

with the pressure:

$$\frac{\partial p}{\partial z} - \rho g = \frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) \quad (1.367)$$

The lefthand side is only a function of  $z$  and the righthand side is only a function of  $r$ . We have succeeded in separating the two variables,  $r$  and  $z$ . Thus, both sides must be equal to the same constant, which we call  $\lambda$  [58]:

$$\frac{\partial p}{\partial z} - \rho g = \frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) = \lambda \quad (1.368)$$

We separated the  $z$  and  $r$  parts of Equation 1.367 into two independent equations that we can solve directly:

$$\frac{\partial p}{\partial z} - \rho g = \lambda \quad (1.369)$$

$$\frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) = \lambda \quad (1.370)$$

Because Equations 1.369 and 1.370 are now ODEs, we change the differentiation from partial differentiation  $\partial/\partial r$ ,  $\partial/\partial z$  to total differentiation  $d/dr$ ,  $d/dz$ . The remaining steps are straightforward:

$$\text{Pressure ODE: } \frac{dp}{dz} - \rho g = \lambda \quad (1.371)$$

$$\frac{dp}{dz} = (\lambda + \rho g) \quad (1.372)$$

$$\int dp = \int (\lambda + \rho g) dz \quad (1.373)$$

$$p = (\lambda + \rho g)z + C_3 \quad (1.374)$$

where  $C_3$  is an integration constant.

$$\text{z-velocity ODE: } \frac{\mu}{r} \frac{d}{dr} \left( r \frac{dv_z}{dr} \right) = \lambda \quad (1.375)$$

$$\frac{d}{dr} \left( r \frac{dv_z}{dr} \right) = \left( \frac{\lambda}{\mu} \right) r \quad (1.376)$$

The solution of Equation 1.376 is discussed in the previous example (compare to Equation 1.356).

## 1.4 Problems

1. Create a list of five real engineering problems or societal challenges that can be addressed with the modeling introduced in this chapter and studied in fluid mechanics.



2. The green hose fills a swimming pool in 4 hours, the red hose fills the same pool in 6 hours, and the yellow hose fills it in 8 hours. With all three hoses running at those rates, how long will it take to fill the pool?
3. What is a typical volumetric flow rate (in gpm and lpm (liters per minute) for household plumbing? What is a typical value of average velocity in a pipe? Assume half-inch type-K copper tubing (see *Perry's Chemical Engineering Handbook* [132] for dimensions).
4. Compare typical values of velocity head, pressure head, elevation head, and friction head. What is a good rule of thumb for velocity differences that are significant in the flow of household water? Assume that the relevant piping is half-inch type K copper tubing (see *Perry's Chemical Engineering Handbook* [132] for dimensions).
5. What are the viscosity and density of glycerin at room temperature? A useful reference for physical-property data is *Perry's Chemical Engineering Handbook* [132].
6. How do the viscosity of sugar-water solutions vary with concentration and temperature? (Find the answer in the literature.) Provide a plot that shows how the data vary; consider carefully how to plot the data so that the trend is displayed meaningfully.
7. Examine the friction factor/Reynolds number relationship for turbulent flow in pipes (see Figure 1.21). Calculate the pressure drop versus the flow rate for turbulent flow in a rough pipe in an existing apparatus at a chemical plant. List the information needed about the pipe to make the calculation. Which factors are the most critical?
8. For household water in steady flow in a half-inch Schedule 40 horizontal pipe at 3.0 gpm (see Figure 1.20), what are the frictional losses over a 100-foot run of pipe? The flow may be laminar or turbulent. (This problem was proposed originally as Example 1.8; on completion of this chapter, we now can solve it.)
9. What is the range of the friction factor for turbulent flow in smooth and rough pipes? What is the range of the friction factor for laminar flow?
10. Water at 25°C flows at  $6.3 \times 10^{-3} \text{ m}^3/\text{s}$  through the irregularly shaped container in Figure 1.48. What is the average fluid velocity at the exit? The apparatus is open to the atmosphere at the entrance and the exit.
11. At a Reynolds number of 10,000, flow in a pipe is turbulent and it is not possible to produce a laminar flow. What is the friction factor for a flow in smooth pipe at this Reynolds number? If somehow we could produce a laminar flow at this Reynolds number, what would the friction factor be? Repeat for  $\text{Re} = 10^5$ . Compare the two answers and discuss.
12. *Piping* and *tubing* are names for conduits of fluids, but the two terms differ in that the outer diameter (OD) of piping is standardized to allow pipefitters to mount pipes into standard-size holders. The tubing OD is not standardized. What are the ID and OD of nominal 1/2-inch, 3/4-inch, and 1-inch Schedule 40 pipes? Give dimensions in both inches and mm. What are the closest metric standard pipe sizes to these three sizes? Search for these answers in the literature.



