1.	/20
2.	/20
3.	/20
4.	/20
5.	/20

Exam 3

CM3110

Thursday 12 November 2020

MORRISON

Name: SOLUTION

Rules:

- Closed book, closed notes.
- Two-page 8.5" by 11" study sheet allowed, double sided; you may use a calculator; you may not search the internet or receive help from anyone.
- Please text clarification questions to Dr. Morrison 906-487-9703. I will respond if I am able.
- All work submitted for the exam must be your own.
- Do not discuss the contents of the exam with anyone before midnight Thursday 12 November 2020.
- ***Please copy the following Honors Pledge onto the first page of your exam submission and sign and date your agreement to it.***

Honor's Pledge:

On my honor, I agree to abide by the rules stated on the exam sheet.

Signature ____________________________

Date ____________________

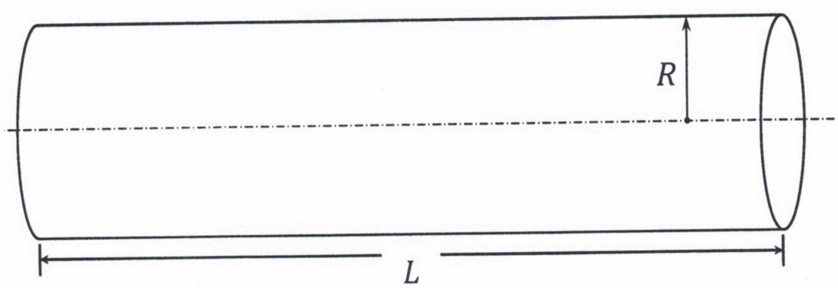
Exam Instructions:

i. You may work on the exam for up to two hours (120 minutes).
ii. Please be neat. Only neat answers will be granted partial credit. Please use a dark pencil or pen so that your work is readable once scanned.
iii. Significant figures always count.
iv. Please box your final answers.
v. Submit your work as a single PDF file; put your name on every page. (Genius Scan is a free app that can create a PDF from photos taken by your phone)
vi. Submit your exam study sheet as a separate PDF file; put your name on the first page (at a minimum)

1. (20 points)
   a. Our exam 3 formula sheet gives three data correlations for Fanning friction factor f versus Reynolds number Re. What are these for? Please limit your answer to at most 3 sentences.
   b. What is the key difference between the flow inside a momentum boundary layer and the flow outside the boundary layer? Please limit your answer to at most 3 sentences.

2. (20 points) A reaction takes place in a fluidized bed reactor. The catalyst is a powder of density $1.2 \times 10^3 kg/m^3$ with mean particle diameter of 0.067 mm. The powder is fluidized by the upward flow of air, which has a viscosity of $1.0 \times 10^{-4}\ Pa\ s$. What is a reasonable estimate of the minimum fluidization velocity? This is also known as the velocity of incipient fluidization. Note that if a fluidized bed's void fraction ε is not known, Denn recommends using the approximation $\left(\frac{\varepsilon^3}{1-\varepsilon}\right) \approx 0.091$.

3. (20 points) Steady, turbulent flow of an incompressible, Newtonian liquid (density ρ and viscosity μ) takes place in a horizontal tube of radius R and length L. The volumetric flow rate is Q. The upstream pressure is P_0 and the downstream pressure is P_L. Using a macroscopic momentum balance and indicating your assumptions, calculate the fluid drag on the inside of the tube.



