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Note: This document represents text was part of the draft that is published in the following book:

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The excerpt is concerning the Mechanical Engineering Balance, especially when the friction term is not negligible.

Mechanical Energy Balance: Intro and Overview

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The mechanical energy balance is a type of energy balance that can tell us a great deal about simple flow systems. We begin with a discussion of conservation of energy and derive the mechanical energy balance (MEB). Finally, we show how to apply the MEB to simple flow systems.

0.1 Energy Balances

The First Law of Thermodynamics expresses a fundamental law of physics: energy is conserved. Energy can be neither created nor destroyed (just like mass and momentum), but energy can move across the boundaries of a system, increasing or decreasing the total system energy.

$$\left(\begin{array}{c}\text{Increase in the}\\ \text{Total Energy}\\ \text{in a system}\end{array}\right) = \left(\begin{array}{c}\text{Net Energy}\\ \text{into the}\\ \text{system}\end{array}\right) \tag{1}$$

Energy can cross system boundaries in a variety of ways. One is in the form of heat, and another is in the form of work. The third way energy enters or leaves a system is when it is carried along by material entering or leaving the system, a mechanism know as convection.

$$\Delta E_{total} + \Delta \dot{E}_{convection} = Q_{in} + W_{on} \tag{2}$$

In the energy balance above, E_{total} is the total energy of the system, Q_{in} is the heat that flows into the system, W_{on} is the total work done on the system, and $\Delta \dot{E}_{convection}$ is the net energy out by convection.[1] The heat that flows out is equal to $-Q_{in}$, and the work done *by* the system is equal to $-W_{on}$.

[1] A term for net-energy-in placed on the right-hand side of equation 2 might seem a better choice for notation. The choice is arbitrary. Net-energy-out is more convenient to use in steady state analysis, as we will see in a moment.

The total energy of the system has contributions from three types of energy, the kinetic energy of the system, the potential energy of the system, and the internal energy of the system (Felder and Rousseau, Tipler; Figure 1). The kinetic energy is the energy due

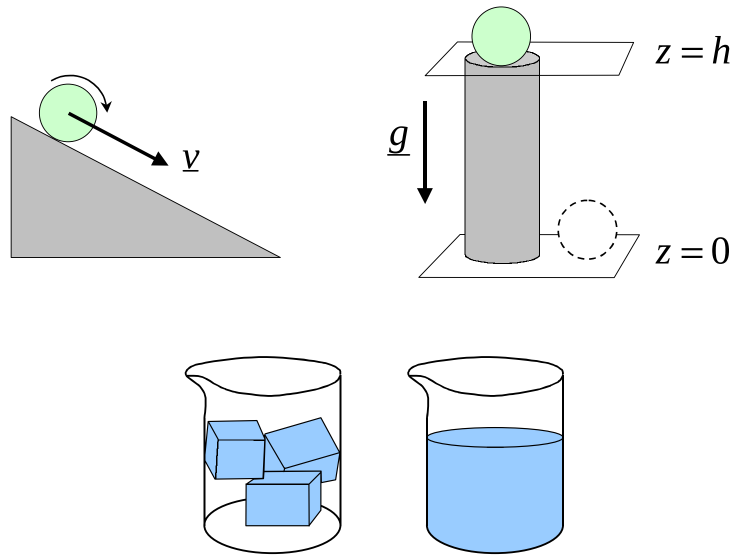

Figure 1: Energy is a property of a system. Energy may be stored in the state of a system, for example, as kinetic energy stored in the speed of the system, as potential energy stored in the position of the system in a potential field, or as internal energy stored in the chemical state of a system.

to the speed at which the system is moving. To calculate the kinetic energy, first we must choose a reference state; for kinetic energy the reference state is the system at rest, $v = 0$. Relative to a system at rest, the kinetic energy of a system moving with speed v is given by

$$\begin{pmatrix} \text{Kinetic Energy} \\ \text{of a system moving} \\ \text{with speed } v \end{pmatrix} = \frac{1}{2} m v^2 = E_k \tag{3}$$

where m is the mass of the system, and v is the speed of the system.

The potential energy is the energy of the system by virtue of the position of the system in a potential field. The most important potential fields are gravity and electromagnetic fields. Potential energy in the Earth s gravitational field is the energy that the system has by virtue of its being at a high elevation. A ball, for example, can roll down a hill and exchange its potential energy (the energy it had stored in it simply by being at the top of the hill) for kinetic energy (speed). Again energy is calculated relative to a reference state. For potential energy we choose a reference elevation (or position), and then measure the elevation of the system relative to that reference elevation. The potential energy of a system is therefore given by

$$\begin{pmatrix} \text{Potential Energy} \\ \text{of a system at} \\ \text{elevation } z \end{pmatrix} = mg(z - z_{\text{ref}}) = E_p \tag{4}$$

where m is the mass of the system, g is the acceleration due to gravity, and $(z - z_{\text{ref}})$ is the elevation of the system relative to the reference elevation z_{ref}. Often z_{ref} is chosen to be $z = 0$, and $E_p = mgz$.

Internal energy is the energy possessed by a system internally, that is, in its molecules and atoms. The temperature of a system is one indicator of its internal energy, but a system may store internal energy in its phase (being a solid versus being a liquid, for example) or in its chemical composition (being a mixture of gasses H_2 and O_2 versus being a beaker of H_2O). Internal energy is kept track of with the defined function U. Again, the value of U reported for a system is always with respect to some chosen reference state.

$$\begin{pmatrix} \text{Internal Energy} \\ \text{of a system} \\ \text{with respect to} \\ \text{a chosen reference} \\ \text{state} \end{pmatrix} = U \tag{5}$$

The reference state for internal energy must fully describe the internal energy of the system. For example we might choose liquid water at temperature 25^oC as the reference state for a calculation involving steam. We must specify temperature (25^oC in this example), phase (liquid), and chemical composition (H_2O) in order to fully specify the internal energy.

The key to getting the most information out of energy balances is making the correct the choice of system on which to base the calculations.

0.1.1 Closed Systems (No Convection)

Balances of many types, for example mass, energy, or momentum, may be performed on any system, but not all systems are equally useful. A system is defined by boundaries drawn around components of a physical situation under consideration. When we write our balance equations we choose the boundaries and then note the quantities of mass, energy, or momentum that cross the boundaries (Figure 2).

A closed system is a system that does not have any mass crossing its boundaries. For closed systems, there is no mass coming in or going out and thus no convection of mass, energy, or momentum.

For a closed system, the energy balance relates two states of the system, an initial state and a final state. The changes in energy between initial and final states of the system are

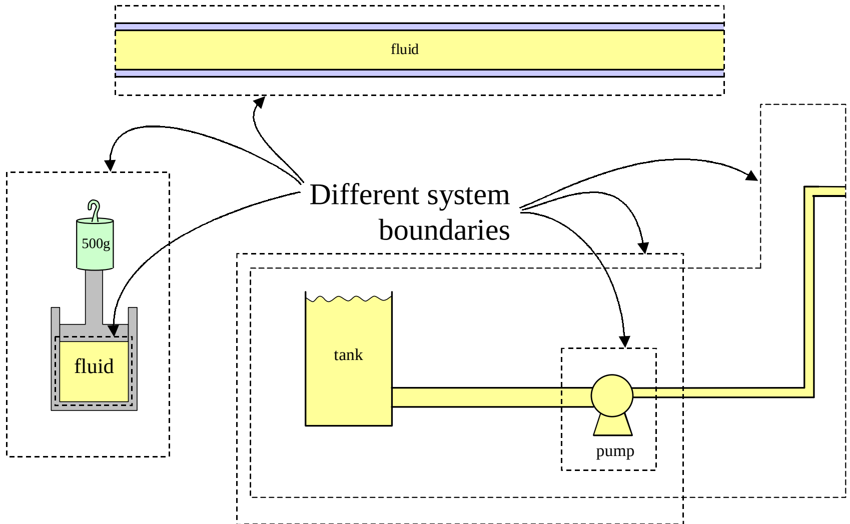

Figure 2: System boundaries are chosen for convenience of the calculation. Usually the system boundaries are chosen so that the inputs and outputs to the system are locations where fluid velocities, pressures, and/or elevations are known. Some problems require multiple balances over different systems.

brought about by additions of energy through heat (Q_{in}) and additions of energy through work done on the system (W_{on}).

$$\Delta E_{total} = Q_{in} + W_{on} \tag{6}$$

The total energy change of the system, ΔE_{total}, is calculated by summing the changes in potential, kinetic, and internal energy.

$$\Delta E_{total} = \left(\begin{array}{c}\text{Final}\\ \text{Total Energy}\\ \text{of a closed system}\end{array}\right) - \left(\begin{array}{c}\text{Initial}\\ \text{Total Energy}\\ \text{of a closed system}\end{array}\right) \tag{7}$$

$$= (E_{p,final} + E_{k,final} + U_{final}) - (E_{p,initial} + E_{k,initial} + U_{initial}) \tag{8}$$

These terms combine to give the macroscopic closed system energy balance.

$$\boxed{\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on}} \quad \begin{array}{c}\text{Macroscopic}\\ \text{Closed System}\\ \text{Energy Balance}\end{array} \tag{9}$$

Δ here signifies final − initial.