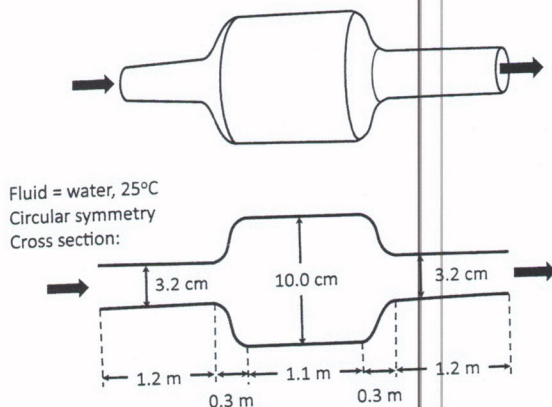


Figure 1.48

Flow through an irregular container (Problem 10).

13. Piping is rated by its nominal size—for example, 1/2-inch or 3/8-inch pipe—but the true ID is not the same as the nominal size. For water flowing in 1/2-inch, Schedule 40 PVC (smooth) pipe at 3.0 gpm, calculate the average velocity and the Reynolds number using the correct, true ID of the pipe. Calculate the average velocity and Reynolds number using 0.5 inch (i.e., the nominal size) as the diameter. Calculate the friction factor based on these two numbers (e.g., using the Colebrook equation or Equation 7.158). Calculate the predicted pressure drop per unit length  $\Delta p/L$  in the two cases. How much error in pressure drop is generated for 100 feet of pipe when the wrong diameter is used?
14. Glycerin at room temperature is made to flow through a pipe (the ID is 1.2 mm) at a Reynolds number of  $1.00 \times 10^2$ . What is the average velocity of the glycerin? What is the average velocity if the fluid is water instead? Which flow generates more friction? Be quantitative in your answer and explain.
15. A 30-gallon bathtub takes about 8.0 minutes to fill. What is the flow rate of water in the pipes (1/2-inch type-K copper tubing) in gpm? What is the flow rate in  $\text{cm}^3/\text{s}$ ?
16. Water (25°C) flows through 1-inch Schedule 40 steel pipe at 2.0 gpm. What is the Reynolds number of the flow? What is the friction factor? Is the flow laminar or turbulent?
17. Water (25°C) flows through 1-1/2-inch Schedule 40 pipe at 2.0 gpm. What is the pressure drop along 5,000 feet of smooth pipe? If the pipe is not smooth but rather commercial steel, what is the pressure drop?
18. Room temperature water comes out of a spigot at 3.0 gpm. How long would it take to fill a 5-gallon bucket?
19. Water at room temperature comes out of a spigot at the maximum speed possible for the flow to still be laminar. What is the flow rate in gpm and in liters/minute? The flow line is 1/2-inch, Schedule 40 smooth pipe.
20. Water (25°C) flows through DN40 (metric pipe size) Schedule 40 smooth pipe at 8.0 liters/minute. What is the pressure drop along 1,500 meters of pipe? If the flow rate doubles to 16 liters/minute, what is the pressure drop?



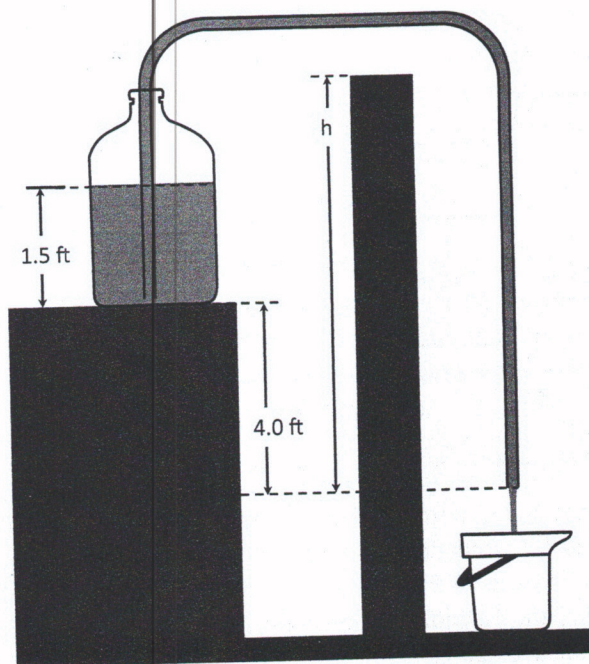


Figure 1.49

Schematic for Problem 23.

