Macroscopic Momentum Balance

$$\beta_{laminar} = 0.75$$
$$\beta_{turbulent} \sim 1$$

$\underline{\text{P}}$ = ***fluid*** momentum

$$\frac{d\underline{\text{P}}}{dt} + \sum_{i=1}^{\#streams} \left[\frac{\rho A\cos\theta\langle v\rangle^2}{\beta}\hat{v}\right]_{A_i} = \sum_{i=1}^{\#streams} [-pA\hat{n}]_{A_i} + \underline{R} + M_{CV}\underline{g}$$

$$\begin{pmatrix} \frac{d\text{P}_x}{dt} \\ \frac{d\text{P}_y}{dt} \\ \frac{d\text{P}_z}{dt} \end{pmatrix}_{xyz} + \sum_{i=1}^{\#streams} \left[\frac{\rho A\cos\theta\langle v\rangle^2}{\beta}\begin{pmatrix} \hat{v}_x \\ \hat{v}_y \\ \hat{v}_z \end{pmatrix}_{xyz}\right]_{A_i}$$

$$= \sum_{i=1}^{\#streams} \left[-pA\begin{pmatrix} \hat{n}_x \\ \hat{n}_y \\ \hat{n}_z \end{pmatrix}_{xyz}\right]_{A_i} + \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}_{xyz} + M_{CV}\begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix}_{xyz}$$

$\underline{R}$ = net force ***on fluid*** due to walls
M_{CV} = mass of control volume
$\hat{n}$ = outwardly pointing unit normal

21Oct2015

See inside front cover of Morrison, 2013