0.1.2 Open Systems

An open system is a system that has mass crossing its boundaries. For open systems, convection or flow contributes to mass, energy, and momentum balances. In open systems, balances are done on energy per time instead of on bare energy. Also, while for a closed system we were concerned with changes in the system between two states, a final state and an initial state, for open systems we will be concerned with the system at all times. We will keep track of the state of the system by following the rate of accumulation of energy with time.

$$\begin{pmatrix} \text{Rate of} \\ \text{Total Energy} \\ \text{Accumulation} \\ \text{in an open system} \end{pmatrix} = \begin{pmatrix} \text{Rate of} \\ \text{Total Energy} \\ \text{into the system} \end{pmatrix} - \begin{pmatrix} \text{Rate of} \\ \text{Total Energy} \\ \text{out of the system} \end{pmatrix} \tag{10}$$

For an open system, energy can enter the system in the same way as it did for a closed system, through the addition of heat or through work performed on the system. The rate of heat added per unit time will be denoted $\dot{Q}_{in}$, and the rate of work done on the system per unit time will be called $\dot{W}_{on}$. In addition, the streams that flow into and out of an open system bring their potential, kinetic, and internal energies with them; these are the convective terms. Equation 10 thus becomes

$$\frac{dE_{total}}{dt} = \begin{pmatrix} \text{Rate of} \\ \text{Total Energy} \\ \text{Accumulation} \\ \text{in an open system} \end{pmatrix} \tag{11}$$

$$= \dot{Q}_{in} + \dot{W}_{on} + \begin{pmatrix} \text{Rate of Energy } \textit{in} \\ \text{through convection} \end{pmatrix} - \begin{pmatrix} \text{Rate of Energy } \textit{out} \\ \text{through convection} \end{pmatrix} \tag{12}$$

At steady state this equation becomes[2]

$$\begin{pmatrix} \text{Rate of} \\ \text{Energy out} \\ \text{through} \\ \text{convection} \end{pmatrix} - \begin{pmatrix} \text{Rate of} \\ \text{Energy in} \\ \text{through} \\ \text{convection} \end{pmatrix} = \dot{Q}_{in} + \dot{W}_{on} \tag{13}$$

$$\Delta \dot{E}_{convection} = \dot{Q}_{in} + \dot{W}_{on} \tag{14}$$

Note that in equation 14 and in the remainder of this section on open systems, Δ refers to out $-$ in.

To express the convective energy term $\Delta \dot{E}_{convection}$, we must take a sum of the energy contributions to each stream. Each stream of mass flow rate $\dot{m}_i$ brings with it an associated

[2]For more on unsteady state balances, see Felder and Rousseau.

kinetic energy per unit mass $\hat{E}_{k,i}$, an associated potential energy per unit mass $\hat{E}_{p,i}$, and an associated internal energy per unit mass $\hat{U}_i$. Thus for each stream

$$\dot{E}_i = \dot{m}_i \hat{E}_{k,i} + \dot{m}_i \hat{E}_{p,i} + \dot{m}_i \hat{U}_i \tag{15}$$

Using i to index the inflow streams and using j to index the outflow streams, we can now sum over all the streams to obtain the net convective contribution $\Delta \dot{E}_{convection}$.

$$\begin{aligned}
\sum_{out} \dot{E}_j &= \sum_{out} \left(\dot{m}_j \hat{E}_{k,j} + \dot{m}_j \hat{E}_{p,j} + \dot{m}_j \hat{U}_j \right) && (16)\\
&= \sum_{out} \dot{m}_j \hat{E}_{k,j} + \sum_{out} \dot{m}_j \hat{E}_{p,j} + \sum_{out} \dot{m}_j \hat{U}_j && (17)\\
\sum_{in} \dot{E}_i &= \sum_{in} \left(\dot{m}_i \hat{E}_{k,i} + \dot{m}_i \hat{E}_{p,i} + \dot{m}_i \hat{U}_i \right) && (18)\\
&= \sum_{in} \dot{m}_i \hat{E}_{k,i} + \sum_{in} \dot{m}_i \hat{E}_{p,i} + \sum_{in} \dot{m}_i \hat{U}_i && (19)\\
\Delta \dot{E}_{convection} &= \sum_{out} \dot{E}_j - \sum_{in} \dot{E}_i && (20)\\
&= \sum_{out} \dot{m}_j \hat{E}_{k,j} + \sum_{out} \dot{m}_j \hat{E}_{p,j} + \sum_{out} \dot{m}_j \hat{U}_j \\
&\qquad - \sum_{in} \dot{m}_i \hat{E}_{k,i} - \sum_{in} \dot{m}_i \hat{E}_{p,i} - \sum_{in} \dot{m}_i \hat{U}_i && (21)\\
&= \left(\sum_{out} \dot{m}_j \hat{E}_{k,j} - \sum_{in} \dot{m}_i \hat{E}_{k,i} \right) + \left(\sum_{out} \dot{m}_j \hat{E}_{p,j} - \sum_{in} \dot{m}_i \hat{E}_{p,i} \right) \\
&\qquad + \left(\sum_{out} \dot{m}_j \hat{U}_j - \sum_{in} \dot{m}_i \hat{U}_i \right) && (22)\\
\Delta \dot{E}_{convection} &= \Delta \dot{E}_k + \Delta \dot{E}_p + \Delta \dot{U} && (23)
\end{aligned}$$

Again, the $\Delta \dot{E}_k$, $\Delta \dot{E}_p$, and $\Delta \dot{U}$ in equation 23 refer to the differences between the sum of contributions from the outlet streams minus the sum of contributions from the inlet streams (out − in).

$$\begin{aligned}
\Delta \dot{E}_k &\equiv \sum_{out} \dot{m}_j \hat{E}_{k,j} - \sum_{in} \dot{m}_i \hat{E}_{k,i} && (24)\\
\Delta \dot{E}_p &\equiv \sum_{out} \dot{m}_j \hat{E}_{p,j} - \sum_{in} \dot{m}_i \hat{E}_{p,i} && (25)\\
\Delta \dot{U} &\equiv \sum_{out} \dot{m}_j \hat{U}_j - \sum_{in} \dot{m}_i \hat{U}_i && (26)
\end{aligned}$$

Putting it all together we obtain a raw form of the open system macroscopic energy balance.

$$\Delta \dot{E}_p + \Delta \dot{E}_k + \Delta \dot{U} = \dot{Q}_{in} + \dot{W}_{on} \tag{27}$$

We can further refine the open system balance by recognizing that in open systems the work term, $\dot{W}_{on}$, contains two contributions, one due to moving parts that intrude into

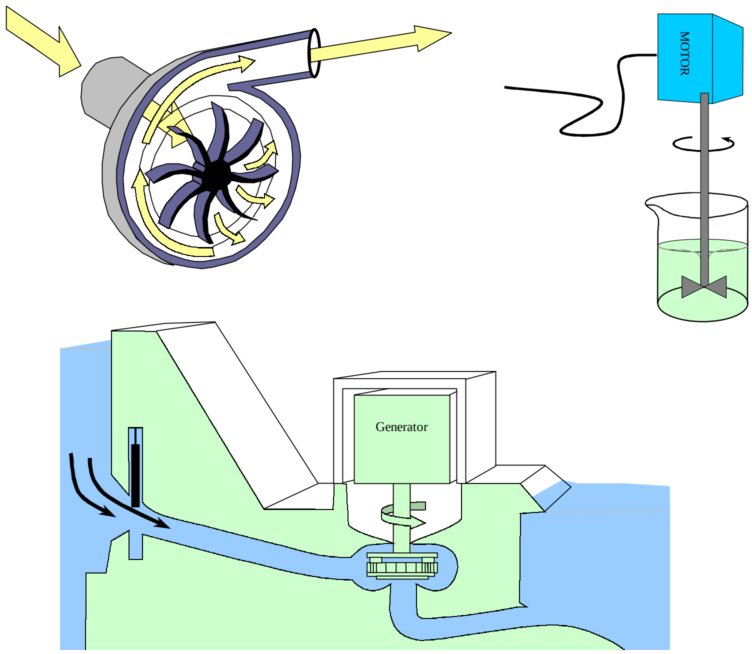

Figure 3: Work is force times displacement, and thus moving parts are one source of work. Work associated with moving parts is called shaft work. Examples of systems with shaft work present are centrifugal pumps, mixers, and turbines used in hydropower generation.

the system, such as shafts, turbines, and the internal workings of pumps (Figure 3). This is called shaft work, and it is given the symbol $\dot{W}_{s,on}$. The other contribution to $\dot{W}_{on}$ in an open system is the work done by the fluid itself as it enters or leaves the system (Figure 4), called flow work. Flow work is usually combined with the convective terms as follows.

