in recent years have come from molecular modeling. For an introduction to molecular modeling in non-Newtonian fluids, refer to the literature [13, 83].

5.4 Summary

Our search in this chapter was for a stress-velocity relationship for molecular forces in fluids. We sought expressions that could describe all (or most) fluids. In reaching that goal, we have been reasonably successful. We arrived at the Newtonian constitutive equation, a rigorous stress-velocity relationship that captures the behavior of thousands of fluids, including the two most common: water and air. The field of fluid mechanics is the field of Newtonian fluid mechanics—that is, the study of fluids that follow the Newtonian constitutive equation.

Non-Newtonian fluids challenge the continuum approach. We can use continuum ideas to develop inelastic constitutive equations and even simple viscoelastic constitutive equations, but we find that for the most complex fluids and complex behaviors, it is more fruitful to return to a molecular approach for stress-velocity calculations.

The pattern of discovery in the quest for a proper molecular-force term in fluids follows a standard discovery pattern in science. When investigating observations, we look at the simplest explanations first, seeking to define when they are suitable. When the simplest systems are well understood, we move on to more complex cases. If the most obvious modifications fail, we move on to more complicated models, always looking for constraints that help narrow down the possible choices.

In the remaining chapters, we apply the Newtonian stress-velocity relationship and hone our skills in solving for velocity and stress fields in flowing liquids. Chapter 6 focuses on generalizing our solution methods and equations; Chapter 7 applies our techniques to flows within boundaries; and Chapter 8 focuses on unbounded flows, known as external flows, which includes the analysis of boundary layers. Chapter 9 takes forward and generalizes the macroscopic balance techniques. Advanced applications of continuum modeling are described in Chapter 10.

5.5 Problems

1. In fluid mechanics, what is a constitutive equation?
2. Is the Newtonian constitutive equation related to molecular forces in a fluid? How?
3. What does it mean to say a fluid is a "Newtonian fluid?" Give examples of non-Newtonian behavior.
4. What is stress? What is pressure? Distinguish between these two concepts in the context of fluid flow.
5. In Chapter 4, molecular force on a surface in a fluid is given by $\iint_S \hat{n} \cdot \underline{\underline{\tilde{\Pi}}}\big|_{\text{surface}}\, dS$. How can we calculate $\underline{\underline{\tilde{\Pi}}}$ for a flow?

6. The total stress tensor $\tilde{\underline{\underline{\Pi}}}$ is given by $\tilde{\underline{\underline{\Pi}}} = -p\underline{\underline{I}} + \tilde{\underline{\underline{\tau}}}$. If the velocity is zero, what is $\tilde{\underline{\underline{\Pi}}}$?
7. Sometimes we see different versions of the total-stress tensor:

$$\text{This text and [174]:} \qquad \tilde{\underline{\underline{\Pi}}} = -p\underline{\underline{I}} + \tilde{\underline{\underline{\tau}}}$$

$$\text{Bird et al. and [12, 104]:} \qquad \underline{\underline{\Pi}} = p\underline{\underline{I}} + \underline{\underline{\tau}}$$

What is the difference between these two equivalent representations?
8. Figure 5.1 and the accompanying text discusses stress at a point in terms of the different stresses that different surfaces experience at the same point in a moving fluid. If the stress tensor $\tilde{\underline{\underline{\Pi}}}$ for a flow was calculated at Point P to be (arbitrary units):

$$\tilde{\underline{\underline{\Pi}}}|_P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{xyz}$$

calculate the force on the following six surfaces of unit area through P: The first surface is oriented with $\hat{n} = \hat{e}_x$. The five remaining surfaces are oriented with unit normals that are obtained from $\hat{e}_x$ by making successive 60-degree rotations around the z-axis. Plot these vectors.
9. For the velocity profile given here, what is $\frac{\partial v_z}{\partial z}$ along the line $x = 0$, $y = 0$? What is the value of that derivative on that line at locations $z = 1$, 2, and 3? What is the value of the velocity vector at those locations? Comment on the connection between the two quantities, $\underline{v}$ and $\frac{\partial v_z}{\partial z}$.