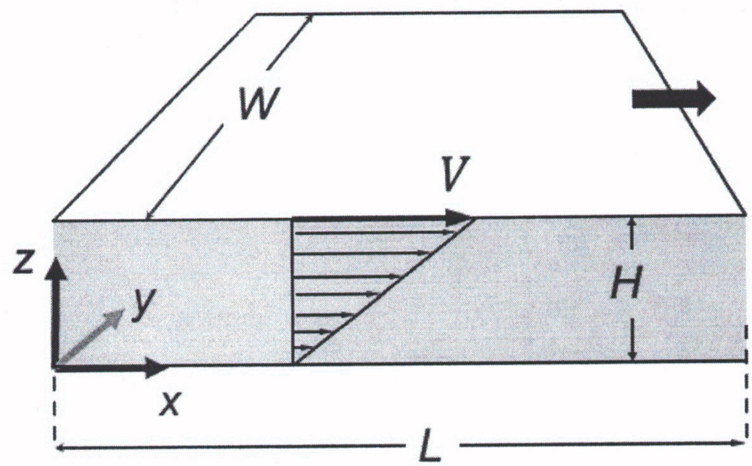
4. (20 points) What is the drag on a 4.0 mm diameter solid sphere if air (density = $1.3\ kg/m^3$, viscosity = $1.0 \times 10^{-4} Pa\ s$) rushes past it in a wind tunnel at 1.9 m/s? Please give your answer in Newtons.

5. (20 points) An incompressible, Newtonian fluid flows between two long, wide parallel plates (see figure below). The plates are separated by a gap H. The top plate moves in the x-direction at a constant velocity V. Using the Navier-Stokes equation, we have determined that the velocity field is given by:

$$\underline{v} = \begin{pmatrix} \frac{V}{H}z \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$p = P_0$$

The pressure is uniform (constant) and equal to P_0 everywhere. What is the vector fluid force on the top plate needed to drive the upper plate? Show how you arrived at your answer beginning with the velocity and pressure fields given above.



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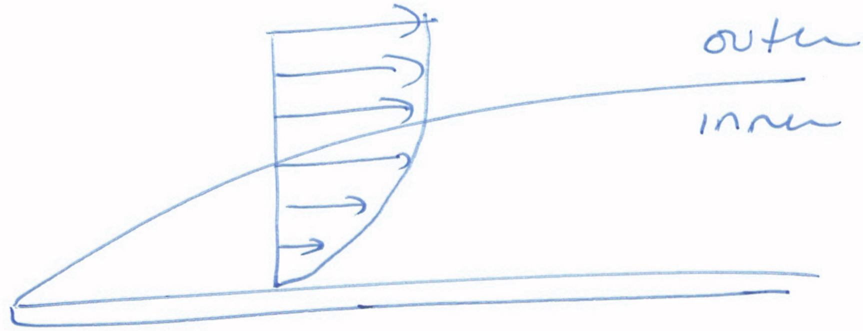
1.) a) $f(Re)$ represent experimental results of dimensionless drag in a pipe versus dimensionless flow rate. They are used to determine the $\Delta P(Q)$ relationship for pipes.

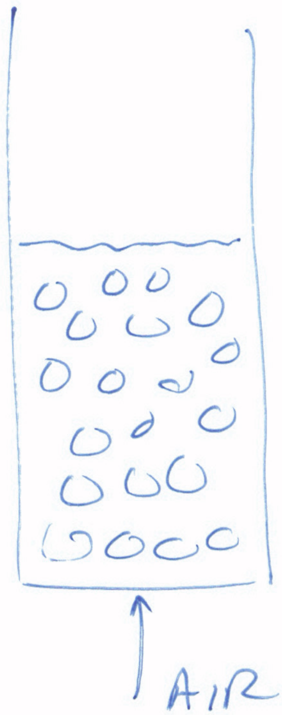
b)



Friction (viscosity) is not important outside of the boundary layer.