A stream entering a chosen open system flows with a pressure $p_{i,in}$ and at a volumetric flow rate of $\dot{V}_{i,in} = v_i A_i$, where v_i is the magnitude of the velocity of the fluid (speed) and A_i is the cross-sectional area of the pipe. Pressure is force per unit area, and work is force multiplied by displacement; thus, just at the system boundary as the fluid enters, the pressure times the cross sectional area of the pipe is a force acting on the fluid, doing work on the fluid as it crosses into the system (Figure 4).

$$\begin{aligned}
\left(\begin{array}{c}\text{Rate of Flow Work}\\ \textit{on}\text{ system at entrance}\\ \text{for } i^{th} \text{ input stream}\end{array}\right) &= (\text{force})\left(\frac{\text{displacement}}{\text{time}}\right)\\
&= \left[\left(\frac{\text{force}}{\text{area}}\right)(\text{area})\right]\left(\frac{\text{displacement}}{\text{time}}\right)\\
&= p_{i,in}A_i v_i \qquad (28)
\end{aligned}$$

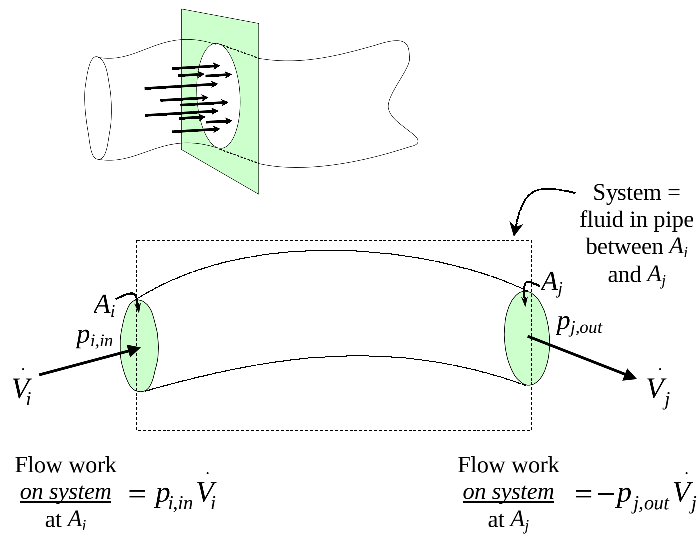

Figure 4: Work is force times displacement, and thus moving fluid is a source of work. Work done by the fluid or on the fluid as it enters or leaves the system is called flow work. The work on the boundaries of a flow system is done by fluid outside the boundary on the fluid inside the system. If the system itself does work on its surroundings, such as at the exit above, then the work on the system is negative.

$$= \quad p_{i,in}\dot{V}_{i,in} \tag{29}$$

A stream exiting a chosen open system flows with a pressure $p_{j,out}$ and at a volumetric flow rate of $\dot{V}_{j,out} = v_j A_j$, where v_j is the speed of the fluid and A_j is the cross-sectional area of the pipe. As before, just at the system boundary as the fluid exits, the pressure times the cross sectional area of the pipe is a force acting on the fluid, but since this stream is an exiting stream, the work is done on fluid that is outside of the chosen system. Thus, the work done *on* the chosen system at the exit is the negative of the force times the fluid displacement at the exit.

$$\begin{pmatrix} \text{Rate of Flow Work} \\ \textit{on} \text{ system at exit} \\ \text{for } j^{th} \text{ stream} \end{pmatrix} = -\begin{pmatrix} \text{Rate of Flow Work} \\ \textit{by} \text{ system at exit} \\ \text{for } j^{th} \text{ stream} \end{pmatrix} \tag{30}$$

$$= -p_{j,out} A_j v_j \tag{31}$$

$$= -p_{j,out}\dot{V}_j \tag{32}$$

We can now sum all the flow-work contributions and rearrange the open system energy balance to include the separation of shaft work and flow work into the different expressions derived above.

$$\Delta\dot{E}_p + \Delta\dot{E}_k + \Delta\dot{U} = \dot{Q}_{in} + \dot{W}_{on} \tag{33}$$

$$= \dot{Q}_{in} + \dot{W}_{s,on} + \sum_{i,in} p_{i,in}\dot{V}_i - \sum_{j,out} p_{j,out}\dot{V}_j \tag{34}$$

$$\Delta\dot{E}_p + \Delta\dot{E}_k + \left[\Delta\dot{U} + \sum_{j,out} p_{j,out}\dot{V}_j - \sum_{i,in} p_{i,in}\dot{V}_i\right] = \dot{Q}_{in} + \dot{W}_{s,on} \tag{35}$$

The two flow-work terms are commonly combined with the internal energy term and expressed in terms of the change in the thermodynamic function enthalpy, as we will now show. Specific enthalpy or enthalpy per unit mass $\hat{H}$ is defined as

$$\boxed{\hat{H} \equiv \hat{U} + p\hat{V}} \qquad \text{Specific Enthalpy} \tag{36}$$

For each of the flow streams in our system we can calculate the amount of enthalpy brought in or taken out, and, summing as we did for kinetic, potential, and internal energy, we can calculate an overall change in enthalpy for our system.

$$\left(\begin{array}{c}\text{Net Rate}\\ \text{of Enthalpy}\\ \text{flow out of}\\ \text{open system}\end{array}\right) = \Delta H = \sum_{j,out} \dot{m}_j\hat{H}_j - \sum_{i,in} \dot{m}_i\hat{H}_i \tag{37}$$

$$= \sum_{j,out}\left(\dot{m}_j\hat{U}_j + \dot{m}_j p_{j,out}\hat{V}_j\right) - \sum_{i,in}\left(\dot{m}_i\hat{U}_i + \dot{m}_i p_{i,in}\hat{V}_i\right) \tag{38}$$

The $mp\hat{V}$ terms can be recognized as the flow-work terms that appeared in equation 35 (see also equation 29).

$$\dot{m} \quad \hat{V} = \dot{V} \tag{39}$$

$$\left(\frac{\text{mass}}{\text{time}}\right)\left(\frac{\text{volume}}{\text{mass}}\right) = \left(\frac{\text{volume}}{\text{time}}\right) \tag{40}$$

$$\left(\begin{array}{c}\text{Net Rate of}\\ \text{Enthalpy flow out}\\ \text{of an open system}\end{array}\right) = \Delta\dot{H} \tag{41}$$

$$= \sum_{j,out}\left(\dot{m}_j\hat{U}_j + p_{j,out}\dot{V}_j\right) - \sum_{i,in}\left(\dot{m}_i\hat{U}_i + p_{i,in}\dot{V}_i\right) \tag{42}$$

$$= \left(\sum_{j,out}\dot{m}_j\hat{U}_j - \sum_{i,in}\dot{m}_i\hat{U}_i\right) + \sum_{j,out} p_{j,out}\dot{V}_j - \sum_{i,in} p_{i,in}\dot{V}_i \tag{43}$$

$$= \Delta\dot{U} + \sum_{j,out} p_{j,out}\dot{V}_j - \sum_{i,in} p_{i,in}\dot{V}_i \tag{44}$$

Equation 44 matches the bracketed terms in equation 35. Returning to equation 35 and combining with equation 44 we obtain the conventional form of the macroscopic, open-system energy balance.

$$\boxed{\Delta \dot{E}_p + \Delta \dot{E}_k + \Delta \dot{H} = \dot{Q}_{in} + \dot{W}_{s,on}} \qquad \begin{array}{c}\text{Macroscopic}\\ \text{Open System}\\ \text{Energy Balance}\\ \text{(steady state)}\end{array} \tag{45}$$

For many heat-transfer systems, separation systems, and reactors, the kinetic and potential energy changes are not important, and there is no shaft work (no mixers, no turbines, no pumps) and the open-system energy balance reduces to

$$\boxed{\Delta \dot{H} = \dot{Q}_{in}} \qquad \begin{array}{c}\text{Open-System Energy Balance}\\ \text{when } \Delta \dot{E}_p,\ \Delta \dot{E}_k,\ W_{s,on} \approx 0\\ \text{(steady state)}\end{array} \tag{46}$$

A way to think about enthalpy, therefore, is as the energy function that changes when heat is added to an open system (mass flows in and out) under the fairly common conditions listed above.

Note that for all the $\Delta-$terms in the open-system balances, Δ refers to out$-$in. Techniques for applying the open-system energy balance are discussed in introductory chemical-engineering textbooks (Felder and Rousseau).

0.1.3 Mechanical Energy Balance (MEB)

The simple form of the open-system macroscopic energy balance discussed above, $\Delta \dot{H} = \dot{Q}_{in}$ (equation 46), is quite common in heat exchangers and reactors, but in the flow of liquids and gasses through conduits, the kinetic energy, potential energy, and shaft work dominate the energy balance. This circumstance is so common, in fact, that a simplified version of the open-system, macroscopic energy balance has been given its own name, the mechanical energy balance, and a simplified form of the mechanical energy balance itself has its own name, the Bernoulli equation. We will discuss these now.

