Surface-tension gradients caused by concentration, temperature, or electrical gradients can drive flows. This effect, called the Marangoni effect, is demonstrated in the NCFMF film that highlights surface-tension effects [112]. Surface tension is important in slow flow through porous media. In most macroscopic engineering flows, surface-tension effects are negligible. There are two dimensionless numbers that can be used to determine whether surface tension is important in a flow—the Bond number and the Weber number:

Ratio of gravity forces and surface-tension forces	Bond number	$Bo = \frac{\rho g L^2}{\sigma}$	(4.443)
Ratio of inertial forces and surface-tension forces	Weber number	$We = \frac{\rho V^2 L}{\sigma}$	(4.444)

The topic of surface tension highlights a significant aspect of the continuum model. Because the continuum picture is a model and not physical reality, it does not reflect all of the physics in a system that it approximates. In this chapter, the idea of a continuous field of matter is valuable for calculations on bulk fluids, but this picture cannot capture the boundary effects that result from the real physics at interfaces. We must think again about the real system and adjust our model to account for this newly appreciated aspect of the system. The cost of this adjustment is that we have a new material parameter—the surface tension—that we must measure and consider in continuum modeling to correctly capture the true behavior of our system.

This circumstance—the need to adjust or to complicate a chosen model—will recur as we seek to apply our models to complex systems. At every new juncture where new or neglected physics intrudes, we revisit and adjust our initial model. This does not mean that the model is wrong; it means only that any model is limited to the circumstances under which it was developed. Dimensional analysis is a tool that helps us to quantify when we need to switch from one description of a physical situation to another.

In Chapter 5, we return to the task of incorporating molecular physics into our description of stress so that we can relate $\underline{\underline{\tilde{\tau}}}$ and $\underline{v}$ in a moving fluid. Once we know the relationship between $\underline{\underline{\tilde{\tau}}}$ and $\underline{v}$, we can complete our balance calculations on moving fluids.

4.5 Problems

1. What is a control volume? Can fluid pass through the walls of a control volume?
2. Thinking about a fluid as a chemist would—as a collection of molecules—which properties of molecules generate contact forces on a control volume in a fluid? Which properties of molecules generate noncontact forces on a control volume in a fluid?

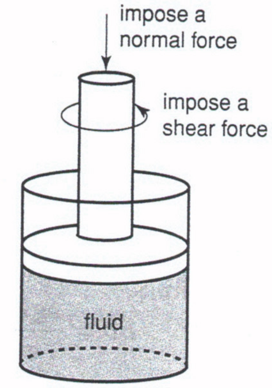
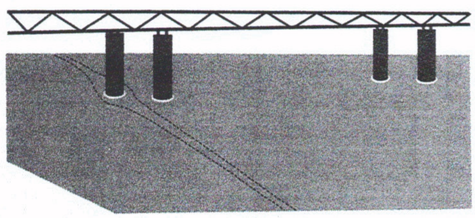


Figure 4.63 Olive oil is confined in the device shown (see Problems 6 and 7).

3. How does the continuum picture differentiate between the flow behavior of a polar molecule like water compared to a nonpolar molecule like methane?
4. How does the continuum picture differentiate between the flow behavior of a long-chain polymer that entangles with other long-chain molecules and a short-chain molecule that is not capable of entangling?
5. Describe how a stiff solid like steel responds to shear forces. Describe for normal forces. How does a soft solid like a block of tofu respond to shear forces? Normal forces?
6. Describe how a confined fluid, such as olive oil in the device shown in Figure 4.63, responds to shear forces imposed by the rotation of the lid. How does the fluid respond to normal forces imposed by pressing down on the lid?
7. For the fluid-filled device in Figure 4.63, we can write the forces imposed on the fluid in a cylindrical coordinate system, with $\hat{e}_z$ vertically upward and parallel to the axis of rotation. In this coordinate system with a pure normal force imposed, what is the vector representation of the normal force $\underline{N}$ on the lid? Which components are zero? Give your answer in both matrix form and component/basis-vector form (with the $\hat{e}_r, \hat{e}_\theta, \hat{e}_z$ basis vectors). In the case of the lid rotating, write the tangential force on the lid in the cylindrical coordinate system. Which components are zero? If the lid is subjected to a force that has both normal and tangential components, what does the force vector look like in the cylindrical coordinate system?
8. River flow in the vicinity of a vertical bridge support induces a molecular force on the rod-shaped support. The forces at Points (a) and (b) (Figure 4.64) and are given by the following two vectors (i.e., arbitrary force units):

$$\underline{f}\Big|_{(a)} = \begin{pmatrix} 320 \\ 210 \\ 0 \end{pmatrix}_{xyz} \qquad \underline{f}\Big|_{(b)} = \begin{pmatrix} 310 \\ -200 \\ -3 \end{pmatrix}_{xyz}$$