21. A Venturi meter with a 1.00-inch diameter throat is to be installed in a 2-inch line (i.e., Schedule 40 piping, smooth) with water flowing at 25°C. If the flow is turbulent and the range of expected flow rates is 0–200 gpm, what is the expected range of pressure drop in the Venturi meter? You may neglect frictional losses.
22. A Venturi meter with a 4.00 mm ID throat is installed in a 25DN line (metric pipe size, Schedule 40, piping, smooth) with water flowing at 25°C. If the flow is turbulent and the maximum flow rate is 40.0 liter/min, what is the pressure drop in the Venturi meter? You may neglect friction.
23. A gasoline tank is connected to a 25-foot hose (ID = 1.50 cm) as shown in Figure 1.49. The ambient temperature is 38°C. What is the maximum height of the barrier over which the gasoline may be siphoned? You may neglect frictional losses. Note the following physical property data: density of gasoline = 5.6 lb<sub>m</sub>/gal and vapor pressure at 38°C is 12.3 psia.
24. A water tank is connected to a 100-foot hose (ID = 1.50 cm), as shown in the top of Figure 1.50. The height  $h$  is 1.8 meters. Calculate the average velocity of water in the hose. Do not neglect friction; you may assume turbulent flow.
25. For the flow setup in Problem 24 ( $h = 1.8$  meters), if we elevate the center of the hose, the flow will continue unabated. At some elevation, however, the pressure inside the elevated part will drop to the vapor pressure of water at 25°C and the water will boil, breaking the siphon. At what height,  $H$ , will the siphon break?
26. A pipeline of diameter  $d$  connects the fluid (density =  $\rho$ ) in an elevated open tank and a closed tank (Figure 1.51). The fluid is motionless. Determine the pressure in the lower tank in terms of the labeled heights.



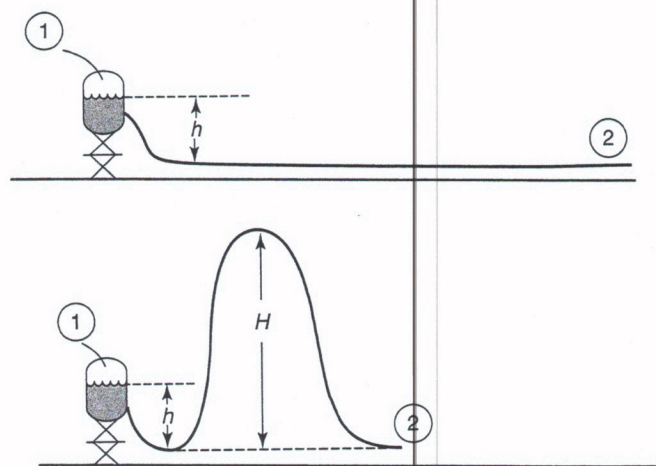


Figure 1.50

Schematic for Problems 24 and 25.

27. Water at  $25^\circ\text{C}$  fills the irregularly shaped container in Figure 1.52. What is the absolute pressure  $P$  in psia at the position noted? The apparatus is open to the atmosphere at the top. The apparatus is 100.0 cm thick into the page.
28. A tall scaffolding is erected next to a lake where a pump is operating. The maximum head deliverable by the pump is  $W_{\text{pump}}/mg = 70$  ft. A long hose is connected to the pump exit, and the pump draws water from the lake. The

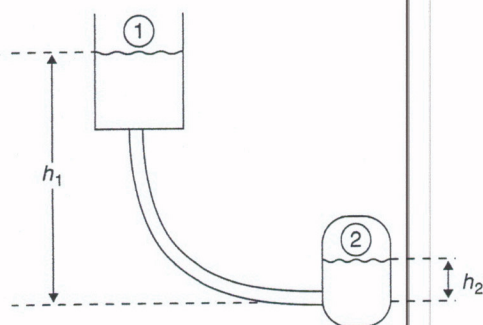


Figure 1.51

Schematic of apparatus for Problem 26.