We consider the special case of a single-input, single-output system such as a liquid pushed through a piping system by a pump (Figure 5), and we apply the open-system energy balance.

$$\Delta \dot{E}_k + \Delta \dot{E}_p + \Delta \dot{H} = \dot{Q}_{in} + \dot{W}_{s,on} \tag{47}$$

For such a system there is only a single mass flow rate, $\dot{m}$, and thus all the summations implicit in the Δ terms of the open-system energy balance become simple differences. We will label the outlet as position 2 and the inlet as position 1. We can further substitute

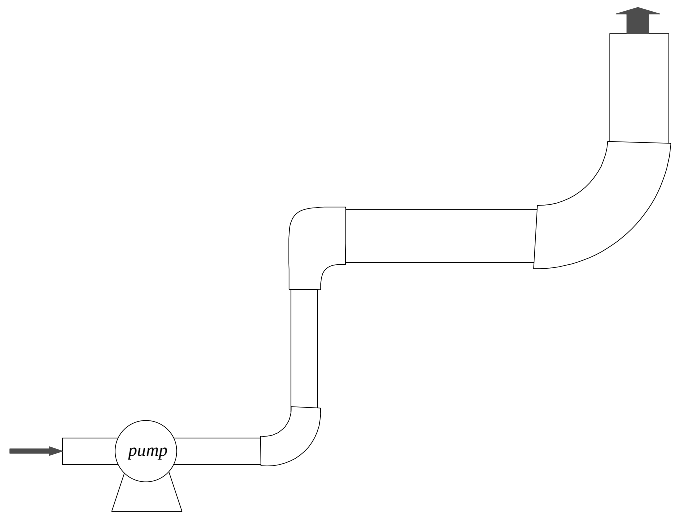

Figure 5: A system that presents itself quite often is one with a single input stream, a single output stream, and in which an incompressible ($1/\hat{V} = \rho =$ constant), non-reacting, nearly isothermal ($\dot{U}$ small) fluid is flowing.

$\hat{E}_k = \dot{E}_k/\dot{m} = v^2/2$ (equation 3) and $\hat{E}_p = \dot{E}_p/\dot{m} = gz$ (equation 4). Each term in the open system energy balance simplifies as shown below.

$$
\begin{aligned}
\Delta \dot{E}_k &\equiv \sum_{out} \dot{m}_j \hat{E}_{k,j} - \sum_{in} \dot{m}_i \hat{E}_{k,i} && (48)\\
&= \dot{m}\hat{E}_{k,2} - \dot{m}\hat{E}_{k,1} && (49)\\
&= \dot{m}\left(\hat{E}_{k,2} - \hat{E}_{k,1}\right) && (50)\\
&= \dot{m}\left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right) && (51)\\
\Delta \dot{E}_p &= \sum_{out} \dot{m}_j \hat{E}_{p,j} - \sum_{in} \dot{m}_i \hat{E}_{p,i} && (52)\\
&= \dot{m}\hat{E}_{p,2} - \dot{m}\hat{E}_{p,1} && (53)\\
&= \dot{m}\left(\hat{E}_{p,2} - \hat{E}_{p,1}\right) && (54)\\
&= \dot{m}g\left(z_2 - z_1\right) && (55)\\
\Delta \dot{H} &= \left(\sum_{out} \dot{m}_j \hat{U}_j - \sum_{in} \dot{m}_i \hat{U}_i\right) + \sum_{j,out} \dot{m}_j p_{j,out} \hat{V}_j - \sum_{i,in} \dot{m}_j p_{i,in} \hat{V}_i && (56)\\
&= \dot{m}\hat{U}_2 - \dot{m}\hat{U}_1 + \dot{m}p_2\hat{V}_2 - \dot{m}p_1\hat{V}_1 && (57)\\
&= \dot{m}\left(\hat{U}_2 - \hat{U}_1 + p_2\hat{V}_2 - p_1\hat{V}_1\right) && (58)
\end{aligned}
$$

$$= \dot{m}\left(\hat{U}_2 - \hat{U}_1 + \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) \tag{59}$$

In the last equation we have used the fact that $\hat{V} = 1/\rho$, where ρ is fluid density. For an incompressible fluid $\rho_1 = \rho_2 = \rho$. We can now use Δ to mean *2 minus 1* and substitute all the results above back into the open-system energy balance and simplify.

$$\Delta\dot{E}_k + \Delta\dot{E}_p + \Delta\dot{H} = \dot{Q}_{in} + \dot{W}_{s,on} \tag{60}$$

$$\dot{m}\left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right) + \dot{m}g\left(z_2 - z_1\right) + \dot{m}\left(\hat{U}_2 - \hat{U}_1 + \frac{p_2}{\rho} - \frac{p_1}{\rho}\right) = \dot{Q}_{in} + \dot{W}_{s,on} \tag{61}$$

$$\left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right) + g\left(z_2 - z_1\right) + \left(\hat{U}_2 - \hat{U}_1\right) + \left(\frac{p_2}{\rho} - \frac{p_1}{\rho}\right) = \frac{\dot{Q}_{in}}{\dot{m}} + \frac{\dot{W}_{s,on}}{\dot{m}} \tag{62}$$

$$\frac{\Delta p}{\rho} + \frac{\Delta\left(v^2\right)}{2} + g\Delta z + \left[\Delta\hat{U} - \frac{\dot{Q}_{in}}{\dot{m}}\right] = \frac{\dot{W}_{s,on}}{\dot{m}} \tag{63}$$

The terms in square brackets are small for the flow of incompressible fluids in pipes since temperature is approximately constant (also no phase or other chemical changes take place and thus $\Delta\hat{U} \approx 0$) and only modest amounts of heat are transferred. We will group these terms together and call them the friction term, F.

$$\boxed{\frac{\Delta p}{\rho} + \frac{\Delta\left(v^2\right)}{2\alpha} + g\Delta z + F = \frac{\dot{W}_{s,on}}{\dot{m}}} \tag{64}$$

Mechanical Energy Balance
(single-input, single-output,
no phase change,
$\Delta T \approx 0$, no reaction)

$\alpha \approx 1$ for turbulent flow (empirical result)
$\alpha = 0.5$ for laminar flow (analytical result)

We have added the constant α to the denominator of the kinetic-energy term of the mechanical energy balance to account for variations in the velocity at different radial positions in the pipe. This effect can be deduced from the study of momentum balances (see Geankoplis). The constant α is approximately equal to one for turbulent flow and is exactly 0.5 for laminar flow.

When the friction term and the shaft work are zero, the mechanical energy balance simplifies still further to a form known as the Bernoulli equation.

$$\boxed{\frac{\Delta p}{\rho} + \frac{\Delta\left(v^2\right)}{2\alpha} + g\Delta z = 0} \tag{65}$$

Bernoulli Equation
(single-input, single-output,
no phase change,
$\Delta T \approx 0$, no reaction
no friction, no shaft work)

The Bernoulli equation is important in the study of hydrodynamics.

The mechanical energy balance gives the relationship between pressure, velocity, elevation, frictional losses, and shaft work for the steady flow of incompressible fluids where there is little heat transfer, no phase changes, no chemical changes, and very little change in temperature. Application of the mechanical energy balance is limited to single-input, single-output systems. Pressure, fluid velocity, and elevation are easily measured in experimental systems, and shaft work is often the quantity to be calculated with the mechanical energy balance. The friction term may sometimes be neglected; when the friction term cannot be neglected, it must be calculated from experimental results, that is from data correlations (see section 0.1.3.2).

Now we will learn to apply the mechanical energy balance.

0.1.3.1 MEB No Friction

We turn first to some examples that make use of the mechanical energy balance with the friction term neglected. We will then turn to the problem of calculating the contribution from fluid friction.