What is the normal force on the support at (a)? What is the tangential force on the support at (a)? Point (a) is located at coordinate point $(R, 3\pi/4, 5)$ in the $r\theta z$-coordinate system. What are the normal and tangential forces on



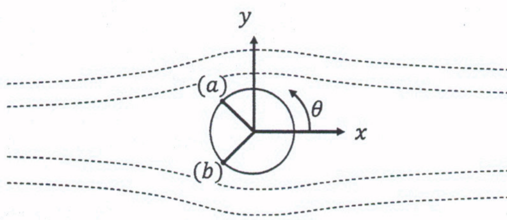


Figure 4.64 A bridge support is subject to normal and tangential forces by the river (Problem 8).

the support at (b)? Point (b) is located at coordinate point $(R, 5\pi/4, 5)$ in the $r\theta z$-coordinate system.

9. A rectangular parallelepiped is subjected to the force $\underline{f}$ below on its top surface (perpendicular to z-axis, arbitrary units). What is the shear force? What is the normal force?

$$\underline{f} = 3\hat{e}_x + 2\hat{e}_y - 0.5\hat{e}_z$$

10. A cylinder of height L is subjected to the force $\underline{f}$ on its side surface at location $(R, \pi/4, L/2)$. What is the shear force on the cylinder surface? What is the normal force? The units of f are arbitrary.

$$\underline{f} = 2\hat{e}_r + 2\hat{e}_\theta + \hat{e}_z$$

11. An ideal gas fills a balloon that has a spherical shape. The coordinate system chosen for the problem is a Cartesian system located at the center of the sphere with x pointing east, y pointing north, and z pointing vertically upward. The temperature of the gas is 305 K, and the molar volume is 12.5 l/mol. What are the forces (your answers should be vectors) on the 1.0 cm^2 areas of balloon surface located as follows:
    (a) At the equator of the balloon and centered where the balloon intersects the x-axis?
    (b) At the equator of the balloon and centered where the balloon intersects the y-axis?
    (c) At the equator of the balloon and centered halfway between the first two areas?
12. A cubical box 0.10 m on a side contains 0.121 moles of ideal gas at 403 K. What is the force on each side? The effect of gravity may be neglected. The answer should be a vector; choose a convenient coordinate system.
13. A standard manometer is used to calibrate a digital pressure meter. For the manometer shown in Figure 4.65, one side is open to atmospheric pressure ($p_{atm} = 76.2$ cm Hg). What is the unknown pressure P? The manometer

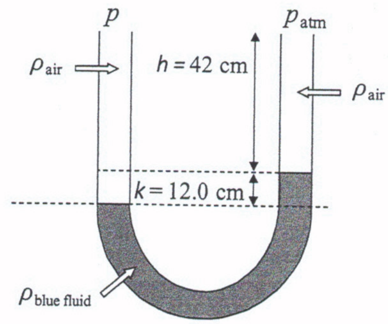


Figure 4.65 Manometer for Problem 13.

fluid is Blue Fluid 175 with a density of 1.75 g/cm^3, and the gas on both sides of the manometer is air at 25°C.

14. A closed manometer (pressure is zero absolute on the closed end) contains a dense nonvolatile liquid. The open end is connected by tubing to a gas process stream at pressure p. The manometer fluid has density $\rho = 13.6$ g/cm^3, and the height difference between the two sides of the manometer is 5.4 mm. What is the pressure in the process stream?
15. A tilted manometer is used to measure very small pressure differences. For the tilted manometer shown in Figure 4.66, what is the pressure difference between the two sides in terms of the variables defined in the figure?
16. A manometer is configured as shown in Figure 4.67. A heavy fluid (Fluid B) has been placed in the bottom of the manometer; a light fluid (Fluid A) has been added to the left side only. The left side of the manometer is connected to a process stream in which water (25°C, Fluid C) is flowing; the right side is open to the atmosphere (Fluid D). The manometer is being used to measure the pressure difference $p_1 - p_2$. The density of the two fluids in the manometer are 1.75 and 13.6 g/cm^3. What is the pressure difference $p_1 - p_2$ in terms of fluid heights and fluid densities? For $h_1 = 2.3$ cm, $h_2 = 2.3$ cm, $h_3 = 1.0$ cm, what is the pressure difference in psi?
17. The pressure in the vapor space of a tank (Figure 4.68) is measured with a mercury manometer that is open to air. The manometer is isolated from the water in the tank by an intermediate section of piping in which oil is trapped.

