1.	/20
2.	/20
3.	/20
4.	/20
5.	/20

Name: SOLUTION

Exam 4

CM3110 Spring
Thursday 8 April 2021

Rules:

- Closed book, closed notes.
- Two-page 85 " by 11" study sheet allowed, double sided; you may use a calculator; you may not search the internet or receive help from anyone.
- Please text clarification questions to Dr. Morrison 906-487-9 7Œ. I will respond if I am able.
- All work submitted for the exam must be your own.
- Do not discuss the contents of the exam with anyone before 11:59pm Thursday, 8 April 2Œ 1.
- *Please copy the following Honors Pledge onto the first page of your exam submission and sign and date your agreement to it.*

Honor Pledge:

On my honor, I agree to abide by the rules stated on the exam sheet.

Signature ______________________________

Date ____________________

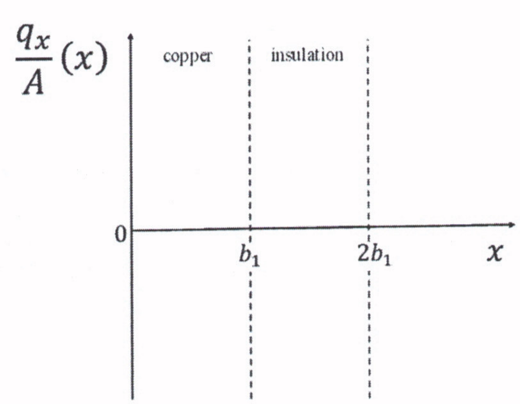
Exam Instructions:

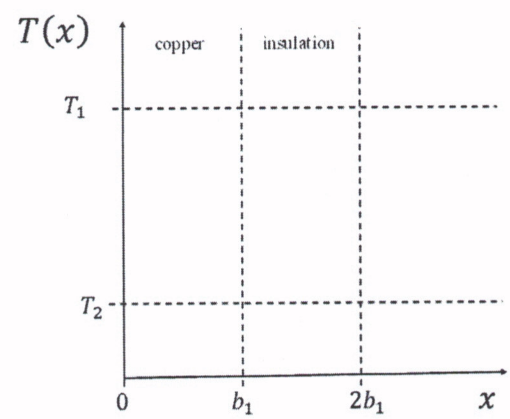
i. You may work on the exam for up to two hours and 30 minutes (150 minutes).
ii. Please submit your exam work within 150 minutes of downloading the exam.
iii. Please be neat. Only neat answers will be granted partial credit. Please use a dark pencil or pen so that your work is readable once scanned.
iv. Significant figures always count.
v. Please box your final answers.
vi. Submit your work as a single PD Ffile; put your name on every page. (Genius Scan is a free app that can create a PD Ffrom photos taken by your phone)
vii. Submit your exam study sheet as a separate PD Ffile; put your name on the first page (at a minimum)

1. (20 points) How much heat (kW) would it take to raise the temperature of a liquid water stream flowing at 3.21 kg/s from 17^oC to 44^oC?

2. (20 points) Define thermal conductivity and explain how this quantity influences the heat-transfer behavior of materials (solids and fluids); contrast thermal conductivity with heat capacity. Please be concise.

3. (20 points) A tall, deep composite wall is formed of copper (thermal conductivity k, thickness b_1) and insulation (much smaller thermal conductivity $k/10$, thickness also b_1) The left side of the composite wall is held at T_1 and the right side ($x = 2b_1$) is held at T_2 with $T_1 > T_2$. Sketch the following over the domain $0 \leq x \leq 2b_1$, using the axes provided below. Draw your profiles carefully.
   a. Steady state flux of heat $\frac{q_x}{A}$ through the composite wall for $0 \leq x \leq 2b_1$
   b. Steady state temperature distribution through the composite wall for $0 \leq x \leq 2b_1$