EXAMPLE *What is the work required to pump 6.0 gallons/min of water in the piping network shown in Figure 6? You may neglect the effect of friction.*

SOLUTION When a flow problem involves the amount of shaft work required to bring about a flow, the mechanical energy balance is the first place to start. The Δ-terms in the MEB refer to out $-$ in. We will choose location 2 to be where the fluid exits the pipe, and location 1 will be the liquid free surface in the tank. For both of these locations we know the pressure, the velocity of the fluid (the velocity of fluid at the surface of the tank is nearly zero), and the elevation. This is all the information we need to calculate $\dot{W}_{s,on}$ from the mechanical energy balance.

$$\begin{aligned} \frac{\Delta p}{\rho} + \frac{\Delta\left(v^2\right)}{2\alpha} + g\Delta z + F &= \frac{\dot{W}_{s,on}}{\dot{m}} \\ \frac{p_2 - p_1}{\rho} + \frac{v_2^2 - v_1^2}{2\alpha} + g(z_2 - z_1) + F &= \frac{\dot{W}_{s,on}}{\dot{m}} \end{aligned}$$

At position 1, $p_1 = 1\ atm$, $z_1 = 0$ (position 1 is chosen to be the reference elevation), and $v_1 \approx 0$. At position 2, $p_2 = 1\ atm$, $z_2 = 75 ft$, and the velocity v_2 may be calculated from the volumetric flow rate and the cross sectional area of the pipe. The frictional term F is equal to zero, as given in the problem statement.

$$\dot{V} = \left(\frac{6.0 gal}{min}\right)\left(\frac{1 ft^3}{7.4805 gal}\right)\left(\frac{min}{60 s}\right)$$

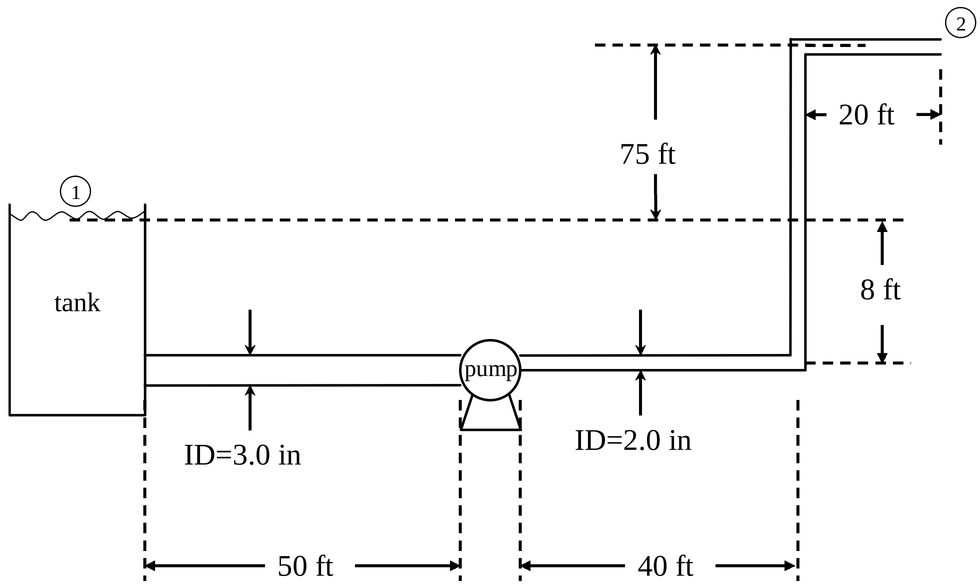

Figure 6: A common problem in engineering involves pumping a fluid from a tank at atmospheric pressure through a piping system. The amount of work required to pump at a chosen flow rate may be calculated with the mechanical energy balance.

$$
\begin{aligned}
&= 0.013368 ft^3/s = \boxed{1.3 \times 10^{-2} ft^3/s} \\
\dot{m} &= \frac{0.013368 ft^3}{s}\left(\frac{62.43 lb_m}{ft^3}\right) \\
&= 0.83456 lb_m/s = \boxed{8.3 \times 10^{-1} lb_m/s} \\
v_2 &= \frac{0.013368 ft^3}{s}\left(\frac{1}{\pi(1.0in)^2}\right)\frac{(12in)^2}{(1ft)^2} \\
&= 0.612748 ft/s = \boxed{6.1 \times 10^{-1} ft/s}
\end{aligned}
$$

The average velocity of the 3-inch inner diameter pipe may be calculated from a mass balance.

$$\begin{aligned}
\begin{pmatrix} \text{mass flow} \\ \text{2-in pipe} \end{pmatrix} &= \begin{pmatrix} \text{mass flow} \\ \text{3-in pipe} \end{pmatrix} \\
\rho v_2 \frac{\pi\left(2in\right)^2}{4} &= \rho v_1 \frac{\pi\left(3in\right)^2}{4} \\
v_1 &= 0.272333 ft/s = \boxed{2.7 \times 10^{-1} ft/s}
\end{aligned}$$

To choose α we need to calculate the Reynolds number, which tells us whether the flow is laminar (Re < 2100) or turbulent (Re > 4000).

$$\begin{aligned}
\mathrm{Re}_{2in\ pipe} &= \left.\frac{\rho v D}{\mu}\right|_{2in\ pipe} = \frac{\frac{62.43 lb_m}{ft^3}\frac{0.612748 ft}{s}\frac{2in}{12in/ft}}{0.8937 cp \frac{6.7197\times10^{-4} lb_m}{ft\cdot s\cdot cp}} \\
&= 10,617 = 1.1 \times 10^4 \\
\mathrm{Re}_{3in\ pipe} &= \left.\frac{\rho v D}{\mu}\right|_{3in\ pipe} = \frac{\frac{62.43 lb_m}{ft^3}\frac{0.272333 ft}{s}\frac{3in}{12in/ft}}{0.8937 cp \frac{6.7197\times10^{-4} lb_m}{ft\cdot s\cdot cp}} \\
&= 7078 = 7.1 \times 10^3
\end{aligned}$$

From the values of Re we can conclude that the flow in both pipe sections is turbulent, and therefore $\alpha = 1$ for our calculations. Now we can assemble the mechanical energy balance and calculate the shaft work.

$$\left[\frac{1-1}{\rho} + \frac{(0.612748 ft/s)^2 - 0^2}{2(1)} + \frac{32.174 ft}{s^2}(75ft - 0ft) + 0\right]\frac{s^2 \cdot lb_f}{32.174 ft \cdot lb_m} = \frac{\dot{W}_{s,on}}{0.83456 lb_m/s}$$

$$\begin{aligned}
\dot{W}_{s,on} &= (5.83484 \times 10^{-3} + 75)(0.83456) \\
&= 62.59687 ft \cdot lb_f/s \left(\frac{1.341 \times 10^{-3} hp}{0.7376 ft \cdot lb_f/s}\right) \\
&= 0.1138046 hp = \boxed{1.1 \times 10^{-1} hp}
\end{aligned}$$

Note that the kinetic-energy contribution (5.8×10^{-3}) is very small compared to the potential energy contribution (75). Note also that significant figures should be considered when reporting values for $\dot{W}_{s,on}$, $\dot{V}$, $\dot{m}$, and v_2 (e.g. $v_2 = 6.1 \times 10^{-1} ft/s$), but when the numbers are needed in carrying forward the calculation, the complete number (all digits) should be used in order to minimize round-off error (e.g. $v_2 = 0.612748 ft/s$).

EXAMPLE *What is the relationship between measured pressure drop and flow rate through a Venturi meter?*

SOLUTION A Venturi meter is a device that allows for the measurement of flow rate in incompressible liquid flow in pipes (Figure 7). The design of a

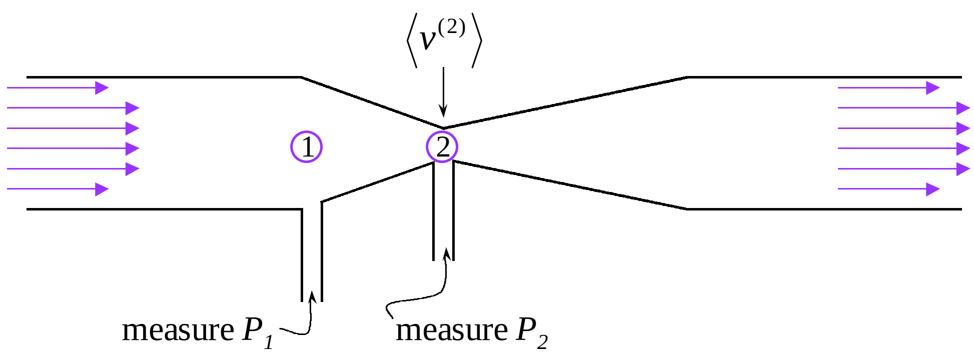

Figure 7: Venturi meters take up a great deal of space, but they do allow for an accurate measurement of flow rate without greatly disturbing the flow. Flow is directed through a gently tapering tube. Pressure is measured before the contraction (1) and at the point of smallest diameter (the throat, 2). The relationship between the measured pressures and the fluid velocity may be deduced from the mechanical energy balance (for systems where friction may be neglected) or from the mechanical energy balance and a calibration specific to the device (if friction effects are to be taken into account).