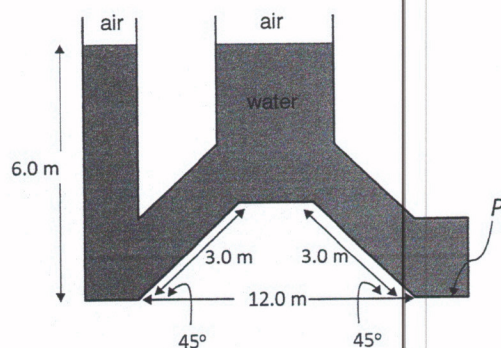


Figure 1.52

Schematic for Problem 27.



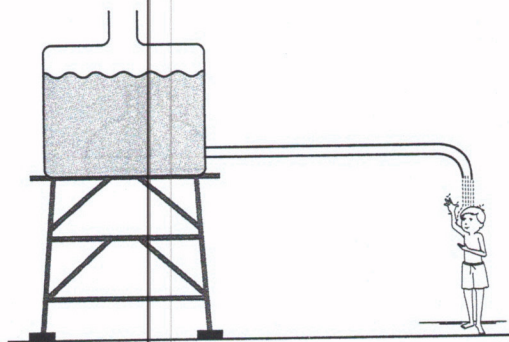


Figure 1.53

Schematic of a rustic shower arrangement in the woods (Problem 29).

pump is running and water is coming out of the hose. You grab the end of the hose and start climbing the scaffolding. How high do you have to climb before the water stops coming out of the hose? Justify your answer using the mechanical energy balance.

29. At a vacation camp in the woods, the owner collects rainwater for washing. She plans to construct a cold-water shower by mounting the collection tank (i.e., 150-gallon, 36-inch diameter) on a platform and using gravity to provide the flow through piping attached to a hole in the side near the bottom of the tank (Figure 1.53). She easily can obtain PEX tubing (i.e., cross-linked polyethylene) in nominal 1/2-inch and 1-inch sizes. What is the flow rate at the pipe exit at the beginning of the shower if she connected 10 feet of the 1/2-inch PEX (ID = 0.632 inches) to a full tank of water? What is the flow rate if the tank were only half full? Do not neglect friction.
30. Your grandfather has a cottage at the lake and wants to install a pump to deliver water to the house. He plans to pump water at night to fill a storage tank that he installed next to the cottage (Figure 1.54). The pipes and fittings he chose to use for the installation are listed in the table given. The pumps in the catalog your grandfather consulted are rated by their value of horsepower (hp). What is the minimum hp rating of a pump capable of providing a flow

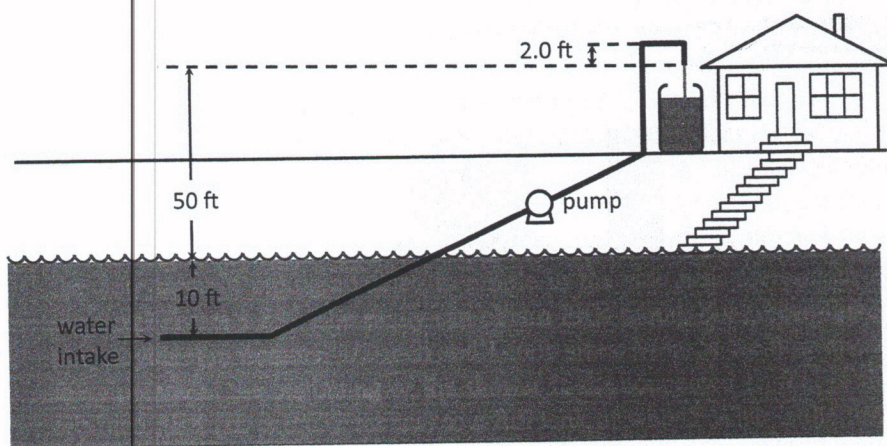


Figure 1.54

Schematic of the water system at cottage (Problem 30).