2)

$$\frac{\varepsilon^3}{1-\varepsilon} \approx 0.091$$



$$\rho_{powder} = 1.2 \times 10^3 \, kg/m^3$$

$$D_p = 0.067 \times 10^{-3} \, m$$

$$\mu = 1.0 \times 10^{-4} \, \overbrace{Pa\,s}^{kg/m\,s}$$

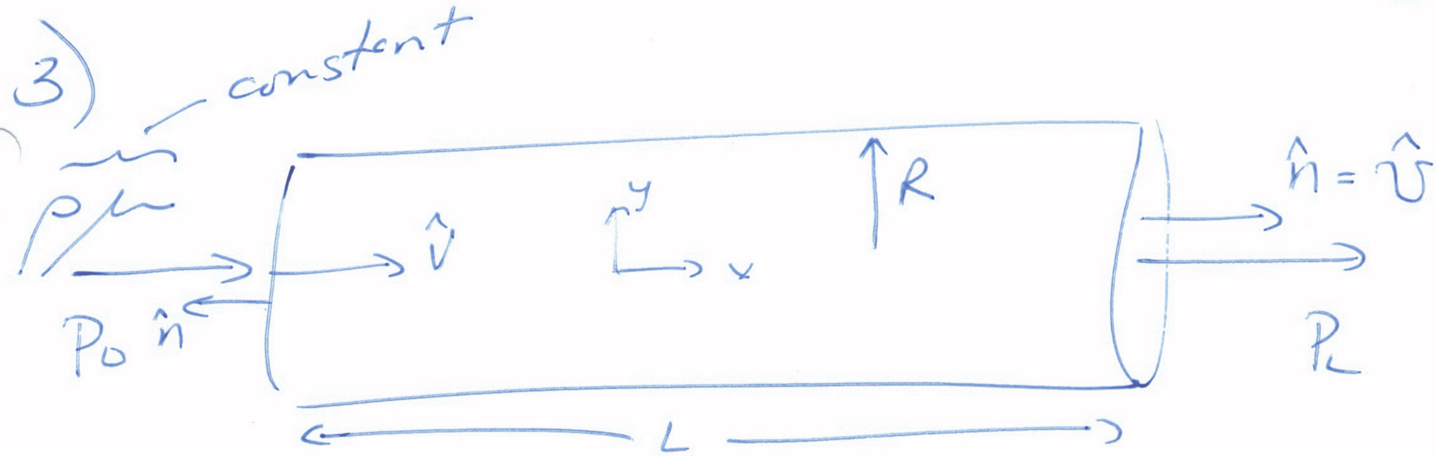
$$V_0 = \frac{(\rho_p - \cancel{\rho_w}_0)\, g\, D_p^2}{150 \mu} \left(\frac{\varepsilon^3}{1-\varepsilon}\right)$$

$$= \frac{\left(1.2 \times 10^3 \frac{\cancel{kg}}{\cancel{m^3}}\right)\left(9.8066 \frac{\cancel{m}}{s^2}\right)\left(0.067 \times 10^{-3} \cancel{m}\right)^2 (0.091)}{(150)\left(1.0 \times 10^{-4} \frac{\cancel{kg}}{m\,s}\right)}$$

$$= 3.20478 \times 10^{-4} \, m/s$$

$$= \boxed{3.2 \times 10^{-4} \, m/s}$$

3)



$\beta = 1$ turbulent

$\frac{dP}{dt} = 0$ steady state

neglect gravity (horizontal tube)

inlet

$\hat{n} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$

$\hat{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$

$\cos\theta = \hat{n} \cdot \hat{v} = -1$

outlet

$\hat{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$

$\hat{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$

$\cos\theta = \hat{n} \cdot \hat{v} = 1$

MACRO MOMENTUM BAL:

$$\cancel{\rho \pi R^2 (-1) \langle v \rangle^2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}} + \cancel{\rho \pi R^2 (1) \langle v \rangle^2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}}$$

$$= -P_0 \pi R^2 \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz} + (-P_L) \pi R^2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