Venturi meter is of a converging section of pipe followed by a diverging section; the changes in cross-section are gradual in order to minimize the frictional losses within the device. We begin our analysis with the mechanical energy balance, and we will neglect the frictional contribution at first.

$$\frac{\Delta p}{\rho} + \frac{\Delta\left(v^2\right)}{2\alpha} + g\Delta z + F = \frac{\dot{W}_{s,on}}{\dot{m}}$$

Point 1 will be chosen as the point of the upstream pressure measurement, and point 2 will be at the throat, the location of the other pressure measurement. There are no moving parts, and therefore $\dot{W}_{s,on} = 0$. As stated above, we will neglect friction and thus $F = 0$. Venturi meters are installed horizontally, and thus $z_1 - z_2 = 0$. The mechanical energy balance simplifies in this case to

$$\frac{p_2 - p_1}{\rho} + \frac{v_2^2 - v_1^2}{2\alpha} = 0$$

We can relate v_1 and v_2 through the mass balance between point 1 and point 2. We are considering the steady flow of an incompressible liquid where the density is constant $\rho_1 = \rho_2 = \rho$.

$$\begin{aligned}
\begin{pmatrix} \text{Mass flow at} \\ \text{point 1} \end{pmatrix} &= \begin{pmatrix} \text{Mass flow at} \\ \text{point 2} \end{pmatrix} \\
\left(\frac{\text{mass}}{\text{volume}}\right)_1 \left(\frac{\text{volume}}{\text{time}}\right)_1 &= \left(\frac{\text{mass}}{\text{volume}}\right)_2 \left(\frac{\text{volume}}{\text{time}}\right)_2 \\
\rho v_1 \pi \frac{D_1^2}{4} &= \rho v_2 \pi \frac{D_2^2}{4} \\
v_1 D_1^2 &= v_2 D_2^2 \\
v_1 &= \left(\frac{D_2}{D_1}\right)^2 v_2
\end{aligned}$$

Substituting this result back into the simplified mechanical energy balance, we obtain the final relationship between the flow rate ($\dot{V} = v_2 \pi D_2^2/4$) and the measured pressure drop ($p_1 - p_2$).

$$\begin{aligned}
\frac{p_2 - p_1}{\rho} + \frac{v_2^2 - v_1^2}{2\alpha} &= 0 \\
\frac{p_2 - p_1}{\rho} + \frac{1}{2\alpha}\left[v_2^2 - \left(\frac{D_2}{D_1}\right)^4 v_2^2\right] &= 0
\end{aligned}$$

$$\begin{aligned}
v_2 &= \sqrt{\frac{\frac{2\alpha(p_1 - p_2)}{\rho}}{\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}} \\
\dot{V} &= v_2 \pi D_2^2 / 4
\end{aligned}$$

$$\boxed{\dot{V} = \frac{\pi D_2^2}{4}\sqrt{\frac{\frac{2\alpha(p_1 - p_2)}{\rho}}{\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}}} \qquad \text{Flow Rate measured by a Venturi Meter (no friction)} \qquad (66)$$

For many Venturi meters when the flow is sufficiently rapid ($\mathrm{Re} > 10^4$) (Geankoplis) this no-friction relationship describes the pressure-drop/flow-rate relationship well. For slower flows, friction is more important to the total energy, and calibration should be performed to determine an empirical friction correction factor C_v:

$$\boxed{\dot{V} = C_v \left(\frac{\pi D_2^2}{4}\right)\sqrt{\frac{\frac{2\alpha(p_1-p_2)}{\rho}}{\left[1-\left(\frac{D_2}{D_1}\right)^4\right]}}} \qquad \begin{array}{c}\text{Flow Rate}\\ \text{measured by a}\\ \text{Venturi Meter}\\ \text{(with friction)}\end{array} \tag{67}$$

0.1.3.2 MEB With Friction

Sometimes the friction term makes an important contribution to the mechanical energy balance. This is true when there are changes in pipe diameter, twists and turns in the pipe, flow obstructions such as an orifice plate, or when there are very long runs of piping.

$$\frac{\Delta p}{\rho} + \frac{\Delta\left(v^2\right)}{2\alpha} + g\Delta z + F = \frac{\dot{W}_{s,on}}{\dot{m}} \tag{68}$$

When friction is important, F must be determined experimentally, much as C_v for Venturi meters is determined experimentally as discussed in the last example. The mechanical energy balance would not be very useful, however, if we had to first build every apparatus of interest to us and do experiments on them in order to know the relationships between pressure, velocity, elevation, friction, and work. We had sought to use the mechanical energy balance to predict the relationships between these variables for systems that are not yet built. We may wish to calculate shaft work of a pump, for example, in a hypothetical flow loop, or we may wish to predict flow rate achieved when a pump of a certain horsepower works on a given flow loop.

The solution to this dilemma is to draw on the experiments of prior researchers in order to estimate the friction for the systems that interest us. If someone has built a flow system just like the one we would like to build, then we can use data on the performance of that other system to understand our system.

What if we have data on a flow system that is *somewhat* similar to the system that interests us but is not exactly the same? Can we use that data? The answer to this is, maybe.

The resolution to the dilemma of how to compare similar, but not identical systems, is found through dimensional analysis. Dimensional analysis is based on the correct observation that the laws of physics (mass conservation, energy conservation, momentum conservation) apply to all systems, simple and complex. For simple systems we can apply the techniques of engineering analysis to calculate whatever quantities interest us. For complex systems this is not always possible, but we do know that the laws of physics apply. From dimensional analysis on the laws of physics, we can deduce how quantities that interest us (such as wall friction in the current case or heat-transfer coefficient in an energy-balance case) vary with certain quantities identified with a given system. We (or others) can then do targeted experiments and publish data correlations that can be used by engineers to calculate quantities of interest on similar systems.

The details of the dimensional analysis process may be found elsewhere (Geankoplis). For mechanical energy balance problems the useful results from dimensional analysis are data correlations for frictional losses in straight pipes, valves, fittings, and other devices. For liquid flows in straight pipes, the frictional losses are correlated in terms of the Fanning friction factor f as a function of the Reynolds number, Re. The Fanning friction factor is defined as a dimensionless wall force in straight tubes, and its relationship to pressure drops, flow rates, and geometric factors may be understood by considering the mechanical energy balance applied to a straight section of pipe.

EXAMPLE *For a Newtonian fluid, what is the friction term F in the mechanical energy balance for steady flow in a tube?*

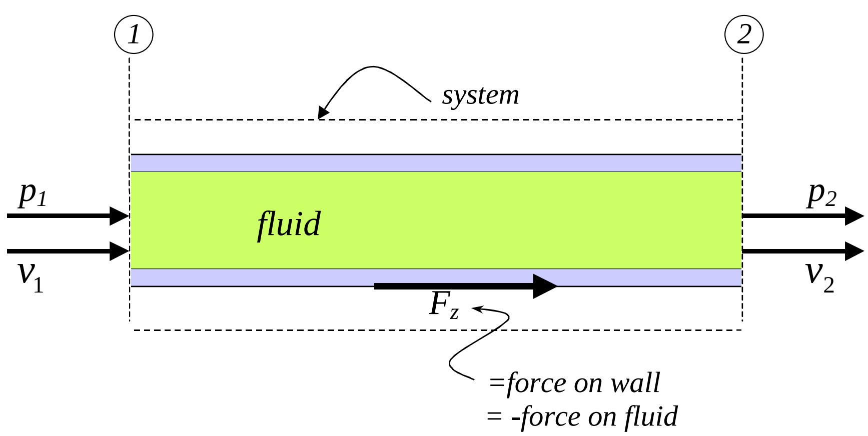

Figure 8: A mechanical energy balance on a straight pipe section yields the expression for the frictional losses in a straight pipe.

SOLUTION We begin with the mechanical energy balance.

$$\frac{\Delta p}{\rho} + \frac{\Delta\left(v^2\right)}{2\alpha} + g\Delta z + F \quad = \quad \frac{\dot{W}_{s,on}}{\dot{m}}$$

We choose as our two points a point upstream where the pressure p_1 is known and a point downstream where the pressure p_2 is known. There is no pump or moving parts in our chosen system, which means $\dot{W}_{s,on} = 0$. The pipe will have a constant flow rate and a constant cross-sectional area; therefore $v_2^2 - v_1^2 = 0$. The pipe will be chosen to be horizontal, and therefore $z_2 - z_1 = 0$. The mechanical energy balance becomes

$$\frac{p_2 - p_1}{\rho} + F \quad = \quad 0$$

The frictional term is therefore found to be

$$\boxed{F_{\text{Straight Pipe}} = \frac{p_1 - p_2}{\rho}} \qquad \begin{array}{c}\text{Friction}\\ \text{in Steady Flow}\\ \text{in Pipes}\end{array} \tag{69}$$

Thus, data can be taken of pressure drop for a variety of flow rates and tube geometries (length, diameter) and for a variety of fluids (with different densities ρ and viscosities μ), and the data could be tabulated and published.