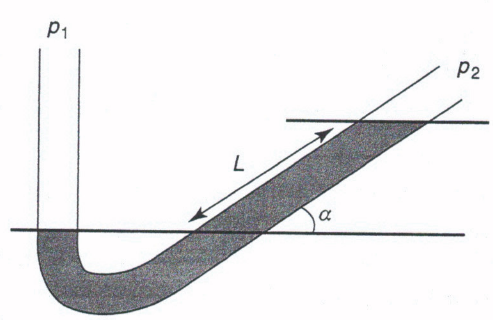


Figure 4.66 Schematic of a tilted manometer (Problem 15).

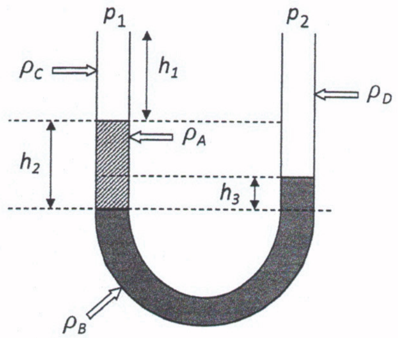


Figure 4.67 A schematic of a manometer that contains two different measurement fluids (Problem 16).

What is the pressure in the head space above the water for the heights and fluids shown in Figure 4.68?

18. In the double-well manometer discussed in this chapter, the same amount of top fluid is placed on each side of the manometer (see Figure 4.36); this is difficult to achieve in practice, however. If slightly more fluid is placed on the lefthand side than on the righthand side, there will be a change in the reading of the manometer, even when the pressures on both sides are the same (Figure 4.69). Show how Equation 4.195 is modified if the initial reading of the double-well manometer is R_0 rather than zero.
19. *Will it float?* In the 1990s on American television, David Letterman's comedy show had a nonsense segment called *Will It Float?* in which Letterman and his bandleader Paul Shafer guessed whether an item would float or sink when dropped into a container of water. For an object that occupies a volume of 4.0 liters, what is the maximum weight that can float?
20. What is the pressure at the bottom of the fluid in the container shown in Figure 4.70 if the angle between the container wall and the horizontal is α? Most of the fluid in this container is not vertically above the bottom surface of the container. Explain how hydrostatic pressure is transferred to that bottom surface in this container.

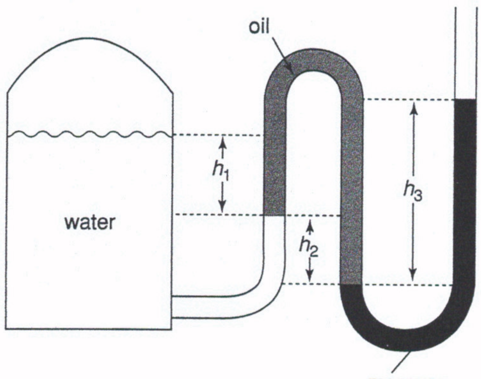


Figure 4.68 We use the static-pressure relationships to relate pressure in a tank to the various heights of fluids as shown (Problem 17).

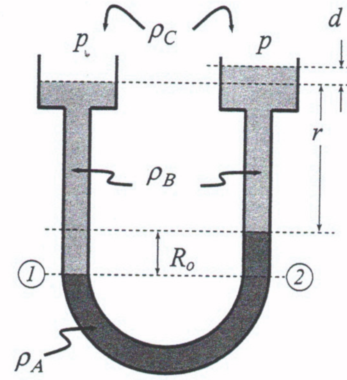


Figure 4.69 If more top fluid is present on the left than on the right, the initial reading of a double-well manometer is R_0 (Problem 18).

