

Data Correlations for Examinations

CM3110 Transport Phenomena I
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I. Flow through Smooth Pipes

A. All Reynolds numbers: Morrison

The correlation from Morrison (2013) fits the smooth pipe data for all Reynolds numbers; beyond $Re = 4000$ this correlation follows the Prandtl equation (see Figure 1; Morrison, equation 7.158). This correlation is explicit in f ; when flow rate is known, Δp may be found directly; when Δp is known, Q or $\langle v \rangle$ must be solved for iteratively.

Morrison (2013)

$$f = \left(\frac{0.0076 \left(\frac{3170}{Re} \right)^{0.165}}{1 + \left(\frac{3170}{Re} \right)^{7.0}} \right) + \frac{16}{Re} \quad (1)$$

B. $4,000 \leq Re \leq 1 \times 10^6$: Prandtl

The Prandtl correlation for $f(Re)$ in turbulent flow is not explicit in friction factor and must be solved iteratively except when f is known (Morrison, equation 7.156). This is good only for $Re > 4,000$ /

Prandtl
or VonKarman-Nikuradse
(Denn, 1980)

$$\frac{1}{\sqrt{f}} = 4.0 \log(Re\sqrt{f}) - 0.40 \quad (2)$$

C. $4,000 \leq Re \leq 1 \times 10^6$: A simplified Correlation

For the turbulent regime, an approximate correlation that is much simpler to work with (with a calculator on an exam, for example) is given here and shown in Figure 2 (Morrison, equation 7.157). This is good only for $Re > 4,000$.

Simplified Turbulent
(White, 1974)

$$f = \frac{1.02}{4} (\log Re)^{-2.5} \quad (3)$$

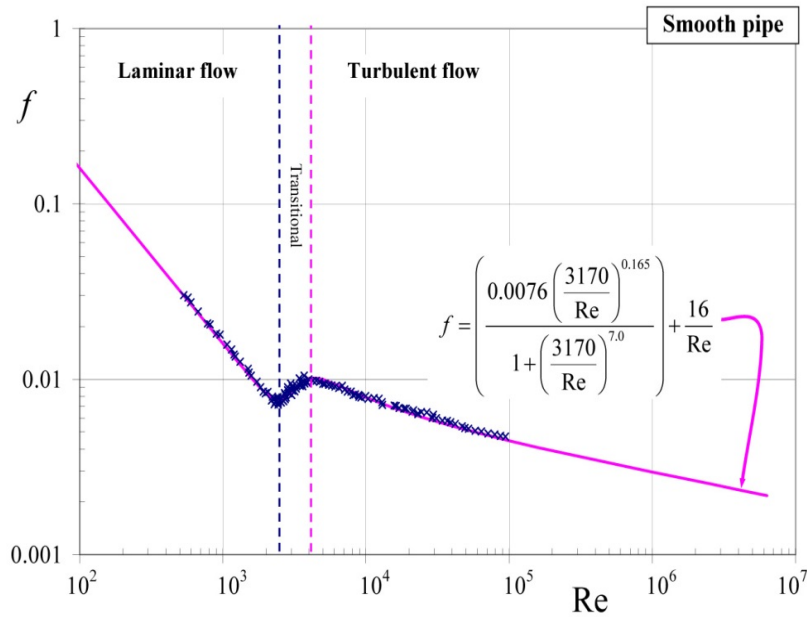


Figure 1: Equation 3 captures smooth pipe friction factor as a function of Reynolds number over the entire Reynolds-number range (Morrison, 2013) and is recommended for spreadsheet use. Also shown are Nikuradse's experimental data for flow in smooth pipes (Nikuradse, 1933). Use beyond $Re = 10^6$ is not recommended; for $Re > 4000$ equation 3 follows the Prandtl equation.

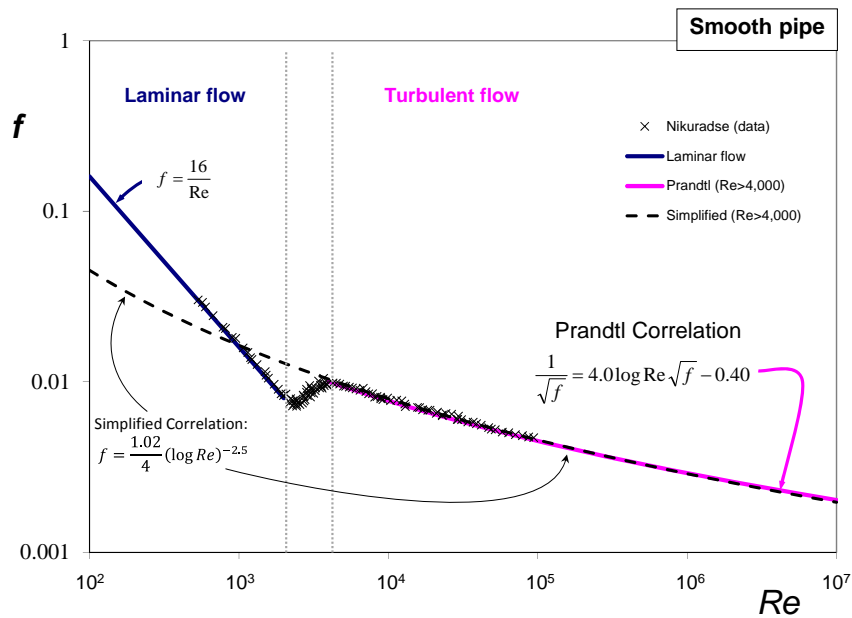


Figure 2: For turbulent flow, the simplified (equation 3) or Prandtl (equation 2) correlations may be used. For work with a calculator, the simplified correlation is perhaps the easiest to work with.