Dimensional analysis can make the collection and reporting of all this pressure-drop-flow-rate data more rational and accessible. From dimensional analysis (see Geankoplis) we find that a useful defined quantity is the Fanning friction factor, f, a dimensionless wall force, which may be used to correlate friction in pipe flows with Reynolds number, a dimensionless flow rate. The Fanning friction factor f is defined as

$$\begin{aligned} f \quad &\equiv \quad \frac{\text{Wall Force}}{(\text{Area})\,(\text{Kinetic Energy})} \\ &= \quad \frac{(p_1 - p_2)\pi R^2}{(2\pi R L)\left[\frac{1}{2}\rho\left(v_{av}\right)^2\right]} \end{aligned}$$

The relation wall force $= (p_1 - p_2)\pi R^2$ was obtained from a momentum balance on the straight pipe system (Geankoplis). Simplifying we obtain,

$$f = \frac{(p_1 - p_2)D}{2L\rho\left(v_{av}\right)^2} \qquad \boxed{\begin{array}{c}\text{Fanning Friction}\\ \text{Factor}\end{array}} \tag{70}$$

Dimensional analysis tells us that for steady flow of a Newtonian fluid in a tube, the Fanning friction factor is only a function of the dimensionless quantity

$\rho v_{av} D/\mu$, which is called the Reynolds number.

$$\boxed{f = f(\text{Re}); \qquad \text{Re} \equiv \frac{\rho v_{av} D}{\mu}} \qquad \begin{array}{c}\text{Dimensional Analysis} \\ \text{Result for Pipe Flow}\end{array} \tag{71}$$

We can therefore calculate the friction term F in the mechanical energy balance for the flow of any fluid in any tube by consulting the data correlations $f(\text{Re})$ that are in the literature and using equations 69 and 70.

$$\boxed{F_{\text{Straight Pipe}} = \frac{p_1 - p_2}{\rho} = \frac{2fL\,(v_{av})^2}{D}} \qquad \begin{array}{c}\text{Friction} \\ \text{in Steady Flow} \\ \text{in Pipes}\end{array} \tag{72}$$

For any flow in a tube, we must calculate the Reynolds number (we need ρ, v_{av}, D, and μ) from which we can get f (from the data correlation in the literature). This in turn can be combined with the other quantities in equation 72 to give us the friction term F from the mechanical energy balance.

The data correlations for f are well established. For laminar flow we can use direct computation to determine f as a function of Reynolds number as we will see below. For turbulent flow the correlations come from careful experiments (see equation 76).

EXAMPLE *What is the relationship between the Fanning friction factor f and the Reynolds number* Re *for steady, laminar flow in a tube?*

SOLUTION As with general flows, the correlation between f and Re for laminar flow in a tube could be determined experimentally. Since laminar flow is a simple flow, however, we can use the techniques of the microscopic momentum balance to derive a relationship between pressure drop and flow rate for this special case. The result for Newtonian fluids is called the Hagen-Poiseuille equation (see Geankoplis).

$$\boxed{p_1 - p_2 = \frac{128 Q \mu L}{\pi D^4} = \frac{32 \mu L v_{av}}{D^2}} \qquad \begin{array}{c}\text{Hagen-Poiseuille equation} \\ \text{(Laminar flow in a tube)}\end{array} \tag{73}$$

where $Q = v_{av} \pi D^2/4$ is the flow rate in the tube, v_{av} is the average fluid velocity, μ is the viscosity of the fluid, L is the length of pipe between points 1 and 2, and D is the diameter of the tube.

Because we have the Hagen-Poiseuille equation, the Fanning friction factor f for the special case of laminar flow in a tube can be calculated directly, and no experiments are needed (except to verify the modeling assumptions).

$$\begin{aligned} f &= (p_1 - p_2)\frac{D}{2L\rho\left(v_{av}\right)^2} \\ &= \left[\frac{32\mu L v_{av}}{D^2}\right]\frac{D}{2L\left(v_{av}\right)^2\rho} \\ &= \frac{16\mu}{\rho v_{av} D} = \frac{16}{\mathrm{Re}} \end{aligned}$$

$$\boxed{f_{\text{Laminar Flow}} = \frac{16}{\mathrm{Re}}} \qquad \begin{array}{c}\text{Fanning friction Factor}\\ \text{in Steady Laminar Flow}\\ \text{in Pipes}\end{array} \qquad (74)$$

The previous two examples show how to calculate the contribution of friction in straight pipes to the friction term F in the mechanical energy balance; for both laminar and turbulent flow F is given by

$$\boxed{F_{\text{Straight Pipe}} = \frac{p_1 - p_2}{\rho} = \frac{2fL\left(v_{av}\right)^2}{D}} \qquad \begin{array}{c}\text{Friction}\\ \text{in Steady Flow}\\ \text{in Pipes}\end{array} \qquad (75)$$

For laminar flow $f = 16/\mathrm{Re}$ and for turbulent flow correlations from the literature supply f. A useful empirical equation for turbulent flow is the Colebrook formula (Denn), which gives f as a function of Reynolds number and k/D, a surface roughness parameter relevant for commercial pipe.

$$\boxed{\frac{1}{\sqrt{f}} = -4.0\log\left(\frac{k}{D} + \frac{4.67}{\mathrm{Re}\sqrt{f}}\right) + 2.28} \qquad \text{Colebrook Formula} \qquad (76)$$

Values of k for various materials are given in Table 1, and the correlation is plotted in Figure 9. Experiments show that laminar flow takes place in straight pipes with a circular crosssection for $\mathrm{Re} < 2100$, and fully turbulent flow occurs for $\mathrm{Re} > 4000$. In between 2100 and 4000 the flow is called transitional flow, which is neither stable laminar nor fully turbulent flow.

material	k (mm)
Drawn tubing (brass, lead, glass, etc.)	1.5×10^{-3}
Commercial steel or wrought iron	0.05
Asphalted cast iron	0.12
Galvanized iron	0.15
Cast iron	0.46
Wood stave	$0.2 - 0.9$
Concrete	$0.3 - 3$
Riveted steel	$0.9 - 9$

Table 1: Surface roughness for various materials; from Denn.

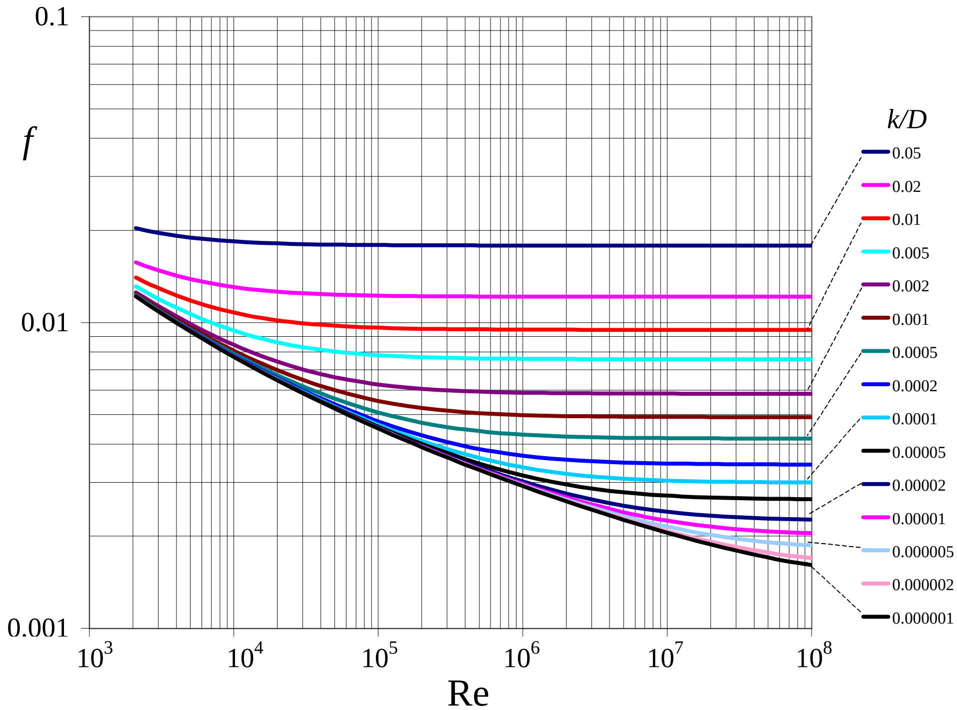

Figure 9: Fanning friction factor versus Reynolds number from the Colebrook formula, equation 76. For Re < 2100, $f = 16/\text{Re}$.

In addition to wall drag in straight pipes, there are many other sources of friction in piping systems: valves, fittings, pumps, expansions, and contractions are all sources of friction. To quantify the friction in these devices we use the same procedure as we used to deduce the result used for straight pipes: we apply the mechanical energy balance to the valve, fitting, or other friction-generating segment of the piping system, then we simplify the resulting equation by using mass and momentum balances as appropriate, and finally we use dimensional analysis to guide experiments to find the appropriate correlations. For valves, fittings, expansions and contractions the data correlations that result from such analyses may be written in the following form:

$$\boxed{F_i = K_i \frac{(v_{av})^2}{2}} \quad \text{Friction from Fittings} \tag{77}$$

The friction coefficients K_i are different for every type of valve, fitting, etc. and some values of K_i may be found in Tables 2 and 3. The friction F for a complete piping system will be equal to the friction due to the straight pipes plus the friction due to each of the valves, fittings, expansions, and contractions that are present in the flow loop.

$$\boxed{F_{\text{Flow Loop}} = \sum_{j,straight\ pipe\ segments} 4f_j \frac{L_j}{D_j}\frac{v_j^2}{2} + \sum_{i,fittings} K_i \frac{v_i^2}{2}} \quad \text{Friction in a Flow Loop} \tag{78}$$

In these correlations note that there is no α in the denominator of the velocity-squared expressions; instead there are different values of K_i depending on whether the flow is laminar or turbulent. The v_i to be used in expansions and contractions is the faster average velocity (the upstream velocity for an expansion and the downstream velocity for a contraction). The v_j to be used in the summation over the straight-pipe segments is the average velocity in the straight pipe, which will be different for different values of D_i, the diameter of the pipe.