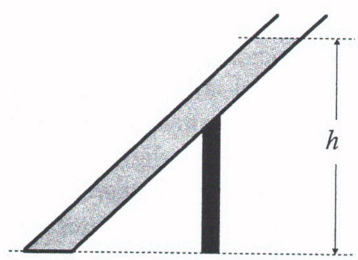


Figure 4.70 A tilted container holds a liquid (Problem 20).

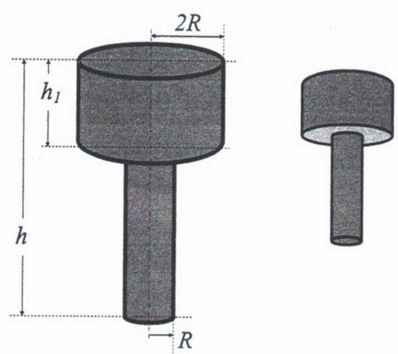


Figure 4.71 An irregularly shaped container holds a liquid (Problem 21).

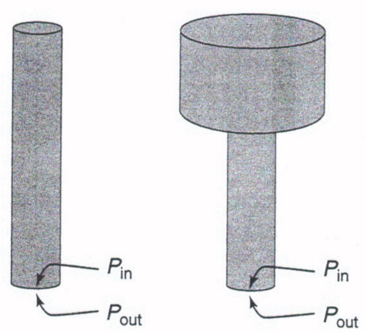


Figure 4.72 Two vessels are constructed and filled with water. We compare the pressure (force/area) near the bottom inside the vessel and the pressure on the solid surface at the bottom of the vessel (Problem 22).

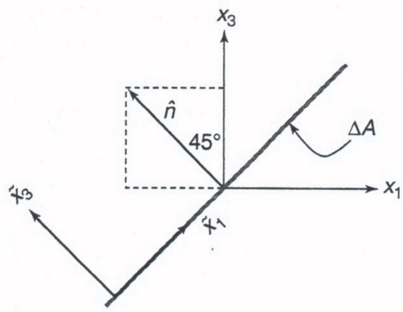


Figure 4.73 Pressure can be calculated in any coordinate system. Problem 23 asks for a calculation in the coordinate system shown.

21. In terms of the dimensions shown in Figure 4.71, what is the pressure on the bottom surface of the container (area $= \pi R^2$)? What is the pressure on the ledge-surface in Figure 4.71? Check whether all of the force due to gravity on the liquid in the container is accounted for by the pressure on these two surfaces.
22. Consider two vessels of different shapes but that have the same height (Figure 4.72). What is the pressure at the bottom of the fluid in each case? What is the pressure at the bottom of the vessel (outside) in each case? Are they the same or different? Explain.
23. What is the total vector force on a $0.550\ \text{m} \times 1.00\ \text{m}$ rectangular plate submerged 13.0 m below the surface of a water tank and oriented as shown in Figure 4.73? Express your answer in the coordinate system shown.
24. As a result of a flood, air is trapped in a room by water of depth 10.0 feet pressing down on a metal sheet as shown in Figure 4.74. How much force (a vector) is the water exerting on the metal sheet? How much force is the weight of the sheet exerting on the portions of the walls that are holding it in place? Is it possible for someone trapped in the room to push up the sheet? The sheet is 4 feet by 6 feet by 0.10 inch thick and made of steel (density $=$ $0.286\ \text{lb}_m/\text{in.}^3$).
25. What is the force on a six inch cube of balsa wood held 5 feet below the surface of water in a tank? The answer must be a vector.

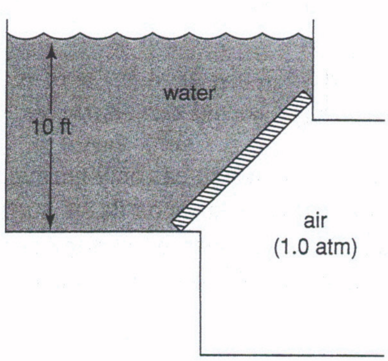


Figure 4.74 A flood creates an air pocket that is protected by a sheet of steel leaning at a 45 degree angle (Problem 24).

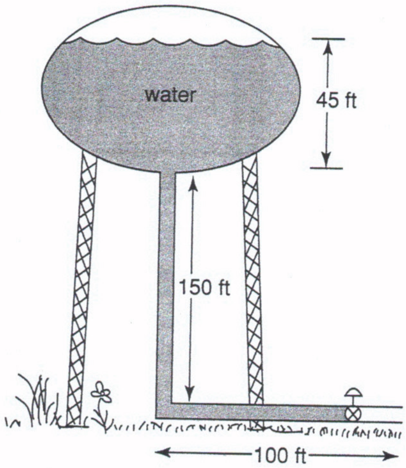


Figure 4.75 A municipal water tank stores water 150 feet in the air (Problem 30).

