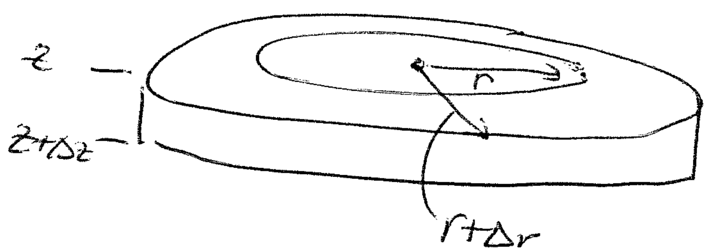
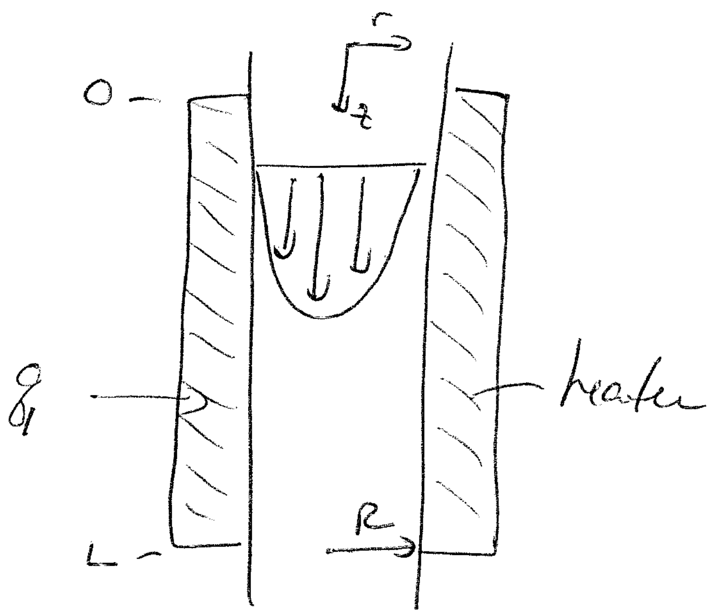
FAITH MORRISON 1
3-28-01

WALL HEATING IN LAMINAR FLOW:





contributions:

radial conduction – $T = T(r)$ – $\boxed{\text{note } q < 0}$
axial conduction – $T = T(z)$
axial convection – $v_z \neq 0$

radial conduction

$$\left.\frac{q_r}{A}\right|_r 2\pi r \Delta z \qquad \text{heat in at } r$$

$$\left.\frac{q_r}{A}\right|_{r+\Delta r} 2\pi (r+\Delta r) \Delta z \qquad \text{heat out at } r+\Delta r$$

NOTE:
ALWAYS
DO SMALLER
COORD.
FIRST,
increasing
r.

axial conduction

$$\left.\frac{q_z}{A}\right|_z \overbrace{2\pi r \Delta r}$$

approximate for $\pi(r+\Delta r)^2 - \pi r^2 \approx 2\pi r \Delta r$

heat in at z

$$\left.\frac{q_z}{A}\right|_{z+\Delta z} 2\pi r \Delta r$$

heat out at z

axial convection

$$\left.\hat{H}\right|_z \cdot \rho \underbrace{v_z \, 2\pi r \Delta r}$$

$$\left(\frac{\text{enthalpy}}{\text{mass}}\right)\left(\frac{\text{mass}}{\text{vol}}\right)\frac{\text{volume}}{\text{time}}$$

heat in at z

$$\left.\hat{H}\right|_{z+\Delta z} \rho v_z \, 2\pi r \Delta r$$

heat out at z

note: $\hat{H} = \int_{T_{ref}}^{T} \hat{C}_p \, dT$

$= \hat{C}_p (T - T_{ref})$

Combine

$$\left.\frac{q_r}{A}\right|_r 2\pi r \Delta z - \left.\frac{q_r}{A}\right|_{r+\Delta r} 2\pi (r+\Delta r)\Delta z$$

$$+ \left.\frac{q_z}{A}\right|_z 2\pi r \Delta r - \left.\frac{q_z}{A}\right|_{z+\Delta z} 2\pi r \Delta r$$

$$+ \rho v_z \hat{C}_p 2\pi r \Delta r \left(T|_z - T_{ref}\right)$$

$$- \rho v_z \hat{C}_p 2\pi r \Delta r \left(T|_{z+\Delta z} - T_{ref}\right) = 0$$

$$\frac{-1}{r} \frac{\left(\frac{q_r}{A} r\right)_{r+\Delta r} - \left.\frac{q_r}{A}\right|_r}{\Delta r} - \frac{\left.\frac{q_z}{A}\right|_{z+\Delta z} - \left.\frac{q_z}{A}\right|_z}{\Delta z}$$

$$- \frac{T|_{z+\Delta z} - T|_z}{\Delta z} \rho \hat{C}_p v_z = 0$$

take limit as $\Delta r, \Delta z \to 0$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{q_r}{A} r\right) + \frac{\partial}{\partial z}\left(\frac{q_z}{A}\right) + \rho \hat{C}_p v_z \frac{\partial T}{\partial z} = 0$$

$$\frac{q_r}{A} = -k\frac{\partial T}{\partial r} \qquad \frac{q_z}{A} = -k\frac{\partial T}{\partial z}$$

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r} r\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) - \frac{\rho \hat{C}_p}{k} v_z(r) \frac{\partial T}{\partial z} = 0}$$

Solve w/ BC's

See R. Siegel
E.M. Sparrow
T.M. Hallman
Appl Science Research
A7 384-392 (1958)

or, Bird, Stewart, Lightfoot
Transport Phenomena
1960 p295.

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