How could we use this solution?
Example: Will my pipes freeze?

The temperature has been 35°F for a while now, sufficient to chill the ground to this temperature for many tens of feet below the surface. Suddenly the temperature drops to -20°F. How long will it take for freezing temperatures (32°F) to reach my pipes, which are 8 *ft* under ground? Use the following physical properties:

$$h = 2.0 \frac{BTU}{h\ ft^2\ {}^oF}$$

$$\alpha_{soil} = 0.018 \frac{ft^2}{h}$$

$$k_{soil} = 0.5 \frac{BTU}{h\ ft\ {}^oF}$$

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①

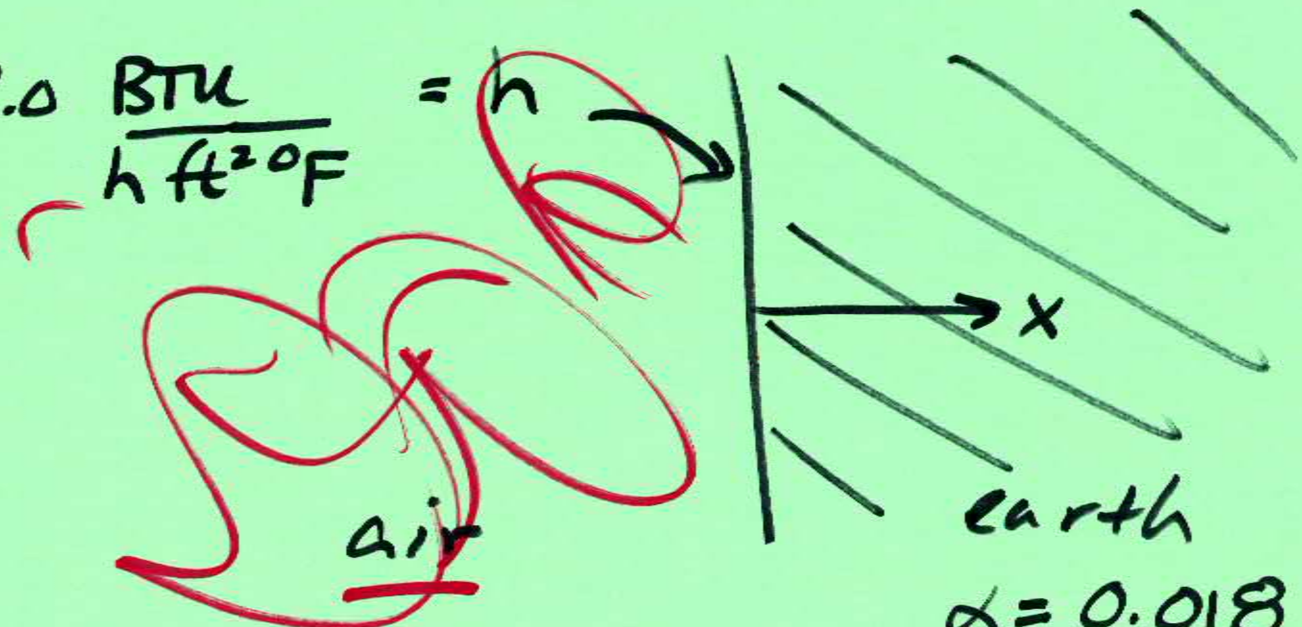
When Will my Pipes Freeze?