26. What is the force on a box of polymer foam (density 330 kg/m^3) held 3 m below the surface of water in a tank? The answer must be a vector. The dimensions of the box are 45 cm by 18.0 cm by 6.0 cm.
27. A ball of radius 20.0 cm is submerged 5.00 m below the surface of the ocean. What is the vector force on this object?
28. An object the shape of half a sphere of radius 20 cm is submerged 2.00 m below the surface of the ocean. What is the vector force on this object?
29. A box made of polymeric foam (density = 330 kg/m^3) is to be weighted so that it will float 4.5 m below the surface of a pool filled with water at 25°C. The dimensions of the box are 20 cm by 25 cm by 6.5 cm. What is the target weight of the box so that it neither sinks nor rises to the surface? If the pool is filled with seawater, what should the new weight be?
30. Figure 4.75 shows a water tower and some piping. For the dimensions shown and assuming that all pipes are Schedule 40 nominal 1.5-inch pipe, calculate the flow rate Q.
31. Pressure is an isotropic normal stress, meaning that it acts perpendicularly to any chosen surface and has the same magnitude for all surfaces. Consider the stress tensor given here, which has only normal stress components. Is this stress tensor isotropic? The stress is in Pa.

$$\underline{\underline{\tilde{\Pi}}} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{123}$$

32. Consider the stress tensor given here, which has only normal stress components. For a cube subjected to this same stress on its six surfaces, calculate the vector force on each surface. The outwardly pointing normal to the top surface is in the x_3-direction and the cube is 20 cm on a side. The stress is in Pa.

$$\underline{\underline{\tilde{\Pi}}} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{123}$$

33. The stress tensor $\underline{\underline{\tilde{\Pi}}}$ at Point P is given in the following matrix form. For a flat square surface centered at P of area 3.1 mm^2 oriented perpendicular to the 13-plane as shown in Figure 4.46, what is the stress on that surface at Point P? Demonstrate that the stress in this problem is not isotropic.

$$\underline{\underline{\tilde{\Pi}}} = \begin{pmatrix} -2.0Pa & 0 & 0 \\ 0 & -4.0Pa & 0 \\ 0 & 0 & -5.0Pa \end{pmatrix}_{123}$$

34. Consider the stress tensor given here (spherical coordinates). For a sphere subjected to this stress field, calculate the vector stress at the following locations (all written in the r, θ, ϕ coordinate system): (R,0,0), (R,$\pi/2$, 0), (R, π,0), and (R, $3\pi/2$, 0). Comment on the results. The stress is in Pa.

$$\underline{\underline{\tilde{\Pi}}} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}_{r\theta\phi}$$

35. Consider the stress tensor given here (Cartesian coordinates, Pa): For a cube (side length is L; located in the first quadrant of the 123-coordinate system with a vertex at the origin) subjected to this stress field, calculate the vector force on the surface at the following locations (all written in the 123-coordinate system): $(\frac{L}{2}, \frac{L}{2}, 0)$, $(\frac{L}{2}, \frac{L}{2}, L)$, $(L, \frac{L}{2}, \frac{L}{2})$, and $(\frac{L}{2}, L, \frac{L}{2})$. Comment on the results.

$$\underline{\underline{\tilde{\Pi}}} = \begin{pmatrix} -3 & 2x_2 & 0 \\ 2x_2 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}_{123}$$

36. For a flow in which the pressure $p = 7$ (all units arbitrary) and the extra-stress tensor is given here, what is the total stress tensor $\underline{\underline{\tilde{\Pi}}}$ in matrix form?

$$\underline{\underline{\tilde{\tau}}} = \begin{pmatrix} 1 & 5 & 2 \\ 5 & 1 & 3 \\ 2 & 3 & 0 \end{pmatrix}_{123}$$

37. For a flow in which the pressure $p = -7x_1 + 3$ (all units arbitrary) and the extra-stress tensor is given here, what is the total stress tensor $\underline{\underline{\tilde{\Pi}}}$ in matrix form?

$$\underline{\underline{\tilde{\tau}}} = \begin{pmatrix} 0 & 5\left(1 - \frac{x_2^2}{9}\right) & 0 \\ 5\left(1 - \frac{x_2^2}{9}\right) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

38. For a flow in which the pressure $p = 3$ (arbitrary units) and the extra-stress tensor $\underline{\underline{\tilde{\tau}}} = 6x_2(\hat{e}_1\hat{e}_2 + \hat{e}_2\hat{e}_1)$, what is the total stress tensor $\underline{\underline{\tilde{\Pi}}}$ in matrix form?

