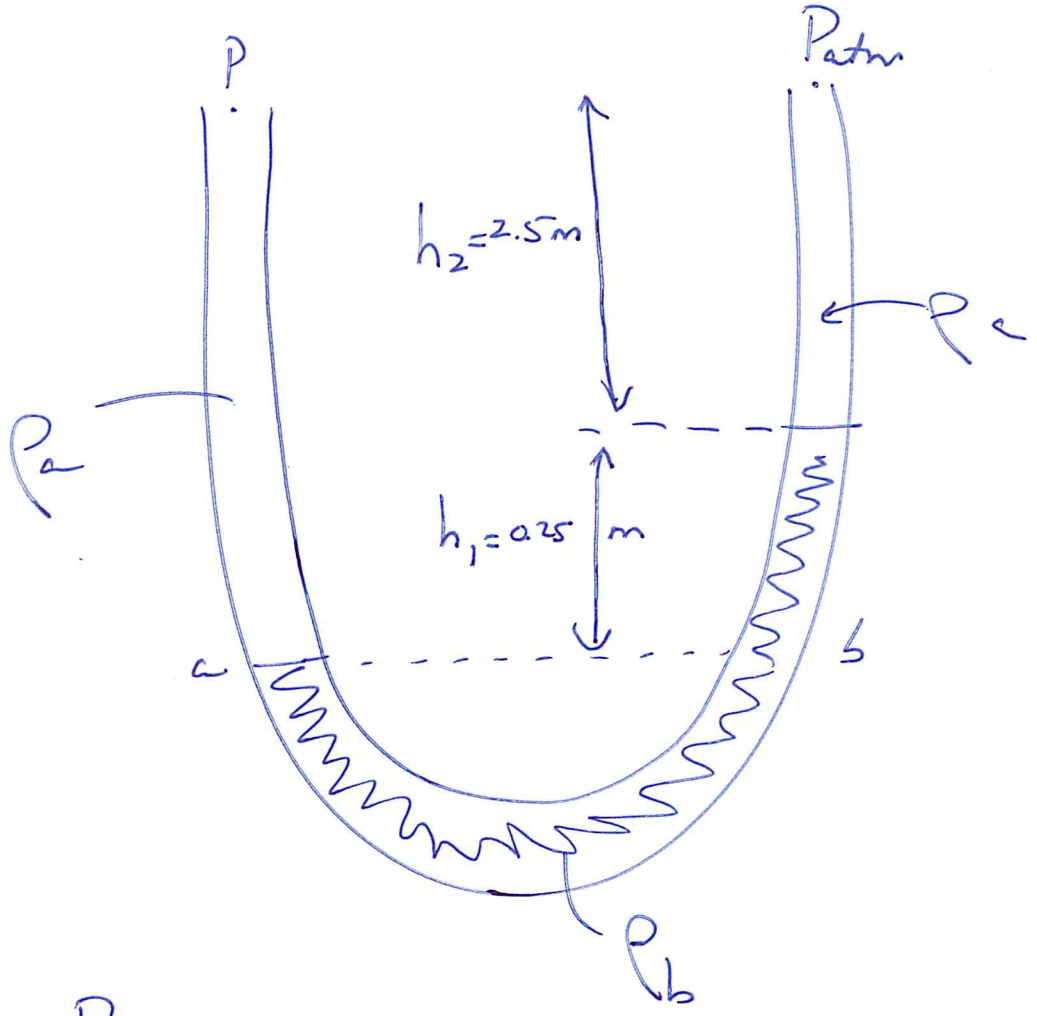
$$\langle v \rangle = \frac{Q}{\pi R^2} = \frac{4Q}{D^2 \pi}$$

$$= \left(\frac{4}{\pi}\right) \frac{(4.1 \text{ gpm})}{(0.542 \text{ in})^2} \left(\frac{2.228009 \times 10^{-3} \text{ ft}^3/\text{s}}{1 \text{ gpm}}\right) \left(\frac{12 \text{ in}}{\text{ft}}\right)^2$$

$$= 5.7013 \frac{\text{ft}}{\text{s}}$$

$$= \boxed{5.7 \frac{\text{ft}}{\text{s}}}$$

2.



RULE 1

$$P_a = P_b$$

RULE 2

$$\begin{cases} P_a = P + (h_1 + h_2)\rho_a g \\ P_b = P_{atm} + \rho_a g h_2 + \rho_b g h_1 \end{cases}$$

equate:

$$P + h_1 \rho_a g + h_2 \rho_a g = P_{atm} + \rho_a g h_2 + \rho_b g h_1$$

$$P = P_{atm} + \rho_b g h_1 - \rho_a g h_1$$

$$\boxed{P = P_{atm} + g h_1 (\rho_b - \rho_a)}$$

$$P = (1.00\ \text{atm}) + \left(9.8066\ \frac{\text{m}}{\text{s}^2}\right)(0.25\ \text{m})$$

$$\frac{\text{N}\,\text{s}^2}{\text{kg m}}\left(\frac{10^2\,\text{cm}}{\text{m}}\right)^3\frac{\text{kg}}{10^3\,\text{g}}\left(13.60 - 1.000\right)\frac{\text{g}}{\text{cm}^3}$$

$$\times \frac{\text{atm}}{1.01325 \times 10^5}\frac{\text{m}^2}{\text{N}}$$

$$P = (1.00\ \text{atm}) + (0.304868\ \text{atm})$$

$$= \boxed{1.30\ \text{atm}}$$

3. a) $\underline{v} \cdot \underline{a} = (2 \quad 1 \quad -1)_{xyz} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}_{xyz}$