(see Heat lecture 3)

$T_0 = 35°F$

$T_1 = -20°F$

$2.0 \frac{BTU}{h\ ft^2\ °F} = h$



$\alpha = 0.018\ ft^2/h$

$k = 0.5 \frac{BTU}{h\ ft\ °F}$

$T(x,t)$:

32°F (freezing) → T; T_0 = 35°F; T_1 = -20°F

$$\frac{T - T_0}{T_1 - T_0} = \operatorname{erfc}(\zeta) - e^{\beta(2\zeta+\beta)}\operatorname{erfc}(\zeta+\beta)$$

$$\beta = \frac{h\sqrt{\alpha t}}{k}$$

$$\zeta = \frac{x}{2(\sqrt{\alpha t})}$$

x ← 8'

t ← what we want

Solve for t

<u>OR</u> use Figure 5.3-3 p 364 ③
Geankoplis
4th Ed
(see lecture 3)

$$\frac{T - T_0}{T_1 - T_0} = \frac{32 - 35}{(-20) - 35} = \frac{-3}{-55}$$

$$= 0.055$$

Soln: guess β
calc t from β
calc t from ζ
compare

$$\beta = \frac{h\sqrt{\alpha t}}{k}$$

$$= \frac{\left(2 \frac{BTU}{h\,ft^2\,°F}\right)\sqrt{\left(0.018 \frac{ft^2}{h}\; t(h)\right)}}{0.5 \frac{BTU}{h\,ft\,°F}}$$

$$\boxed{\beta = 0.536656\sqrt{t(h)}}$$

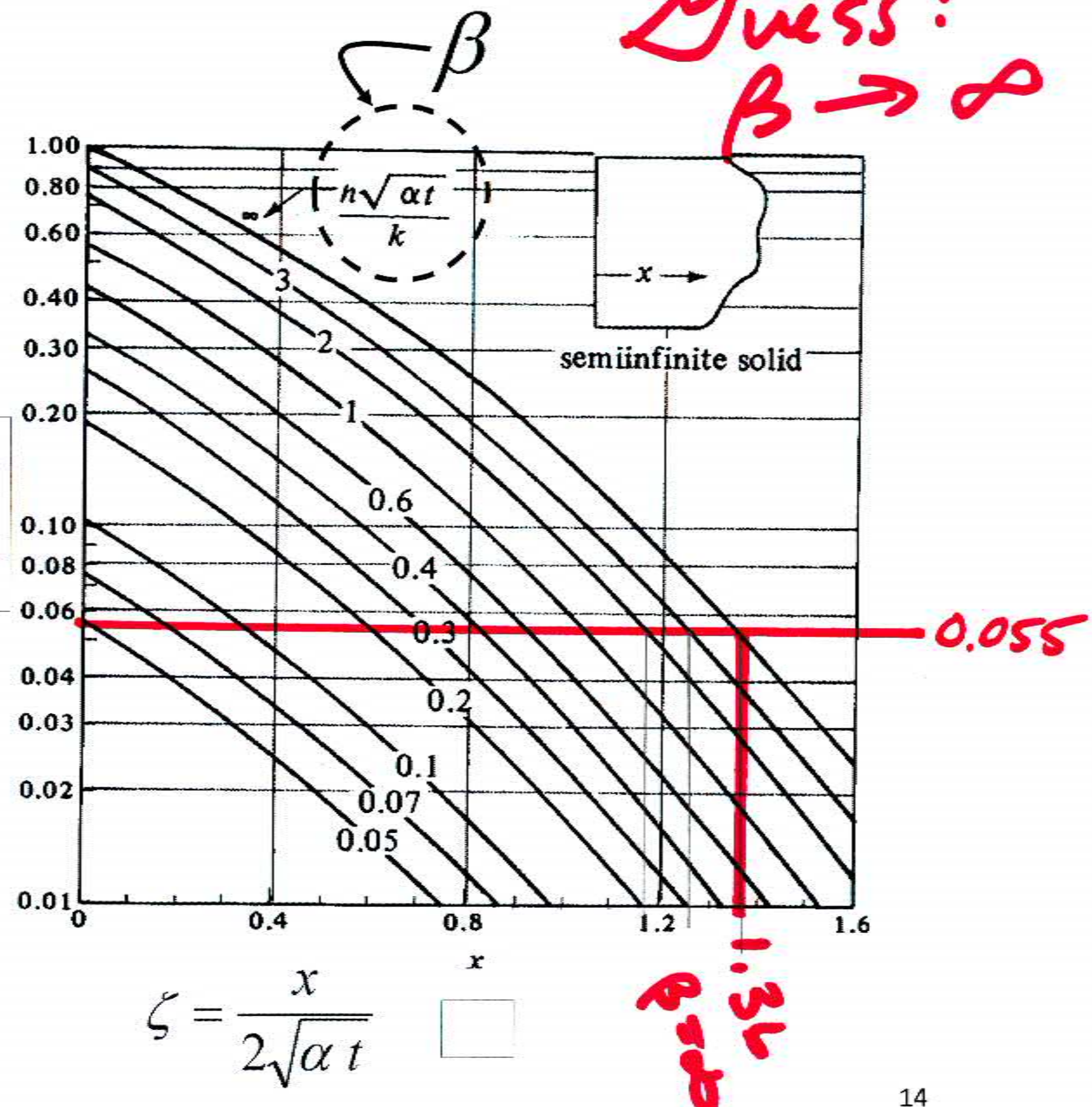
$$\zeta = \frac{x}{2\sqrt{\alpha t}} = \frac{8\,ft}{2\sqrt{0.018 \frac{ft^2}{h}\, t(h)}}$$