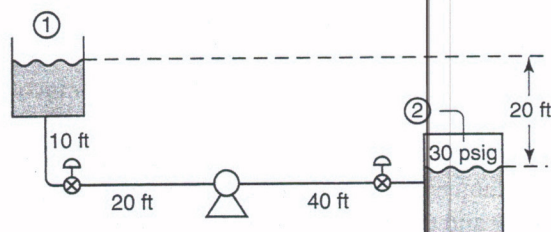
rate of 5 gpm of water at the tank? Assume that the pump is 65 percent efficient (i.e., of the energy put out by the pump, only 65 percent goes toward work on the fluid) and that the pipe is PVC (i.e., polyvinyl chloride, a polymeric material that is assumed to be smooth).

Fitting	Number of fittings
straight pipe, 1 inch, Schedule 40	95 feet
coupling	8
globe valve	1
gate valve	4
disk water meter	1

31. Pressure-drop versus flow-rate data were taken on water (room temperature) flowing in a 30.0 m section of old 1-inch Schedule 40 pipe (Table 1.7). Calculate the friction factor versus the Reynolds number for these data. How do the results compare to the standard correlation for the friction factor (i.e., the Colebrook equation)? Be quantitative. If we assume that there has been some scaling (i.e., deposition of hard deposits on the inner walls) that has decreased the effective pipe ID, can we improve the correspondence between the data and the literature correlation? Discuss.
32. A pump is connected between two tanks as shown in Figure 1.55. Calculate the pressure head, the velocity head, the elevation head, and the friction head

Table 1.7. Data for flow in a pipe for Problem 31

$\Delta p$ (kPa)	$Q$ (cm <sup>3</sup> /s)
8.0	350
20	560
45	880
88	1,400
230	2,200
470	3,500



All piping 1 in. Schedule 40  
2 gate valves  
1 90° bend

Figure 1.55

Schematic of flow between an open and a closed tank (Problem 32).



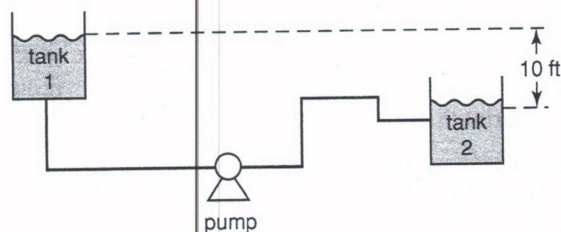


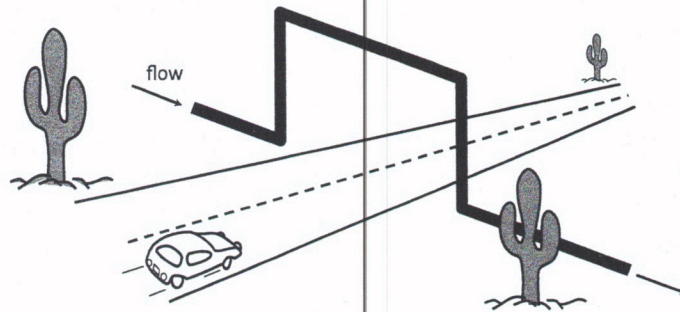
Figure 1.56

Schematic of flow for Problem 34.