$$\underline{v} = -9x\hat{e}_x - 9y\hat{e}_y + 18z\hat{e}_z$$

10. For the flow in Figure 5.18, describe the motion of a fluid particle traveling on the streamlines shown. Are the fluid particles accelerating? How do you know?
11. Drag flow is a flow with velocity vector $\underline{v} = v_x(y)\hat{e}_x$ when written in the Cartesian coordinate system (i.e., flow in the x-direction). Calculate the velocity gradient tensor $\nabla\underline{v}$ (a 3×3 matrix) for this flow.
12. For the two-dimensional flow shown in Figure 4.77, write the velocity gradient tensor $\nabla\underline{v}$ in matrix form, indicating which coefficients and derivatives are zero. Are the fluid particles accelerating?
13. For a flow (Newtonian fluid) that may be written as $\underline{v} = v_x\hat{e}_x + v_z\hat{e}_z$, calculate the 3×3 matrix $\nabla\underline{v} + (\nabla\underline{v})^T$. What is $\tilde{\underline{\underline{\tau}}}$ for this flow?
14. For a flow (Newtonian fluid) that may be written as $\underline{v} = v_\theta\hat{e}_\theta$ in a cylindrical coordinate system, calculate the 3×3 matrix $\nabla\underline{v} + (\nabla\underline{v})^T$. What is $\tilde{\underline{\underline{\tau}}}$ for this flow?
15. For a flow (Newtonian fluid) that may be written as $\underline{v} = v_r\hat{e}_r + v_\theta\hat{e}_\theta$ in a cylindrical coordinate system, calculate the 3×3 matrix $\nabla\underline{v} + (\nabla\underline{v})^T$. What is $\tilde{\underline{\underline{\tau}}}$ for this flow?
16. In Example 5.1, we use the x-component of the momentum balance in a simple shear flow to show that the shear stress is constant in simple shear

flow. What conclusions can we draw from the y- and z-components of the momentum balance?

17. In the development of Newton's law of viscosity in Section 5.1.2, we discuss experimental results that show that in steady shear flow, F/LW is proportional to V/H, where F is the tangential pulling force on the top plate, A is the area of the plate, V is the speed of the plate, and H is the gap between the plates. Use your own experience with fluids to describe how force depends on gap in shear flow for constants A and V. Also, describe a situation that illustrates how force depends on area for constants H and V.
18. A fluid is made to flow in a parallel-plate apparatus with a narrow gap of 1.0 mm. The tangential force to move the top plate at 0.012 mm/s is 13 mN. What is the viscosity of the fluid in the gap? Give the answer in centipoise and American engineering units. The plate area is 9.1 cm^2.
19. What is the vector velocity field in a simple shear flow produced in the flow between two large parallel plates? The top plate is moved at 1.2 mm/s and the gap between the plates is 0.5 mm. What is the velocity (magnitude and direction) in the plane halfway between the plates?
20. What is the shear stress if peanut butter is sheared in a narrow-gap parallel plate device? The gap is set at 0.8 mm and the velocity of the upper plate is 0.1 mm/s. What is the shear stress if the gap doubles?
21. In a narrow-gap parallel-plate device, a Newtonian fluid is made to flow in steady-drag flow. If the shear rate (i.e., shear rate = velocity/gap in this flow) is cut in half, what happens to the stress on the upper plate?
22. What is the extra-stress tensor for a Newtonian fluid undergoing the uniform flow described by the velocity profile given here? U_∞ is a constant.

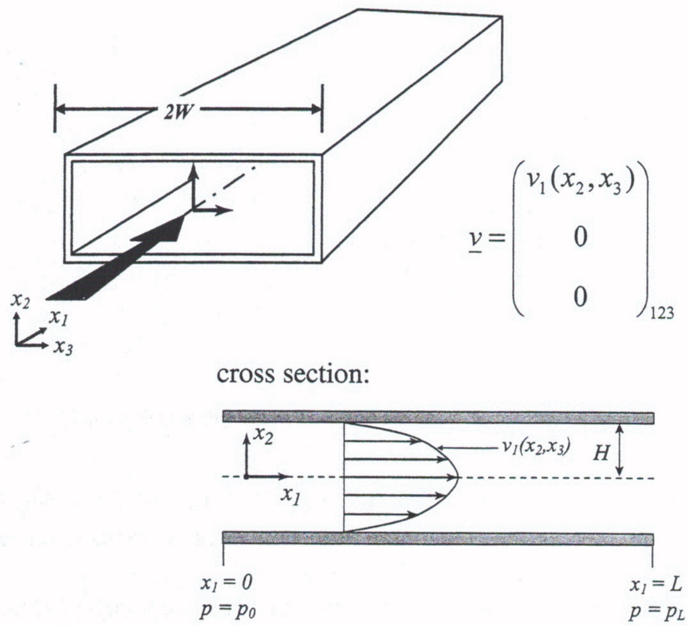
$$\underline{v} = \begin{pmatrix} U_\infty \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