$= (2)(1) + (1)(2) + (-1)(2)$

$= \boxed{2}$

b) $|\underline{v}| = \sqrt{\underline{v} \cdot \underline{v}}$

$\underline{v} \cdot \underline{v} = (2 \quad 1 \quad -1)_{xyz} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}_{xyz}$

$= (4) + 1 + 1 = 6$

$\boxed{|\underline{v}| = \sqrt{6}}$

c) $\underline{\underline{M}} \cdot \underline{a}$

$$= \begin{pmatrix} 0 & 2 & 2 \\ -1 & 1 & 3 \\ 2 & 0 & 5 \end{pmatrix}_{xyz} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}_{xyz}$$

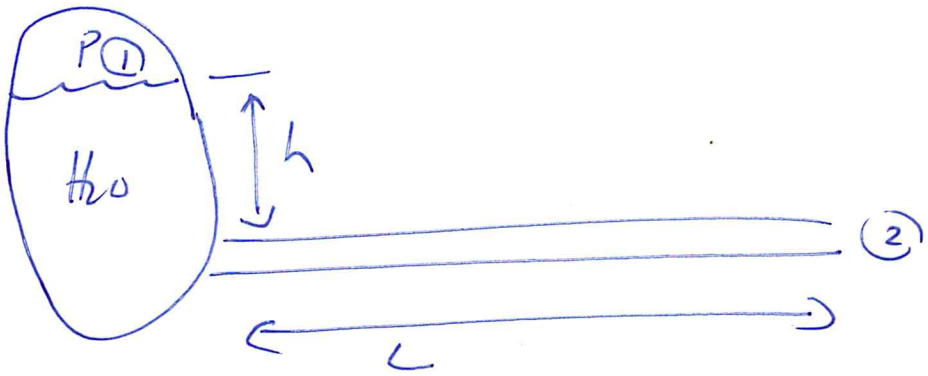
$$= \begin{pmatrix} (0)(1) + (2)(2) + (2)(2) \\ (-1)(1) + (1)(2) + (3)(2) \\ (2)(1) + (0)(2) + (5)(2) \end{pmatrix}_{xyz} \quad (3\times 1 \text{ vector})$$

$$= \boxed{\begin{pmatrix} 8 \\ 7 \\ 12 \end{pmatrix}_{xyz}}$$

d) $\frac{d}{dr}\left(4r^2 - \frac{2}{r}\right) = 8r - 2\left(-\frac{1}{r^2}\right)$

$$= \boxed{8r + \frac{2}{r^2}}$$

4.)



MEB

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2\alpha} + g(z_2 - z_1) + F = W_{s,on} - W_{s,off}$$

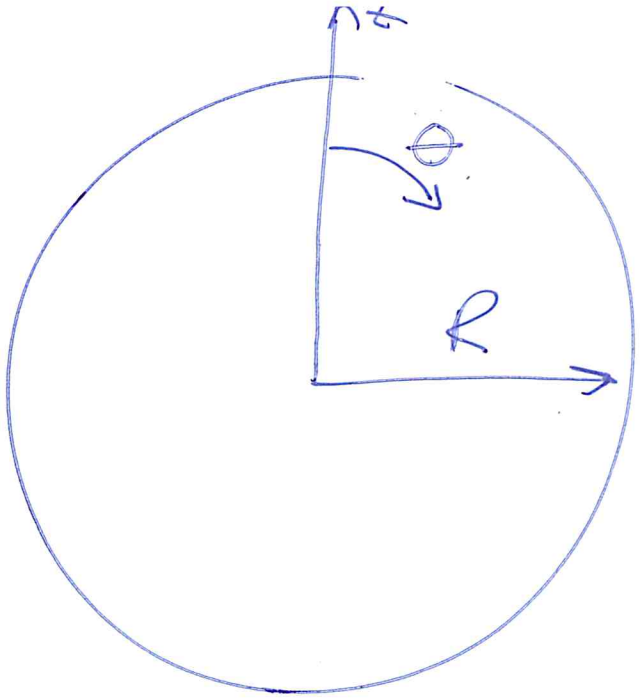
$V_1 \approx 0$; $\alpha = 1$ (turbulent); $z_2 - z_1 = -h$; $F = F_{2\text{-}1}$ (friction); $W_{s,on} = 0$ (no shaft work); $W_{s,off} = W_s$

Solve for V_2.

$$\frac{V_2^2}{2} = \frac{P_1 - P_2}{\rho} + gh$$

$$V_2 = \sqrt{\left(\frac{P - P_{atm}}{\rho} + gh\right)(2)}$$

5.



$0 < \theta < \pi$

$0 < \phi < 2\pi$ (around the equator)

Surface Area?

$$SA = \int_0^{2\pi} \int_0^{\pi} R^2 \sin\theta \, d\theta \, d\phi$$

$$= R^2 \left(\underbrace{\int_0^{2\pi} d\phi}_{2\pi} \right) \left(\int_0^{\pi} \sin\theta \, d\theta \right)$$

$$-\cos\theta \Big|_0^{\pi}$$

$$\underbrace{-(-1-1)}_{-2}$$

$$\underbrace{\qquad\qquad}_{2}$$

$$= \boxed{4\pi R^2} \;//$$