- between the outlet and the inlet for a flow at 5.0 gpm. Calculate the pumping head  $W_{\text{pump}}/mg$  for this flow.
33. For the flow loop shown in Figure 1.55, develop an equation that gives the friction head loss  $h_f \equiv F_{21}/g$  in feet as a function of flow rate  $Q$  in gpm. The answer is an approximately quadratic equation. Plot your answer as friction head versus capacity (i.e., flow rate) for turbulent flow rates up to 10 gpm.
34. Pumps are rated in terms of fluid head (i.e., energy per unit weight of the fluid that they are pumping). A pump is connected between two open tanks as shown in Figure 1.56. The shaft work delivered by the pump at 6.0 gpm is measured at  $W_{\text{pump}}/mg = 75$  feet, where  $W_{\text{pump}}$  is the shaft work done by the pump,  $m$  is the mass flow rate of the fluid being pumped, and  $g$  is the acceleration due to gravity. What is the friction loss of the system between Points 1 and 2? Give your answer in feet of head. The frictional losses of the pump already have been accounted for and should not be included in the calculations.
35. A run of water piping crosses a field where a road is to be built. The piping will be routed temporarily over the road as shown in Figure 1.57. How is the load on the pump affected by the temporary change? Estimate the additional load on the pump as a function of flow rate for the dimensions and fittings shown in Figure 1.57. Both new valves are ball valves.
36. For the piping system shown in Figure 1.58, what is the average fluid velocity at the pipe discharge? Write the answer in terms of the variables defined in the figure. You may neglect friction in the solution. The tank is not open to the atmosphere; the pipe discharges fluid to the atmosphere.  $P$  is the absolute pressure inside the vapor space over the fluid in the tank, and  $P$  is held constant.
37. Modify the solution for the discharge velocity of a siphon (see Example 1.5) by accounting for the friction term. Assume that the friction factor is approximately constant and that flow is in the turbulent regime ( $0.002 < f < 0.010$ ; see the Moody chart, Figure 1.21 [103]). What is the error involved in neglecting friction in a siphon?
38. Water at  $25^\circ\text{C}$  flows at 3.2 gpm through the multipath pipeline in Figure 1.59. Calculate the volumetric flow rate in each branch and the pressure drop between points (a) and (b). Note: the pressure drop across each branch is the same and is equal to the pressure drop from (a) to (b). Equation 1.93 shows us that since  $\Delta P$  is the same, then the head loss  $h_f = \frac{\Delta P}{\rho g} = \frac{2fLV^2}{Dg}$  in each branch is the same. The mass balance provides a second relationship between the two velocities, allowing the problem to be solved.



Pipeline crosses desert road:



40 ft 2 in. Schedule 40 steel  
2 new ball valves  
4 new 90° bends

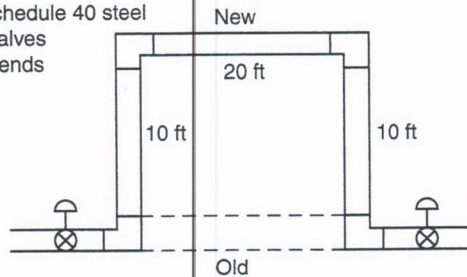


Figure 1.57

Schematic of circumstances described in Problem 35.

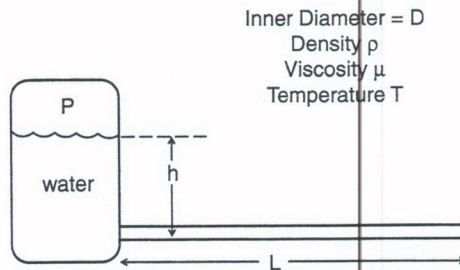


Figure 1.58

Schematic for Problem 36.

All piping is 1/2 inch Schedule 40  
Length branch (1) = 245 ft  
Length branch (2) = 540 ft

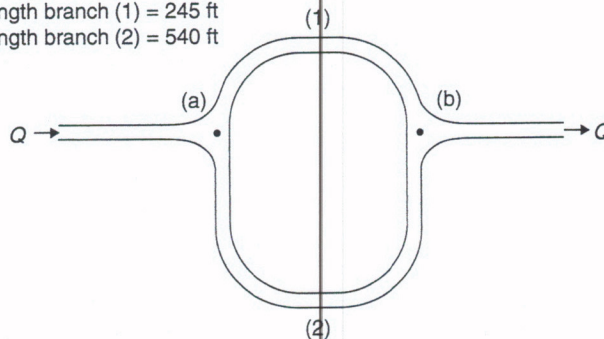


Figure 1.59

Schematic for Problem 38.



39. What is the effect of viscosity on the operation of a siphon?
40. Sketch the cylindrical coordinate system basis vectors  $\hat{e}_r$ ,  $\hat{e}_\theta$ , and  $\hat{e}_z$  at the following points  $(r, \theta, z)$ :  $(3, 0, 0)$ ,  $(3, \frac{\pi}{2}, 0)$ ,  $(3, \frac{3\pi}{4}, 0)$ ,  $(6, 0, 0)$ ,  $(6, \frac{\pi}{2}, 0)$ , and  $(6, \frac{3\pi}{4}, 0)$ . Sketch the Cartesian basis vectors  $\hat{e}_x$ ,  $\hat{e}_y$ ,  $\hat{e}_z$  at the same locations. Comment on your sketches.
41. For the vector  $\underline{v} = U\hat{e}_\theta$  written in the cylindrical coordinate system, what is the component of  $\underline{v}$  in the  $\hat{e}_x$  direction?  $U$  is a constant.
42. For the vector  $\underline{v} = Ur\hat{e}_\theta$  written in the cylindrical coordinate system, what is the component of  $\underline{v}$  in the  $\hat{e}_y$  direction?  $U$  is a constant and  $r$  is the coordinate variable of the cylindrical coordinate system.
43. For the following vectors  $\underline{v}$  and  $\underline{a}$ , what is the component of the velocity  $\underline{v}$  (m/s) in the direction of vector  $\underline{a}$ ?