23. What is the extra-stress tensor for a Newtonian fluid undergoing the shear flow described by the velocity profile given here?

$$\underline{v} = \begin{pmatrix} -8y \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

24. What is the extra-stress tensor for a Newtonian fluid undergoing the pipe flow described by the velocity profile given here? V and R are constants.

$$\underline{v} = \begin{pmatrix} 0 \\ 0 \\ V\left(1 - \frac{r^2}{R^2}\right) \end{pmatrix}_{r\theta z}$$



$$\underline{v} = \begin{pmatrix} v_1(x_2, x_3) \\ 0 \\ 0 \end{pmatrix}_{123}$$

Figure 5.37 Unidirectional flow in a rectangular channel varies in three dimensions (Problem 30).

25. What is the extra-stress tensor for a Newtonian fluid undergoing the elongational flow described by the velocity profile given here?

$$\underline{v} = \begin{pmatrix} -3x \\ -3y \\ 6z \end{pmatrix}_{xyz}$$

26. What is the extra-stress tensor for a Newtonian fluid undergoing the flow in the spiral vortex tank with the velocity profile given here? K is a constant. See Chapter 6 for more details on calculating velocity profiles in flow.

$$\underline{v} = \begin{pmatrix} 0 \\ \frac{K}{r} \\ 0 \end{pmatrix}_{r\theta\phi}$$

27. When a velocity field in a flow has been calculated, it is straightforward subsequently to calculate the stress tensor. For creeping flow around a sphere, calculate $\underline{\underline{\tilde{\Pi}}}$ from the solution given in Equations 5.101 and 5.102.
28. For the problem of a film flowing down an inclined plane discussed in this chapter, what is the maximum value of the velocity? What is the average flow rate?
29. For the problem of a film flowing down an inclined plane discussed in this chapter, what is the maximum value of the shear stress? How does shear stress vary across the film thickness?
30. How do the velocity components simplify for the unidirectional flow of water through a rectangular channel (Figure 5.37)? For a Newtonian fluid, how does the extra-stress tensor simplify as a result of this?

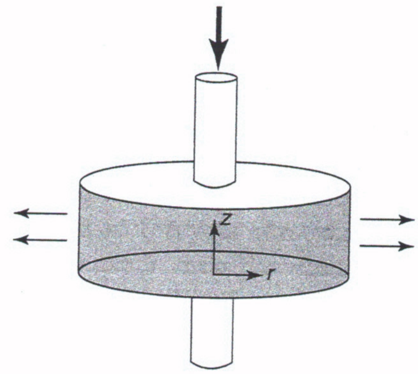


Figure 5.38 Squeeze flow between two parallel plates is a common flow test geometry (Problem 31).

31. How do the velocity components simplify for the squeezing flow shown in Figure 5.38? For a Newtonian fluid, how does the extra-stress tensor simplify as a result of this?
32. How do the velocity components simplify for the wide-slit planar-contraction flow shown in Figure 5.39? For a Newtonian fluid, how does the extra-stress tensor simplify as a result of this?
33. How do the velocity components simplify for axisymmetric-contraction flow shown in Figure 5.40? For a Newtonian fluid, how does the extra-stress tensor simplify as a result of this?
34. The problem of a thin film falling down an incline is discussed in this chapter. Figure 5.41 is a version of this problem in which a Cartesian coordinate system is proposed. Write the velocity vector in this coordinate system (i.e., which components are zero?) and the boundary conditions. How does the

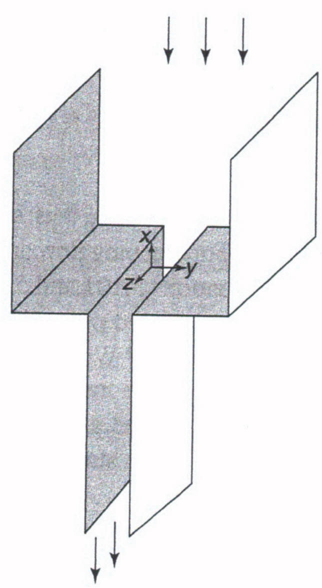


Figure 5.39 Planar contraction flow occurs when a larger reservoir drains through a slot (Problem 32).