$$\boxed{\zeta = \frac{29.8142}{\sqrt{t(h)}}}$$

Unsteady State Heat Conduction in a Semi-Infinite Slab

Guess: $\beta \to \infty$

$$\left(\frac{0-35}{-20-35}\right) = \left(\frac{T-T_o}{T_1-T_o}\right)$$



$$\zeta = \frac{x}{2\sqrt{\alpha t}}$$

Geankoplis 4th ed., Figure 5.3-3, page 364

for $\beta > 5 \to \infty$ there is one line.

let's guess $\beta \to$ large

For $\frac{T-T_0}{T_1-T_0} = 0.055$, β large $\Rightarrow \zeta = 1.36$

$$1.36 = \frac{29.8142}{\sqrt{t}}$$

$$t = 480\ h$$

$$= \boxed{20\ \text{DAYS}}$$

Was β large?

$$\beta = 0.5366556\sqrt{480} = 12\ \checkmark$$

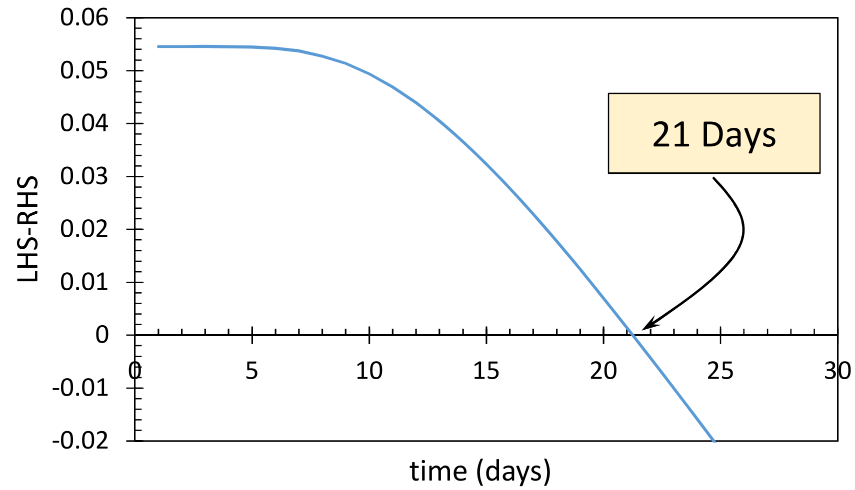
time (days)	time (hours)	Beta (unitless)	Zeta (unitless)	LHS (unitless)	RHS (unitless)	LHS-RHS (unitless)
1	24	2.629068276	6.085806	0.054545	2.23505E-18	0.054545
2	48	3.718064012	4.303315	0.054545	5.26109E-10	0.054545
3	72	4.553679831	3.513642	0.054545	3.71023E-07	0.054545
4	96	5.258136552	3.042903	0.054545	1.04E-05	0.054535
5	120	5.878775383	2.721655	0.054545	7.90616E-05	0.054466
6	144	6.439875775	2.48452	0.054545	0.00031099	0.054234
7	168	6.955860838	2.300219	0.054545	0.000836691	0.053709
8	192	7.436128025	2.151657	0.054545	0.001771924	0.052774
9	216	7.887204828	2.028602	0.054545	0.003195275	0.05135
10	240	8.313843876	1.924501	0.054545	0.005144603	0.049401
11	264	8.71963302	1.83494	0.054545	0.007623557	0.046922
12	288	9.107359661	1.756821	0.054545	0.010611099	0.043934
13	312	9.479240476	1.687899	0.054545	0.014070448	0.040475
14	336	9.837072735	1.6265	0.054545	0.01795615	0.036589
15	360	10.18233765	1.571348	0.054545	0.022219167	0.032326
16	384	10.5162731	1.521452	0.054545	0.026810283	0.027735
17	408	10.8399262	1.476025	0.054545	0.031682233	0.022863
18	432	11.15419204	1.434438	0.054545	0.03679093	0.017755
19	456	11.45984293	1.39618	0.054545	0.042096082	0.012449
20	480	11.75755077	1.360828	0.054545	0.047561394	0.006984