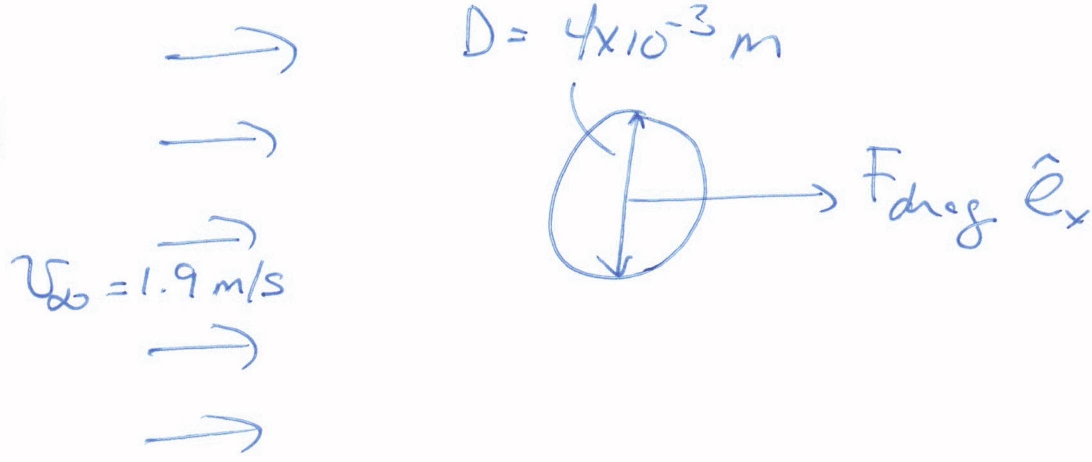
$$+ \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}_{xyz} + M_{cv} \begin{pmatrix} 0 \\ -g \\ 0 \end{pmatrix}_{xyz}$$

x-component:

$$\boxed{-R_x = F_{\substack{\text{drag on} \\ \text{walls}}} = (P_0 - P_L) \pi R^2}$$

The drag is the force in the direction of flow.

4)



AIR

$\rho = 1.3\ kg/m^3$

$\mu = 1.0\times10^{-4}\ \frac{kg}{ms}$

What is F_{drag}?

$$Re = \frac{\rho V_\infty D}{\mu} = \frac{\left(1.3\frac{\cancel{kg}}{\cancel{m^3}}\right)\left(1.9\frac{\cancel{m}}{\cancel{s}}\right)\left(4\times10^{-3}\cancel{m}\right)}{1.0\times10^{-4}\frac{\cancel{kg}}{\cancel{ms}}}$$

$$Re = 98.8$$

Data correlation: $0.1 \le Re \le 10^3$

$$C_D = \frac{24}{Re}\left(1 + 0.14\,Re^{0.7}\right)$$

$$\boxed{C_D = 1.08997}$$

$$C_D = \frac{F_{drag}}{\frac{1}{2}\rho V_o^2 \left(\pi R^2\right)}$$

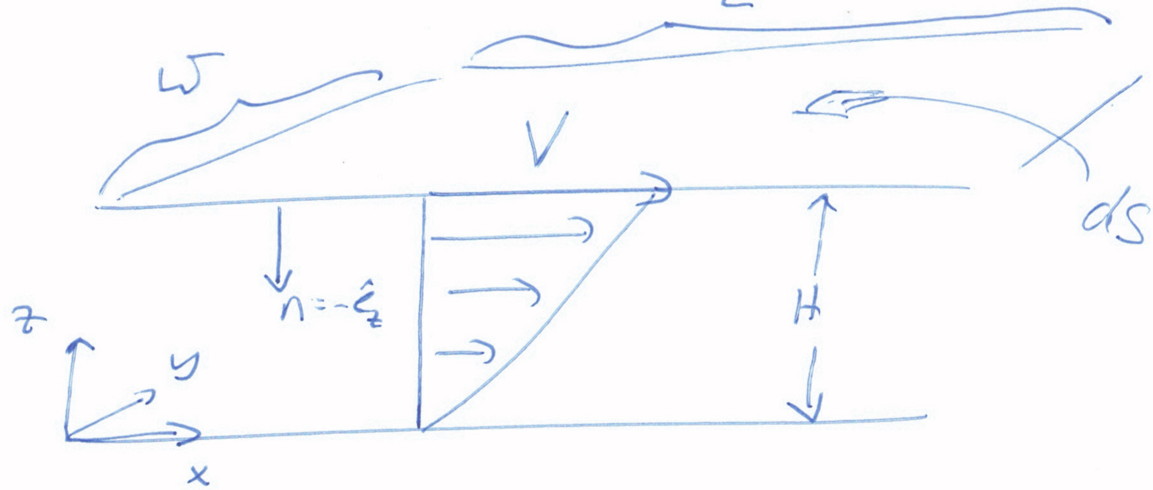
$$F_{drag} = (1.08997)\left(\tfrac{1}{2}\right)\left(1.3\ \frac{\cancel{kg}}{\cancel{m^3}}\right)\left(1.9\ \cancel{m}/\cancel{s}\right)^2 \pi\left(2\times10^{-3}\ \cancel{m}\right)^2$$