$$\underline{v} = 3\hat{e}_x + 2\hat{e}_y + 7\hat{e}_z = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}_{xyz}$$

$$\underline{a} = 6\hat{e}_z = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}_{xyz}$$

44. For the following vector and tensor (matrix), what is  $\hat{n} \cdot \underline{\underline{\tilde{\tau}}}$ ? Both expressions are written in the cylindrical coordinate system.

$$\hat{n} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}_{r\theta z} \quad \underline{\underline{\tilde{\tau}}} = \begin{pmatrix} 0 & 12 & 0 \\ 12 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{r\theta z}$$

45. What is the dot product of the following two vectors? Both vectors represent properties at the point  $(1, 0, 0)_{xyz}$ . Note: The expressions here are written in two different coordinate systems.

$$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}_{xyz} \quad \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}_{r\theta z}$$

46. What is the dot product of the following two vectors? Both vectors represent properties at the point  $(0, 1, 0)_{xyz}$ . Note: The expressions here are written in two different coordinate systems.

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}_{xyz} \quad \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}_{r\theta z}$$

47. What is the cross product of the following two vectors? Both vectors represent properties at the point  $(1, 1, 0)_{xyz}$ . Note: The expressions here are written in



two different coordinate systems.

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}_{xyz} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{r\theta z}$$

48. Write the vector  $\underline{w} = 4\hat{e}_1 - \hat{e}_2 + \hat{e}_3$  in cylindrical coordinates.
49. Write the vector  $\underline{v} = (1 - 4y^2)\hat{e}_x$  in cylindrical coordinates.
50. Write the vector  $\underline{w} = -3\hat{e}_1 - \hat{e}_2 + \hat{e}_3$  in spherical coordinates.
51. Write the vector  $\underline{v} = (1 - 2y^2)\hat{e}_y$  in spherical coordinates.
52. The solution for the velocity field for steady, pressure-driven flow in a tube is provided in Chapter 7 (see Equation 7.23). Convert this solution, which is given in cylindrical coordinates, to Cartesian coordinates,  $x, y, z, \hat{e}_x, \hat{e}_y$ , and  $\hat{e}_z$ .
53. The solution for the velocity field for steady, uniform flow around a sphere is provided in Chapter 8 (see Equation 8.23). Convert this solution, which is given as a vector written in the spherical coordinate system, to a vector written in the Cartesian coordinate system. You may leave your answer in terms of spherical coordinate variables  $r, \theta, \pi$ . What relationships between  $r, \theta, \phi$  and  $x, y, z$  do we need to complete the conversion to the Cartesian coordinate system?
54. The solution for the velocity field for steady, pressure-driven flow in a slit is provided in Chapter 7 (see Equation 7.188). Convert this solution, which is given in Cartesian coordinates centered in the middle of the slit, to Cartesian coordinates anchored on the bottom wall.
55. What is a boundary condition? Why are boundary conditions needed when solving differential equations?
56. How many boundary conditions on  $x$  are needed for the following partial differential equation? How many boundary conditions are needed on  $y$ ?

$$\alpha \frac{\partial v}{\partial x} = \frac{\partial^2 v}{\partial y^2}$$

57. For the steady laminar flow of water through a long pipe, calculate the flow rate  $Q$  from the velocity profile, which is given here. Show your work. The following quantities are constants:  $R, L, \rho, g, P_0, P_L, \mu$ ;  $r$  is the coordinate variable in the cylindrical coordinate system.

$$\underline{v} = v_z \hat{e}_z$$

$$v_z = \frac{R^2(L\rho g + p_0 - p_L)}{4\mu L} \left(1 - \frac{r^2}{R^2}\right)$$

58. Using a computer program (i.e., spreadsheet or other), plot the velocity profile given in Equation 1.140, which represents the velocity profile between two long vertical plates separated by a narrow gap. The flow is caused by natural convection: One plate is hotter than the other.
59. For the natural-convection velocity profile given in Equation 1.140, calculate the second derivative of the velocity-profile function and evaluate the second



derivative at the extrema of the function. What does the second derivative tell us about the extrema?