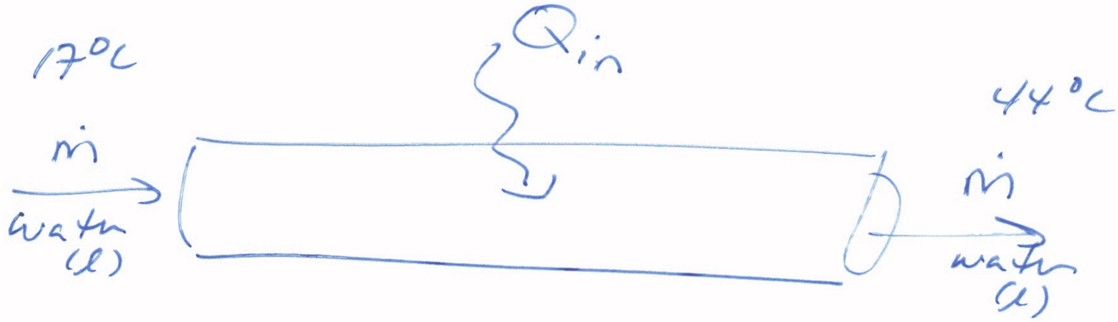


4. (20 points) A tall, wide door constructed of steel (thermal conductivity k, heat capacity $\hat{C}_p$, density ρ) is part of a vacuum oven. The thickness of the door is D and the inside temperature of the door is T_1. The vacuum oven sits in a large room that has a bulk air temperature of T_b; note that $T_1 > T_b$. There are fans running in the room, and the heat transfer coefficient h between the oven door and the room air has been determined. What is the steady state temperature profile $T(x)$ through the thickness of the door? You may leave your answer as an equation with integration constants in it. For full credit you must clearly write the two equations needed to evaluate the integration constants (2 equations, 2 unknowns)

5. (20 points) Water flows steadily in a Schedule 40 steel pipe (inner diameter = 1.05 $inches$, outer diameter 1.315 $inches$, length = $7.00ft$) in laminar flow (Reynolds number = 1.9×10^3) The inlet bulk temperature of the water is 15.0^oC and the outlet bulk temperature of the water is 60.6^oC. What is the heat transfer coefficient that quantifies the heat transfer from the inner pipe surface to the bulk fluid? The temperature variation of the fluid viscosity near the pipe wall may be omitted from consideration in your calculations. Give your answer in W/m^2K.

CM3110
Exam 4 (Morrison)
Spring 2021
Solution

1)



MACRO ENERGY BAL

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

not significant

no shafts

$$Q_{in} = \Delta H = \sum_{outs} \dot{m}_i \hat{H}_i - \sum_{ins} \dot{m}_i \hat{H}_i$$

$$= \dot{m} (\hat{H}_{out} - \hat{H}_{in})$$

$$= \dot{m} \hat{C}_p (T_{out} - T_{in})$$ (liquid) $\hat{C}_p \sim$ constant

$$= \left(3.21 \frac{\cancel{kg}}{s}\right)\left(4.182 \frac{\cancel{kJ}}{\cancel{kg}\,\cancel{K}}\right)(44 - 17)\cancel{K} \frac{kW}{\cancel{kJ/s}}$$

$= \underline{362 \text{ kW}}$

$= \boxed{360 \text{ kW}}$

2) Fourier's Law:

$$\frac{q_x}{A} = -k \frac{dT}{dx}$$

k — thermal conductivity

The thermal conductivity is the transport coefficient that relates heat flux $\frac{q_x}{A}$ in a chosen direction (x) to the gradient in temperature in that direction $\left(\frac{dT}{dx}\right)$. For homogeneous liquids or solids we can determine the temperature distribution from Fourier's law + microscopic energy balance. Heat capacity relates the temperature of a material relative to a reference temperature to the enthalpy that it would take to make the material undergo that temperature change