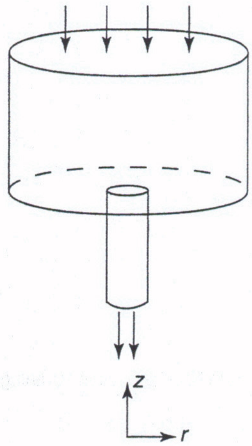


Figure 5.40 Axisymmetric-contraction flow occurs when a larger round reservoir drains through a pipe (Problem 33).

Newtonian constitutive equation simplify? Comment on the chosen coordinate system.

35. We solved the incline problem using a coordinate system that sits on the solid surface (Figure 5.12). Here, choose instead to solve the same problem with a coordinate system in which the coordinate position $x = 0$ is located at the free surface (i.e., the top of the film) (Figure 5.42). What is the gravity acceleration vector in this coordinate system? What are the flow-boundary conditions? Are there any advantages or disadvantages to this choice of coordinate system? Discuss your observations.
36. In a drag flow in the x-direction with the gradient in the y-direction, the force on the top plate is measured as $\underline{f} = 16\hat{e}_x + 2\hat{e}_y$. Is the fluid Newtonian? How do you know one way or the other?
37. The text indicates that the generalized Newtonian models cannot predict normal stresses in shear; show that this is true. (Hint: Write $\underline{\underline{\dot{\gamma}}}$ for shear flow and calculate stresses from the constitutive equation.)

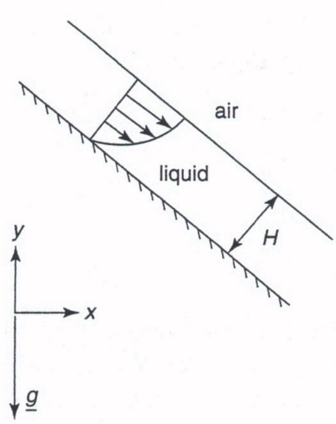


Figure 5.41 Flow problems may be solved in any coordinate system. Some choices are much better than others, however (Problem 34).

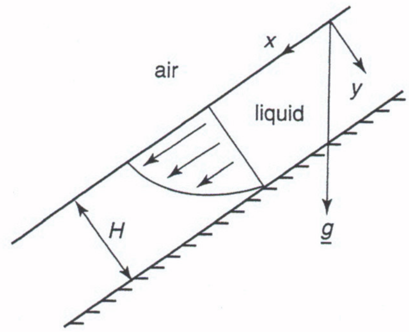


Figure 5.42 Flow problems may be solved in any coordinate system. The choice here has hidden advantages (Problem 35).

38. What are the units of the power-law parameters m and n in the power-law model for viscosity, $\eta = m\dot{\gamma}^{n-1}$ (see Equation 5.242)? Why are the units so strange? Is there an error?
39. For a high molecular-weight polymer, the stress response is modeled with the generalized Newtonian fluid constitutive equation, $\underline{\underline{\tilde{\tau}}} = \eta(\dot{\gamma}) \left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$. The viscosity function $\eta(\dot{\gamma})$ is measured as:

$$\eta[\text{Pa s}] = \left(3.4 \times 10^5\right) \dot{\gamma}^{-0.43} = \left(3.4 \times 10^5\right) \left|\frac{\partial v_1}{\partial x_2}\right|^{-0.43}$$