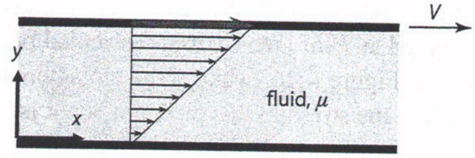


Figure 4.76 Drag flow is the classic geometry for determining viscosity (Problem 40).

39. Consider the stress tensor given here (Cartesian coordinates, Pascals):

$$\underline{\underline{\tilde{\Pi}}} = \begin{pmatrix} -3 & 7x_2 & 0 \\ 7x_2 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}_{123}$$

For a sphere subjected to this stress field, calculate the vector stress at the following locations (all written in the $r\theta\phi$ coordinate system with θ the angle between the x_3-axis and r): (R, 0, 0), (R, $\pi/2$, 0), (R, π, 0), and (R, $3\pi/2$, 0). Comment on the results.

40. For the flow shown in Figure 4.76, a fluid is trapped between two long, wide plates and the upper plate is made to move at a speed V in the x-direction. This is Newton's experiment, called *drag flow*. The pressure is everywhere atmospheric. It takes some force to move the upper plate. Does the pressure (normal force/area) have a role in determining the force that it takes to move the top plate? State your reasoning.

41. For the flow shown in Figure 4.77, a fluid jet impinges on a wall. The pressure around the jet is everywhere atmospheric. The jet produces a force on the wall. Does the fluid pressure p (normal force/area) have a role in determining the force on the wall? State your reasoning.

42. A surface of interest in a flow has a unit normal of $\hat{n} = \frac{1}{\sqrt{6}}(\hat{e}_x + 2\hat{e}_y - \hat{e}_z)$ and an area of 8.0 cm^2. If the pressure is 1.02×10^6 Pa and the extra-stress tensor $\underline{\underline{\tilde{\tau}}}$ in mega Pa is given here, what is the force on the surface? Give the

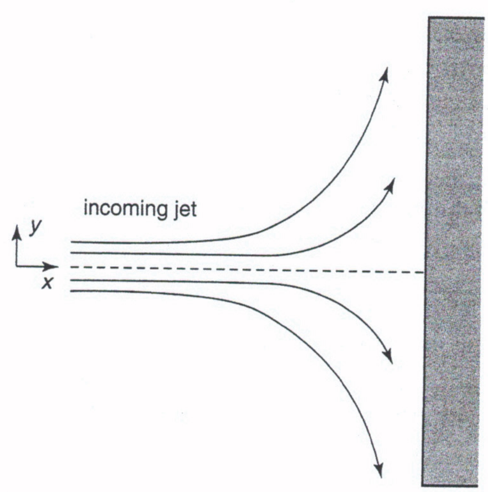


Figure 4.77 A jet produces a force on a wall (Problem 41).

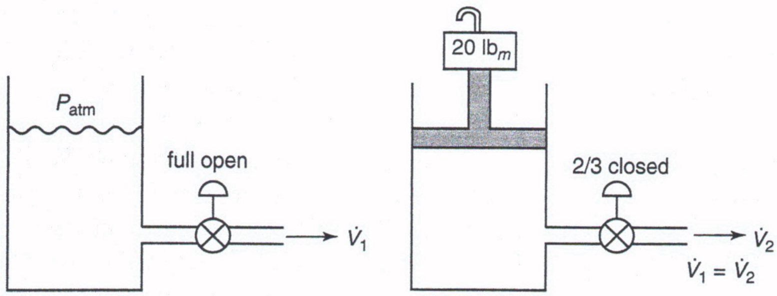


Figure 4.78 Water drains from two tanks at the same rate. However, the tank is open to the atmosphere in one case; in the other case, an external pressure is imposed as shown (Problem 43).

complete force vector.

$$\underline{\underline{\tilde{\tau}}} = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{xyz}$$