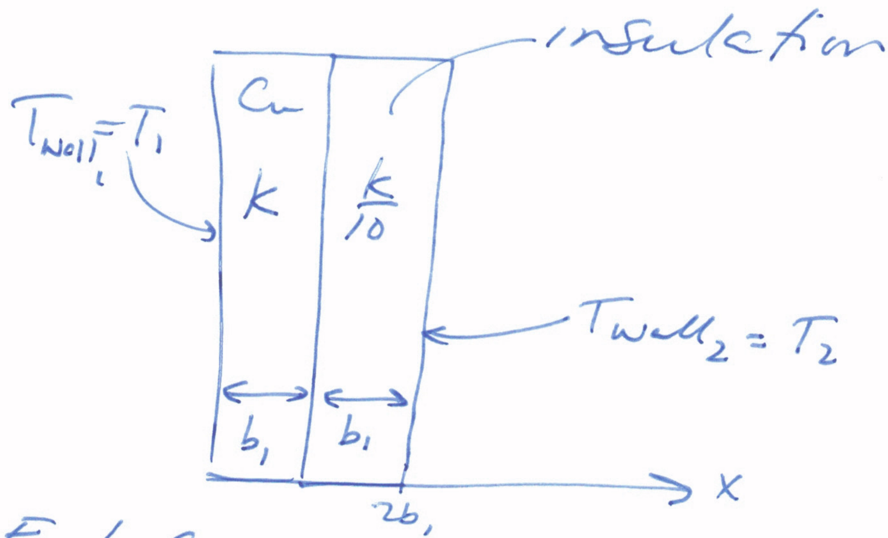
$$\Delta H = \int_{T_{ref}}^{T} m\,\hat{C}_p(T)\,dT$$

related to:

$\hat{C}_p$ capacity to store enthalpy

k rate of transfer of heat.

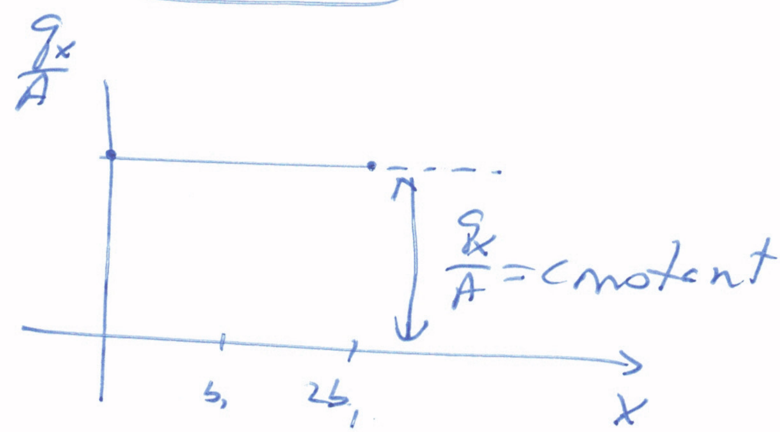
3.)



micro E-bal:

1D rectangular steady heat conduction, in a solid ($\underline{v} = 0$), w/ no rxn + no electric current:

$$\Rightarrow \boxed{\frac{q_x}{A} = C_1} = -k\frac{dT}{dx}$$



Also, the temp profile will be a line (Fourier's Law).

$$\frac{dT}{dx} = -\frac{C_1}{k} \Rightarrow \boxed{T = \left(-\frac{C_1}{k}\right)x + C_2}$$

5)

in copper, Fourier's law:

$$C_1 = \frac{q_x}{A} = -k\left(\frac{dT}{dx}\right)_{copper}$$

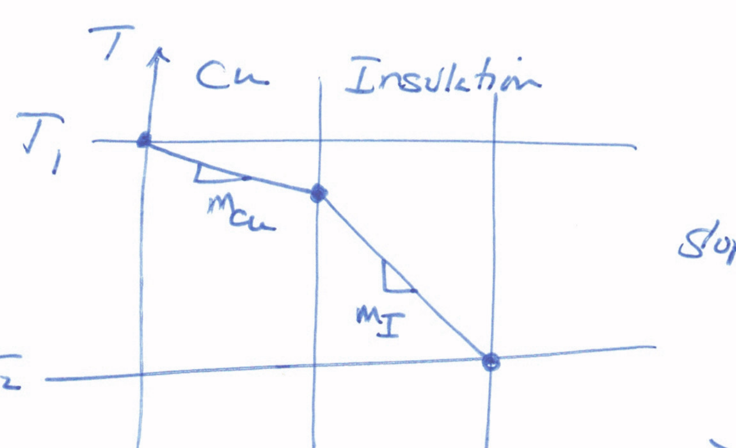
in insulation, Fourier's law:

$$C_1 = \frac{q_x}{A} = -\frac{k}{10}\left(\frac{dT}{dx}\right)_{insulation}$$

equate:

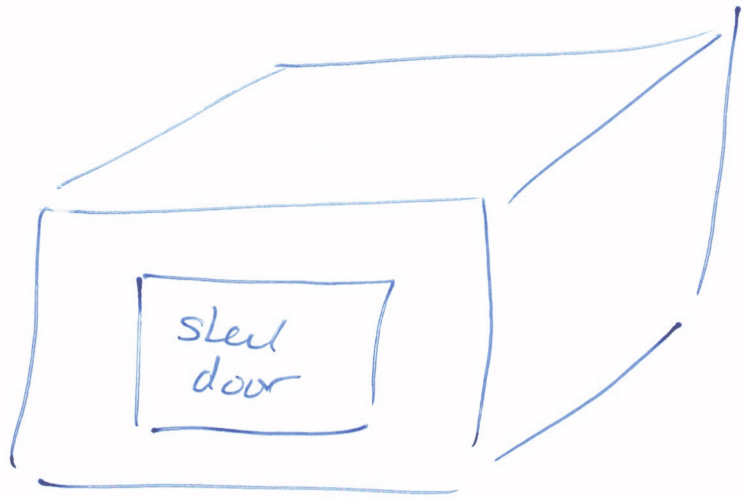
$$\cancel{-k}\left(\frac{dT}{dx}\right)_{copper} = \cancel{-}\frac{\cancel{k}}{10}\left(\frac{dT}{dx}\right)_{insulation}$$

$$\left(\frac{dT}{dx}\right)_{copper} = \frac{1}{10}\left(\frac{dT}{dx}\right)_{insulation}$$



$slope_{cu} = \frac{1}{10}\, slope_{Insulation}$

4)