where $\dot{\gamma} = \left|\frac{\partial v_1}{\partial x_2}\right|$ is given in units of s^{-1}. Plot the viscosity function (i.e., log-log plot). In a flow with $\underline{v} = (0.90\ 1/\text{s})x_2\hat{e}_1$, what is $\tilde{\tau}_{21}$?
40. Plot the viscosity-versus-shear rate for a power-law generalized Newtonian fluid. The parameters of the model are m = 3,200 Pa $\text{s}^{0.54}$ and n = 0.54. Choose a range of shear rate that is physically reasonable. The plot should be logarithmic on both axes.
41. Plot the viscosity versus shear rate for a Carreau–Yasuda Generalized Newtonian fluid. The parameters of the model are listed here. Choose a range of shear rate that is physically reasonable. The plot should be logarithmic on both axes:

$$\eta_0 = 3{,}500 \text{ Pa s}$$
$$\hat{n} = 0.42$$
$$a = 2.0$$
$$\eta_\infty = 0$$
$$\lambda = 11 \text{ s}$$

42. The units of the power-law model do not follow accepted rules in physics. What are the units on m in the power-law model? We can normalize the units of this model by introducing a parameter λ to nondimensionalize the shear rate $\dot{\gamma}$ [36]:

$$\text{Revised power-law model:}\quad \eta = \tilde{\eta}\,(\lambda\dot{\gamma})^{n-1}$$

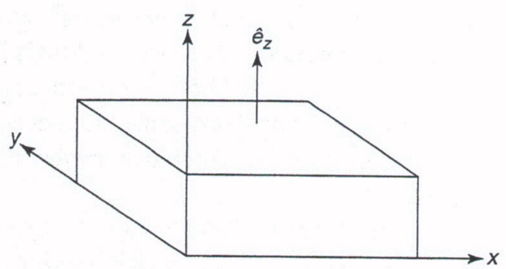


Figure 5.43 A control volume shaped like a rectangular parallelepiped (Problem 44).

For viscosity in units of Pa s, what are the units of the parameters $\tilde{\eta}$ and n of this revised power-law model? Comment on the relative desirability of the revised power-law model compared to the traditional model.

43. What is the extra-stress tensor for a non-Newtonian power-law fluid undergoing the elongational flow described by the velocity profile given here?

$$\underline{v}[\text{m/s}] = \begin{pmatrix} -4x \\ -4y \\ 8z \end{pmatrix}_{xyz}$$

44. A control volume shaped like a rectangular parallelepiped is shown in Figure 5.43. For the velocity field given in Problem 43, how would you calculate the total fluid force on the z-surface of the control volume for a power-law GNF? What additional information would you need? The dimensions of the box in the x-, y-, and z-directions are L, W, and H, respectively.

45. What is the extra-stress tensor for a non-Newtonian power-law fluid undergoing the uniform flow described by the velocity profile given here? U_∞ is a constant.

$$\underline{v} = \begin{pmatrix} U_\infty \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

46. What is the extra-stress tensor for a non-Newtonian power-law fluid undergoing the shear flow described by the velocity profile given here? The parameter a is a constant.

$$\underline{v} = \begin{pmatrix} -ay \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

47. What is the extra-stress tensor for a non-Newtonian power-law fluid undergoing the pipe flow described by the velocity profile given here? V and R are constants.

$$\underline{v} = \begin{pmatrix} 0 \\ 0 \\ V\left(1 - \left(\frac{r}{R}\right)^{2.85}\right) \end{pmatrix}_{r\theta z}$$

48. For Newtonian fluids, the extra-stress tensor is given by $\tilde{\underline{\underline{\tau}}} = \mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$, where μ is a constant—the viscosity. Fluids like "oobleck" (i.e., cornstarch and water) exhibit unusual behavior that does not follow this equation. Investigate the behavior of oobleck on the Internet and indicate the features of the Newtonian constitutive equation that prevent it from describing the behavior of oobleck.
49. For Newtonian fluids, the extra-stress tensor is given by $\tilde{\underline{\underline{\tau}}} = \mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$, where the viscosity μ is a constant. Fluids like pizza dough exhibit unusual behavior that does not follow this equation. Investigate and indicate what must be changed in the Newtonian constitutive equation to correctly describe the behavior of pizza dough.
50. For Newtonian fluids, the extra-stress tensor is given by $\tilde{\underline{\underline{\tau}}} = \mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$, where the viscosity μ is a constant. Fluids like high molecular-weight polymer melts exhibit behavior that does not follow this equation. Investigate and indicate what must be changed in the Newtonian constitutive equation to correctly describe the behavior of entangled polymer melts.