43. Figure 4.78 is a tank with a spout from which water can drain in a controlled manner. When the water surface is open to the atmosphere, the water drains at flow rate $\dot{V}$ liters/min. If a piston that seals with the sides of the tank is added to the top and a 20-kg weight is added, the flow rate increases. If we close the valve, we can reduce the flow rate until it is again $\dot{V}$. We have two situations, then, in which water drains from the tank at flow rate $\dot{V}$; what is the difference in the state of the fluid between the two situations? If we poke a hole in the side of the tank, will the flow from the hole be the same in the two different situations? How?
44. We can use a technique called *quasi-steady-state problem solving* to estimate the time it takes a tank to drain completely. Consider the tank in Figure 4.79.
    (a) When the drain is first opened, what is the instantaneous flow rate Q from the tank in terms of the fluid height h?
    (b) The height of the fluid in the tank is changing as the tank drains. What is the speed of the fluid surface in terms of h? As the tank drains, what is the flow rate through a cross section at the middle of the tank?

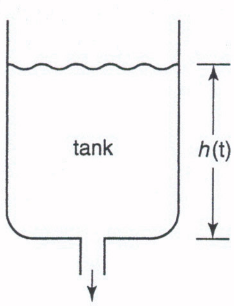


Figure 4.79 Schematic of a tank draining. This problem is solved with the mechanical energy balance and quasi-steady-state methods (Problem 44).

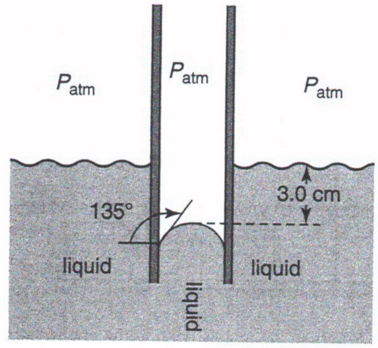


Figure 4.80 When liquids have a repulsive interaction with solids, capillary depression is observed (Problem 49).

(c) Equating the instantaneous flow rate from Part (a) with the flow rate in the tank from Part (b), obtain a differential equation for h and solve. Note that the initial height of the fluid is h_0.

45. In the water-droplet example (see Example 4.24), we concentrated on the z-component of the momentum balance. What information do the x- and y-components of the momentum balance convey?
46. In Example 4.24, we examined the pressure inside a spherical water droplet. For water droplets of various sizes between 0.02 and 3.0 mm, what is the pressure inside the droplet? What is the pressure inside droplets of mercury for this range of sizes?
47. How high will acetone rise in a glass capillary of diameter 0.03 mm? The angle between the vertical and the meniscus is unknown. Make a reasonable estimate for this angle.
48. A carbon-dioxide bubble is motionless at the bottom of a glass of carbonated beverage. Estimate the pressure inside the bubble.
49. Intermolecular forces at solid–liquid–gas interfaces can cause capillary rise or capillary depression. Consider a glass capillary tube submerged in a fluid that does not wet glass—that is, a fluid that has repulsive intermolecular interactions with glass. This situation creates capillary depression as shown in Figure 4.80. If the depression $h = -0.3$ cm and the angle of the meniscus is 135 degrees, what is the surface tension of the liquid? The fluid density is 1200 kg per cubic meter and the capillary is 1.0 mm in diameter.
50. A laminar jet of an oil inside a bath of a second oil forms a cylindrical column of fluid. Because the two oils are not the same material, there is an effect

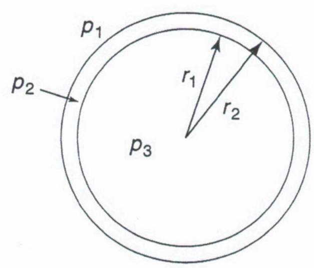


Figure 4.81 A soap bubble can be modeled as shown; the film thickness has been greatly exaggerated (Problem 51).

of surface tension in the formation of the interface. What is the pressure difference due to surface tension between the inside and the outside of a cylindrical column of fluid? Hint: Think of the cylindrical column as a fluid shape with two radii of curvature.

51. What is the pressure difference between the inside and the outside of a soap bubble? Hint: We can draw a soap bubble as shown in Figure 4.81. We write the liquid pressure within the film p_2 in terms of the atmospheric pressure p_1. We subsequently can write the inside air pressure p_3 in terms of the pressure in the film. Taking the limit that $r_1 = r_2$ gives the final result.