$$\times\ \frac{N\ \cancel{s^2}}{\cancel{kg}\ \cancel{m}}$$

$$\boxed{F_{drag} = 3.2\times10^{-5}\ N}$$

7

5.



$$\underline{V} = \begin{pmatrix} \frac{V}{H} z \\ 0 \\ 0 \end{pmatrix}_{xyz} \qquad \hat{n} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}_{xyz}$$

$$ds = dx\,dy$$

$$p = P_0$$

$$\underline{F} = \iint_{S'} \left(\hat{n} \cdot \underline{\underline{\tilde{\Pi}}} \right)_{\substack{at \\ surface}} dS$$

$$= \int_0^W \int_0^L \left(\hat{n} \cdot \underline{\underline{\tilde{\Pi}}} \right)_{z=H} dx\,dy$$

$$\underline{\underline{\tilde{\Pi}}} = -p\underline{\underline{I}} + \underline{\underline{\tau}}$$

8

The Newtonian Constitutive Equation in Cartesian, Cylindrical, and Spherical coordinates

Prof. Faith A. Morrison, Michigan Technological University

Cartesian Coordinates

$$\begin{pmatrix} \tilde{\tau}_{xx} & \tilde{\tau}_{xy} & \tilde{\tau}_{xz} \\ \tilde{\tau}_{yx} & \tilde{\tau}_{yy} & \tilde{\tau}_{yz} \\ \tilde{\tau}_{zx} & \tau_{zy} & \tilde{\tau}_{zz} \end{pmatrix}_{xyz} = \mu \begin{pmatrix} \cancel{2\frac{\partial v_x}{\partial x}} & \cancel{\frac{\partial v_x}{\partial y}} + \cancel{\frac{\partial v_y}{\partial x}} & \frac{\partial v_x}{\partial z} + \cancel{\frac{\partial v_z}{\partial x}} \\ \cancel{\frac{\partial v_x}{\partial y}} + \cancel{\frac{\partial v_y}{\partial x}} & \cancel{2\frac{\partial v_y}{\partial y}} & \cancel{\frac{\partial v_y}{\partial z}} + \cancel{\frac{\partial v_z}{\partial y}} \\ \frac{\partial v_x}{\partial z} + \cancel{\frac{\partial v_z}{\partial x}} & \cancel{\frac{\partial v_y}{\partial z}} + \cancel{\frac{\partial v_z}{\partial y}} & \cancel{2\frac{\partial v_z}{\partial z}} \end{pmatrix}_{xyz}$$

Cylindrical Coordinates

$$\begin{pmatrix} \tilde{\tau}_{rr} & \tilde{\tau}_{r\theta} & \tilde{\tau}_{rz} \\ \tilde{\tau}_{\theta r} & \tilde{\tau}_{\theta\theta} & \tilde{\tau}_{\theta z} \\ \tilde{\tau}_{zr} & \tau_{z\theta} & \tilde{\tau}_{zz} \end{pmatrix}_{r\theta z} = \mu \begin{pmatrix} 2\frac{\partial v_r}{\partial r} & r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} & \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \\ r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} & 2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) & \frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \\ \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} & \frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} & 2\frac{\partial v_z}{\partial z} \end{pmatrix}_{r\theta z}$$