II. Flow Around a Sphere

A. All Reynolds Numbers: Morrison

The correlation from Morrison (2013) fits the flow around a sphere for all Reynolds numbers (Figure 3; Morrison equation 8.83); beyond $Re = 10^6$ this correlation follows the curve shown in Figure 3.

$$\text{Morrison (2013)} \quad C_D = \frac{24}{Re} + \frac{2.6 \left(\frac{Re}{5.0}\right)}{1 + \left(\frac{Re}{5.0}\right)^{1.52}} + \frac{0.411 \left(\frac{Re}{263,000}\right)^{-7.94}}{1 + \left(\frac{Re}{236,000}\right)^{-8.00}} + \frac{0.25 \left(\frac{Re}{10^6}\right)}{1 + \left(\frac{Re}{10^6}\right)} \quad (4)$$

Simplified Correlations

The correlations below (Morrison, 2013; equation 8.82) are simpler relationships more suitable to calculator/exam work.

$$Re < 2 \quad C_D = \frac{24}{Re} \quad (5)$$

$$0.1 \leq Re \leq 1,000 \quad C_D = \frac{24}{Re} (1 + 0.14Re^{0.7}) \quad (6)$$

$$1,000 \leq Re \leq 2.6 \times 10^5 \quad C_D = 0.445 \quad (7)$$

$$2.8 \times 10^5 \leq Re \leq 10^6 \quad \frac{\log C_D}{\left(\frac{Re}{10^6}\right)} = 4.43 \log Re - 27.3 \quad (8)$$

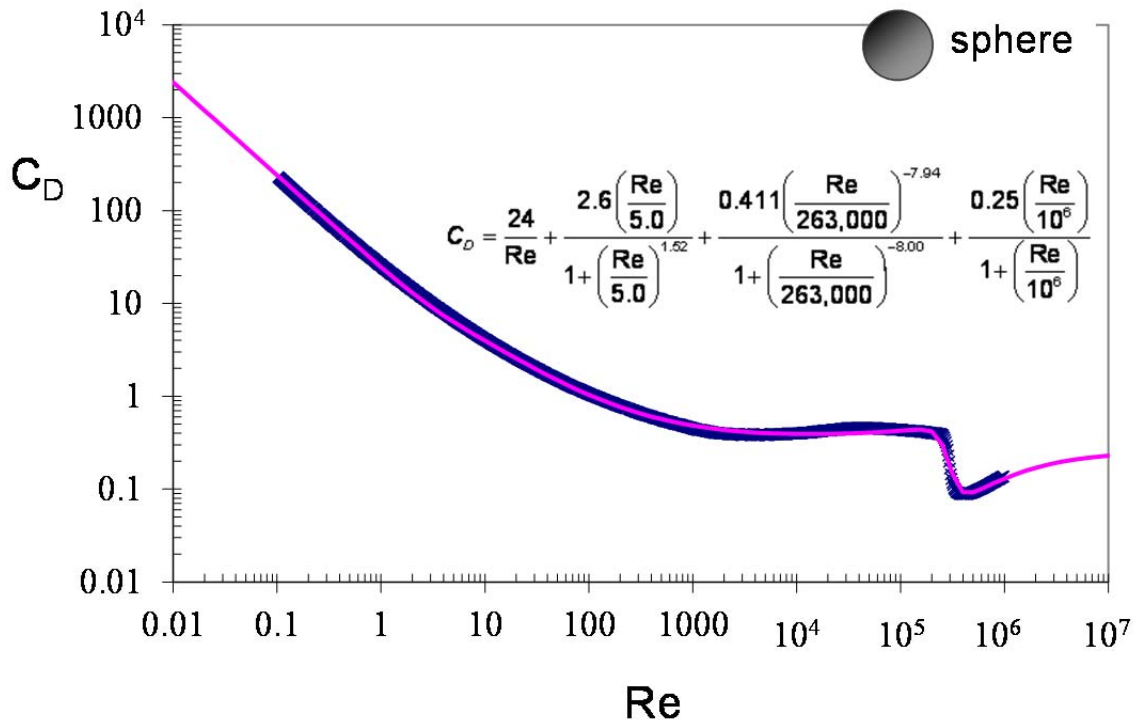


Figure 3: Equation 4 captures flow around a sphere as a function of Reynolds number over the entire Reynolds-number range (Morrison, 2013) and is recommended for spreadsheet use. Also shown are experimental data from White (1974). Use beyond $Re = 10^6$ is not recommended.

References

- M. Denn, *Process Fluid Mechanics* (Prentice Hall, Englewood Cliffs, NJ, 1980)
- F. A. Morrison, *An Introduction to Fluid Mechanics* (Cambridge University Press, New York, 2013).
- F. M. White, *Viscous Fluid Flow* (McGraw-Hill, Inc.: New York, 1974).