21	504	12.04790438	1.328032	0.054545	0.053154527	0.001391
21.2	508.8	12.1051394	1.321753	0.054545	0.054285914	0.00026
21.3	511.2	12.13365567	1.318646	0.054545	0.054853026	-0.00031
22	528	12.33142328	1.297498	0.054545	0.058846908	-0.0043
23	552	12.60856852	1.268978	0.054545	0.064613457	-0.01007
24	576	12.87975155	1.24226	0.054545	0.070432274	-0.01589
25	600	13.14534138	1.217161	0.054545	0.07628433	-0.02174
26	624	13.40567044	1.193525	0.054545	0.082153147	-0.02761
27	648	13.66103949	1.171214	0.054545	0.088024518	-0.03348

T0=	35	F
T1=	-20	F
T=	32	F

h=	2	BTU/h ft2 F
alpha=	0.018	ft2/h
k=	0.5	BTU/h ft F
x=	8	ft

How long until my pipes freeze?

$$\frac{(T-T_0)}{(T_1-T_0)} = \operatorname{erfc}(\zeta) - e^{\beta(2\zeta+\beta)}\operatorname{erfc}\big((\zeta+\beta)\big)$$



time (days)	time (hours)	Beta (unitless)	Zeta (unitless)	LHS-RHS (unitless)
21.24579	509.8989	12.11820494	1.320328	-1.8E-08

time (days)	time (hours)	Beta (unitless)	Zeta (unitless)	LHS (unitless)	RHS (unitless)	LHS-RHS (unitless)
1	24	2.629068276	6.085806	0.054545	2.23505E-18	0.054545
2	48	3.718064012	4.303315	0.054545	5.26109E-10	0.054545
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18	432	11.15419204	1.434438	0.054545	0.03679093	0.017755
19	456	11.45984293	1.39618	0.054545	0.042096082	0.012449
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T=	32	F

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alpha=	0.018	ft2/h
k=	0.5	BTU/h ft F
x=	8	ft

Faith Morrison:
2/1/2019 This is the Solver solution. If your first guess is on that low time plateau (see plot) it will not find the solution. If your first guess is on the slope, however, it rapidly finds the right answer (510 hours or 21 days). It's actually just a good idea to do the plot.

How long until my pipes freeze?

$$\frac{(T-T_0)}{(T_1-T_0)} = \text{erfc}(\zeta) - e^{\beta(2\zeta+\beta)}\,\text{erfc}\big((\zeta+\beta)\big)$$