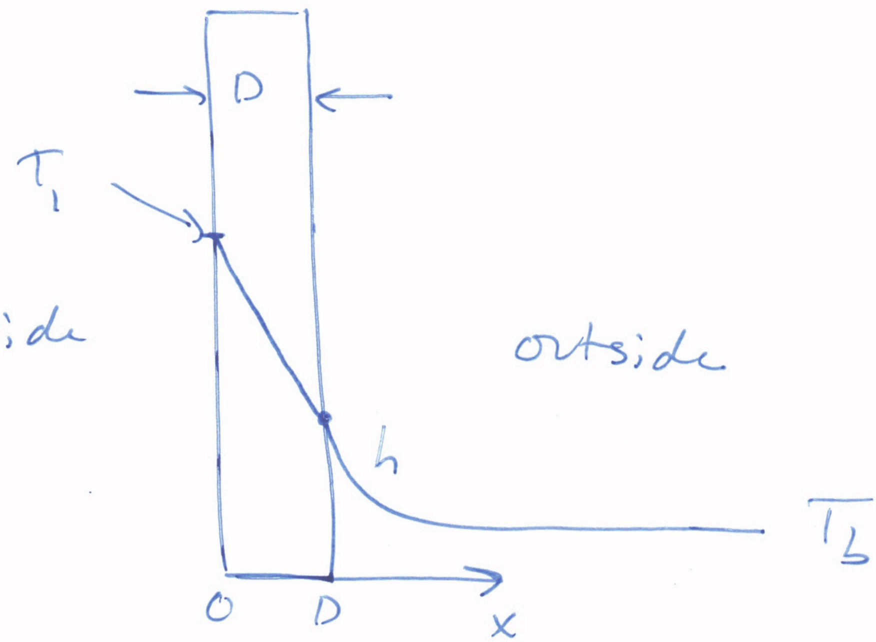


steel

k

$\hat{C}_p$

ρ



What is $T(x)$ $\Rightarrow$ microscopic energy balance

BC: 1 $x = 0 \quad T = T_1$

2 $x = D$ Newton's law of cooling

$$\Rightarrow x = D \quad -k \left.\frac{dT}{dx}\right|_D = h\left(T_{wall} - T_b\right) = h\left(T(D) - T_b\right)$$

7

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2T+S_e$$

no elec current
no reaction

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\cancel{\frac{\partial T}{\partial t}}+\cancel{v_x\frac{\partial T}{\partial x}}+\cancel{v_y\frac{\partial T}{\partial y}}+\cancel{v_z\frac{\partial T}{\partial z}}\right)=k\left(\frac{\partial^2T}{\partial x^2}+\cancel{\frac{\partial^2T}{\partial y^2}}+\cancel{\frac{\partial^2T}{\partial z^2}}\right)+\cancel{S_e}$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

steady

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2T}{\partial\theta^2}+\frac{\partial^2T}{\partial z^2}\right)+S_e$$

x, y, z

$\underline{v}=0$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

(steel)

$\Rightarrow$ 1D

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial\phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2T}{\partial\phi^2}\right)+S_e$$

conduction

$$\Rightarrow\quad 0=k\frac{d^2T}{dx^2}$$

$$\frac{d}{dx}\Big(\underbrace{\frac{dT}{dx}}_{=\Phi}\Big)=0 \qquad \frac{d\Phi}{dx}=0$$

$$\boxed{\Phi=C_1}$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

$$\Phi = \frac{dT}{dx} = C_1$$

1

$$\boxed{T = C_1 x + C_2}$$

$x = 0 \quad T = T_1$

$$\boxed{T_1 = C_2}$$

$$x = D \quad -k \underbrace{\frac{dT}{dx}} = h\left(\underbrace{T(D)} - T_b\right)$$

2

$$\boxed{-kC_1 = h\left(C_1 D + C_2 - T_b\right)}$$

full credit to here

substitute $C_2 = T_1$:

$$-kC_1 - hDC_1 = h\left(T_1 - T_b\right)$$

$$-C_1\left(k + hD\right) = h\left(T_1 - T_b\right)$$

$$\boxed{C_1 = \frac{h\left(T_b - T_1\right)}{k + hD}}$$

$$T = C_1 x + C_2$$

$$\boxed{T = \left(\frac{h(T_b - T_1)}{k + hD}\right) x + T_1}$$

check BC: $x = 0 \quad T = T_1$ ✓

$$x = D \qquad -k\frac{dT}{dx} = h\left(T(D) - T_b\right)$$

$$-k\left(\frac{\cancel{h}(T_b - T_1)}{\cancel{k + hD}}\right) \stackrel{?}{=} \cancel{h}\left(\frac{hD(T_b - T_1)}{k + hD} + T_1 - T_b\right)$$