$\frac{\partial v_x}{\partial x} = 0$

$\frac{\partial v_x}{\partial y} = 0$

$\frac{\partial v_x}{\partial z} = \frac{V}{H}$

$V_z = 0$

Spherical Coordinates

$V_y = 0$

$$\underline{v} = \begin{pmatrix} \frac{V}{H} z \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$\begin{pmatrix} \tilde{\tau}_{rr} & \tilde{\tau}_{r\theta} & \tilde{\tau}_{r\phi} \\ \tilde{\tau}_{\theta r} & \tilde{\tau}_{\theta\theta} & \tilde{\tau}_{\theta\phi} \\ \tilde{\tau}_{\phi r} & \tau_{\phi\theta} & \tilde{\tau}_{\phi\phi} \end{pmatrix}_{r\theta\phi}$$

$$= \mu \begin{pmatrix} 2\frac{\partial v_r}{\partial r} & r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) \\ r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} & 2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) & \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} \\ \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) & \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} & 2\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right) \end{pmatrix}_{r\theta\phi}$$

These expressions are general and are applicable to three-dimensional flows. For unidirectional flows they reduce to Newton's law of Viscosity. Reference: Faith A. Morrison, *An Introduction to Fluid Mechanics* (Cambridge University Press: New York, 2013)

16 April 2014

(9)

$$\tilde{\underline{\underline{\tau}}} = \begin{pmatrix} 0 & 0 & \mu\frac{V}{H} \\ 0 & 0 & 0 \\ \mu\frac{V}{H} & 0 & 0 \end{pmatrix}_{xyz}$$

$$-p\,\underline{\underline{I}} = \begin{pmatrix} -P_0 & 0 & 0 \\ 0 & -P_0 & 0 \\ 0 & 0 & -P_0 \end{pmatrix}_{xyz}$$

$$\tilde{\underline{\underline{\Pi}}} = \begin{pmatrix} -P_0 & 0 & \mu\frac{V}{H} \\ 0 & -P_0 & 0 \\ \mu\frac{V}{H} & 0 & -P_0 \end{pmatrix}_{xyz}$$

$$\left(\hat{n}\cdot\tilde{\underline{\underline{\Pi}}}\right)\Big|_{z=H} = \begin{pmatrix} 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -P_0 & 0 & \mu\frac{V}{H} \\ 0 & -P_0 & 0 \\ \mu\frac{V}{H} & 0 & -P_0 \end{pmatrix}$$

$$= \begin{pmatrix} -\mu\frac{V}{H} & 0 & P_0 \end{pmatrix}_{xyz}$$

$$\underline{F} = \int_0^W \int_0^L \underbrace{\begin{pmatrix} -\mu \frac{V}{H} \\ 0 \\ P_0 \end{pmatrix}_{xyz}}_{\text{all constant}} dx\, dy$$

$$\int_0^L dx = L$$

$$\int_0^W dy = W$$

$$\boxed{\underline{F}_{\substack{\text{on}\\\text{fluid}}} = \begin{pmatrix} -\frac{VLW}{H}\mu \\ 0 \\ LWP_0 \end{pmatrix}_{xyz} = -\underline{F}_{\substack{\text{on}\\\text{wall}}}}$$

check units: $\frac{\mu VLW}{H} \; [=] \; \frac{\cancel{kg}}{\cancel{m}\, \cancel{s}} \frac{\cancel{m}}{s} \frac{m^2}{\cancel{m}} \frac{N s^2}{\cancel{kg}\, m}$

$[=] \; N \; \checkmark$