60. Which of the following expressions is  $\underline{v} \cdot \nabla \underline{v}$ ? Explain how you arrive at your answer.

$$\left( \begin{array}{l} v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \\ v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \\ v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \end{array} \right)_{xyz} \quad \text{OR} \quad \left( \begin{array}{l} v_x \frac{\partial v_x}{\partial x} + v_x \frac{\partial v_y}{\partial y} + v_x \frac{\partial v_z}{\partial z} \\ v_y \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_y \frac{\partial v_z}{\partial z} \\ v_z \frac{\partial v_x}{\partial x} + v_z \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_z}{\partial z} \end{array} \right)_{xyz}$$

61. What is the substantial derivative  $D\underline{v}/Dt$  of the steady-state velocity field represented by the following velocity vector? Note that the answer is a vector. Explain how you arrive at your answer.

$$\underline{v}(x_1, x_2, x_3, t) = \begin{pmatrix} 1 - 9x_2^2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

62. Under the pull of gravity, a Newtonian fluid drains from a cylindrical tank through a small hole in the center of the bottom of the tank. The tank has radius  $R$  and is of height  $H$ . Which coordinate system do you choose for solving for the flow field in this problem? In your chosen coordinate system, what is the general expression for the velocity field  $\underline{v}$ ? Are any of the components of  $\underline{v}$  zero in your coordinate system? If so, why? Of what variables is  $\underline{v}$  a function?
63. A Newtonian fluid flows past a stationary sphere. Upstream of the sphere, the flow is uniform; that is, the velocity is constant in both magnitude and direction. The radius of the sphere is  $D/2$ . Which coordinate system do you choose for solving for the flow velocity field in this problem? In the chosen coordinate system, what is the general expression for the velocity field  $\underline{v}$ ? Are any of the components of  $\underline{v}$  zero in your coordinate system? If so, why? Of what variables is  $\underline{v}$  a function?
64. A Newtonian fluid flows under a driving pressure gradient and down the axis of a duct with a rectangular cross section. The width of the duct is  $2W$  and the height is  $2H$ . The duct has a length of  $L$ . Which coordinate system do you choose for solving for the flow in this problem? In the chosen coordinate system, what is the general expression for the velocity field  $\underline{v}$ ? Are any of the components of  $\underline{v}$  zero in your coordinate system? If so, why? Of what position variables is  $\underline{v}$  a function?
65. A Newtonian fluid flows under a driving pressure gradient down the axis of a duct with a circular cross section. The radius of the duct is  $D/2$  and the length is  $L$ . Which coordinate system do you choose for solving for the flow in this problem? In the chosen coordinate system, what is the general expression for the velocity field  $\underline{v}$ ? Are any of the components of  $\underline{v}$  zero in your coordinate system? If so, why? Of what variables is  $\underline{v}$  a function?
66. A Newtonian fluid flows under a driving pressure gradient down the axis of a duct with an elliptical cross section. The longer axis of the ellipse is  $a$  and the shorter axis is  $b$ . The length of the duct is  $L$ . Which coordinate system do you choose for solving for the flow in this problem? In the chosen coordinate



system, what is the general expression for the velocity field  $\underline{v}$ ? Are any of the components of  $\underline{v}$  zero in your coordinate system? If so, why? Of what variable is  $\underline{v}$  a function?

67. A Newtonian fluid flows past a three-dimensional stationary object that is a simplified version of a modern automobile. Upstream of the object, the flow is uniform; that is, the velocity is constant in both magnitude and direction. The object presents a cross section to the flow of  $A_p$ . Which coordinate system do you choose for solving for the flow velocity field in this problem? In the chosen coordinate system, what is the general expression for the velocity field  $\underline{v}$ ? Are any of the components of  $\underline{v}$  zero in your coordinate system? If so, why? Of what variable is  $\underline{v}$  a function?