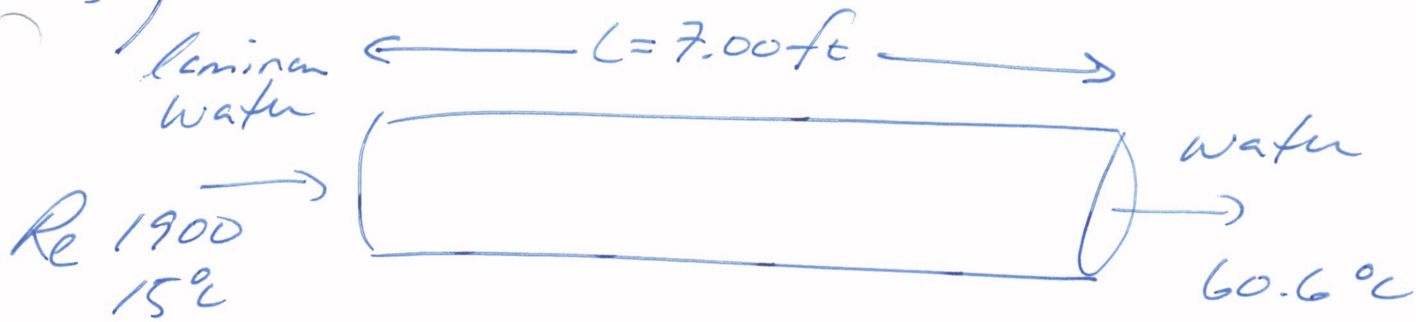
$$= \frac{hD(T_b - T_1) - (k + hD)(T_b - T_1)}{\cancel{k + hD}}$$

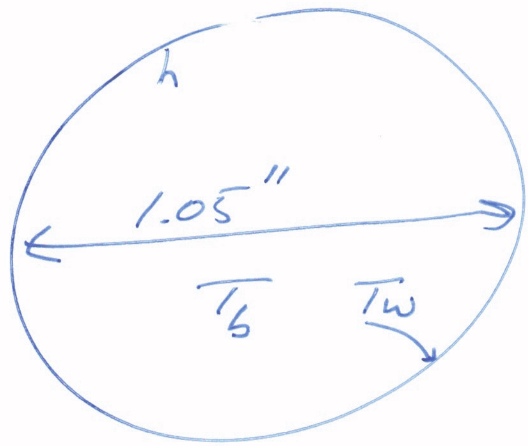
$$-k(T_b - T_1) \stackrel{?}{=} (\cancel{hD} - k - \cancel{hD})(T_b - T_1)$$

$$= -k(T_b - T_1)$$ ✓

(whew!)

5) steady





What is h? $[W/m^2 K]$

may omit $\left(\frac{\mu_b}{\mu_w}\right)$ term

We need the data correlation for $Nu = \frac{hD}{K}$ for laminar flow in a tube.

Sieder + Tate (laminar Re=1900)

11

$$Nu_a = \frac{h_a D}{k} = 1.86\left(Re\, Pr\, \frac{D}{L}\right)^{\frac{1}{3}}\cancel{\left(\frac{\mu_b}{\mu_w}\right)^{0.14}}$$

neglect

need material data
at mean bulk temperature:

$$\bar{T} = \frac{T_{b,in} + T_{b,out}}{2} = \boxed{37.8^\circ C = \bar{T}}$$

need: Pr, k (Geankoplis A.2-11)

$Pr = 4.51$

$k = 0.6283$ W/mK

need $D = \frac{1.05 \text{ in}}{12 \text{ in/ft}} = \boxed{0.0875 \text{ ft} = D}$

$= (0.0875\ \cancel{ft})\left(\frac{0.3048 \text{ m}}{\cancel{ft}}\right)$

$$\boxed{D = 0.02667 \text{ m}}$$

(12)

$$Nu_a = (1.86)\left(\underbrace{(1900)(4.51)\left(\frac{0.0875\,\cancel{ft}}{7.00\,\cancel{ft}}\right)}_{107.1125}\right)^{\frac{1}{3}}$$

$$\underbrace{\qquad\qquad}_{4.749226}$$

$$\boxed{Nu_a = 8.833681}$$

$$= \frac{h_a D}{k}$$

$$h_a = \frac{Nu\,k}{D}$$

$$= \frac{(8.833681)\left(0.6283\,\frac{W}{mK}\right)}{0.02667\,m}$$

$$= 208.099\ W/m^2K$$

$$= \boxed{210\ W/m^2K}$$

Re = 1900 = 2 sig figs