The values of K_i for expansions and contractions are given Tables 2 and 3; note these are slightly different from those given in Geankoplis in that equations 79 and 80 include the α in the equations for the K_i.

$$K_{exp} = \frac{1}{\alpha}\left(1 - \frac{A_1}{A_2}\right)^2 \tag{79}$$

$$K_{cont} = \frac{0.55}{\alpha}\left(1 - \frac{A_2}{A_1}\right) \tag{80}$$

where A_1 is the upstream cross-sectional area and A_2 is the downstream cross-sectional area. The values for K_i for other fittings are also given in Table 2.

With the development of equation 78 for the friction term in the mechanical energy balance, we are now ready to do a mechanical energy balance calculation with friction.

fitting, i	**Friction-Loss Factor,** K_i
Elbow, 45^o	0.35
Elbow, 90^o	0.75
Tee	1
Return bend	1.5
Coupling	0.04
Union	0.04
Gate valve, wide open	0.17
Gate valve, half open	0.45
Globe valve, wide open	6.0
Globe valve, half open	9.5
Check valve, ball	70.0
Check valve, swing	2.0
Water meter, disk	7.0
Expansion from A_1 to A_2	$\frac{1}{\alpha}\left(1-\frac{A_1}{A_2}\right)^2$
Contraction from A_1 to A_2	$\frac{0.55}{\alpha}\left(1-\frac{A_2}{A_1}\right)$

Table 2: Friction-loss factors for turbulent flow ($\alpha = 1$) through valves, fittings, expansions and contractions (Geankoplis). The expressions for expansions and contractions may also be used for laminar flow, for which $\alpha = 0.5$.

fitting, i	$\mathrm{Re}_i = 50$	100	200	400	1000	Turbulent
Elbow, 90^o	17	7	2.5	1.2	0.85	0.75
Tee	9	4.8	3.0	2.0	1.4	1.0
Globe valve	28	22	17	14	10	6.0
Check valve, swing	55	17	9	5.8	3.2	2.0

Table 3: Friction-loss factors K_i for laminar flow through valves, fittings, expansions and contractions (Geankoplis).

EXAMPLE *What is the work required to pump 6.0 gallons/min of water in the piping network shown in Figure 6? You must take into account the effect of friction. The piping may be considered to be smooth pipe.*

SOLUTION The solution is the same as it was in the previous case except that we must calculate the frictional contribution F.

$$F = \sum_{j,straight\ pipe\ segments} 4f_j \frac{L_j}{D_j}\frac{v_j^2}{2} + \sum_{i,fittings} K_i \frac{v_i^2}{2}$$

We have two types of straight-pipe segments, one type that is 50 ft long with inner diameter of 3 inches, and one type that is a total of $40+8+75+20 = 143$ ft long with inner diameter of 2 inches. The average velocities in the pipes were calculated in the previous example to be

$$\begin{aligned} v_{2in\ pipe} &= 0.612748 ft/s \\ v_{3in\ pipe} &= 0.272333 ft/s \end{aligned}$$

The Fanning friction factor f for each of the two types of straight-pipe segments may be different. Fanning friction factor is a function of Reynolds number and may be obtained from the appropriate correlations (i.e. $f = 16/\mathrm{Re}$ for laminar flow and the Colebrook formula for turbulent flow). We previously calculated the Reynolds numbers for the two pipe sizes.

$$\begin{aligned} \mathrm{Re}_{2in\ pipe} &= \left.\frac{\rho v D}{\mu}\right|_{2in\ pipe} = 10,617 = 1.1 \times 10^4 \\ \mathrm{Re}_{3in\ pipe} &= \left.\frac{\rho v D}{\mu}\right|_{3in\ pipe} = 7078 = 7.1 \times 10^3 \end{aligned}$$

The flow is everywhere turbulent ($\mathrm{Re} > 4000$). The Fanning friction factors are then found from the Colebrook formula to be $f = 0.007603$ for the 2-inch pipe and $f = 0.00848$ for the 3-inch pipe.

The fittings for our flow loop are two 90^o elbows, and two contractions, one from the tank to the inlet of the 3-in pipe and one just upstream of the pump. For the contraction from the tank to the 3-in pipe the velocity to use is the velocity in the 3-in pipe (the larger velocity). For the contraction to 2-in and for the two elbows, the velocity to use is the velocity in the 2-in pipe. For the fittings in our system, the friction-loss factors K_i obtained from Table 2 are listed below.

fitting	K_i
Contraction (tank to 3-in pipe, $A_1/A_2 = \infty$)	0.55
Contraction (3-in to 2-in), $A_2/A_1 = 4/9$	0.305556
90^o elbow	0.75

We can now calculate the friction contribution to the mechanical energy balance for this system.

$$\begin{aligned}
F &= \sum_{j,straight\ pipe\ segments} 4f_j \frac{L_j}{D_j}\frac{v_j^2}{2} + \sum_{i,fittings} K_i \frac{v_i^2}{2} \\
&= (4)(0.00848)\left(\frac{50ft}{3in}\frac{12in}{ft}\right)\frac{(0.272333)^2}{2} \\
&\quad + (4)(0.007603)\left(\frac{143ft}{2in}\frac{12in}{ft}\right)\frac{(0.612748ft/s)^2}{2} \\
&\quad + 0.55\frac{(0.272333)^2}{2} \\
&\quad + (0.305556 + (2)0.75)\frac{(0.612748ft/s)^2}{2} \\
&= 5.50946\frac{ft^2}{s^2}
\end{aligned}$$

We can now combine this with equation 66 from the previous example to arrive at the final value for the shaft work.

$$\begin{aligned}
\frac{\dot{W}_{s,on}}{0.83456lb_m/s} &= \left[\frac{1-1}{\rho} + \frac{(0.612748ft/s)^2 - 0^2}{2} + \frac{32.174ft}{s^2}(75ft - 0ft)\right. \\
&\quad \left. + 5.50946\frac{ft^2}{s^2}\right]\frac{s^2 \cdot lb_f}{32.174ft \cdot lb_m} \\
\dot{W}_{s,on} &= 62.73978ft \cdot lb_f/s\left(\frac{1.341 \times 10^{-3}hp}{0.7376ft \cdot lb_f/s}\right) = 0.1140646
\end{aligned}$$

$$\boxed{\dot{W}_{s,on} = 1.1 \quad hp}$$

Note that the result calculated in the last example was the same, within two significant figures, as the calculation without friction. If we examine the contributions to the shaft work, we see that in this flowloop, the 75 ft elevation rise (potential energy) dominates the kinetic energy change and the frictional losses. If we convert the kinetic energy and frictional contributions into equivalent feet of elevation change, we can begin to build an intuition about how these various types of energy contribute to the load on the pump. We can do this by factoring out the acceleration due to gravity in the MEB calculations done in the last example, as we show below.

$$\frac{\dot{W}_{s,on}}{0.83456lb_m/s} = \left[\frac{1-1}{\rho} + \frac{(0.612748ft/s)^2 - 0^2}{2} + \frac{32.174ft}{s^2}(75ft - 0ft)\right.$$

$$
\begin{aligned}
& \left. +5.50946\frac{ft^2}{s^2}\right] \frac{s^2 \cdot lb_f}{32.174 ft \cdot lb_m} \\
&= \left[\frac{(0.612748 ft/s)^2}{(2)(32.174 ft/s^2)} + 75 ft + \frac{(5.50946 ft^2/s^2)}{(32.174 ft/s^2)}\right] \frac{32.174 ft}{s^2} \frac{s^2 \cdot lb_f}{32.174 ft \cdot lb_m} \\
\dot{W}_{s,on} &= \left[\frac{(0.612748)^2}{(2)(32.174)} ft + 75 ft + \frac{5.50946}{32.174} ft\right] 0.83456 \frac{lb_f}{s} \\
\dot{W}_{s,on} &= \left[5.8348 \times 10^{-3} ft + 75 ft + 0.171240 ft\right] 0.83456 \frac{lb_f}{s}
\end{aligned}
$$

When we write the kinetic energy, potential energy, and friction terms all in the same units (ft of elevation or ft of *head*, as it is called) we can easily compare the magnitudes of the terms and, conveniently, compare them in units in which we have some intuition, that is, the energy stored in raising the fluid by one foot of elevation. Looking at the contributions in terms of fluid head, we see that the kinetic energy makes the smallest contribution at $5.8 \times 10^{-3} ft$, but the friction head, $0.17 ft$, while much larger than the velocity contribution, is nearly as negligible compared to the substantial elevation head of $75 ft$. Engineers have often found the concept of head to be quite useful in calculations of this sort.

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