unknown). We use Equation 9.431 to eliminate $\langle v\rangle_2$ from Equation 9.452:

$$\frac{y_1\langle v\rangle_1}{g}\left(\frac{\langle v\rangle_1 y_1}{y_2}-\langle v\rangle_1\right)=\frac{1}{2}\left(y_1^2-y_2^2\right) \tag{9.453}$$

$$\frac{y_1\langle v\rangle_1^2}{gy_2}(y_1-y_2)=\frac{1}{2}(y_1+y_2)(y_1-y_2) \tag{9.454}$$

Canceling (y_1-y_2), multiplying through by y_2/y_1^2, and simplifying, we obtain:

$$\frac{2\langle v\rangle_1^2}{gy_1}=\left(\frac{y_2}{y_1}\right)+\left(\frac{y_2}{y_1}\right)^2 \tag{9.455}$$

This is a quadratic equation for y_2/y_1, which we now solve:

Height of a hydraulic jump:
$$\frac{y_2}{y_1}=-\frac{1}{2}+\frac{1}{2}\sqrt{1+8\left(\frac{\langle v\rangle_1^2}{gy_1}\right)} \tag{9.456}$$

We choose the root that gives a positive y_2/y_1. The quantity $\frac{\langle v\rangle_1^2}{gy_1}$ is the Froude number for this flow (see Chapter 7):

Froude number:
$$\boxed{\mathrm{Fr}\equiv\frac{V^2}{gD}}\quad \text{ratio of } \frac{\text{(inertial forces)}}{\text{(gravity forces)}} \tag{9.457}$$

where $D=y_1$ is the characteristic lengthscale of this flow. We can calculate the height of the hydraulic jump for our problem using the final result in Equation 9.456:

Height of the spillway jump:
$$y_2=-\frac{y_1}{2}+\frac{y_1}{2}\sqrt{1+8\left(\frac{\langle v\rangle_1^2}{gy_1}\right)} \tag{9.458}$$

$$=\boxed{3.6\text{ ft}} \tag{9.459}$$

The Froude number for this flow is 30.

For more information on open-channel flows, consult the literature [178, 183].

9.3 Problems

1. The name "mechanical energy balance" implies, perhaps, that "mechanical energy" exists. Does it? Does it balance? Explain the meaning of the name of this important equation.
2. What is "macroscopic" about the macroscopic mass, momentum, and energy balances? How are these balances different compared to their microscopic analogs?

3. In the mechanical energy balance, the symbol Δ signifies "out" minus "in." If we mistakenly write "in" minus "out," what are the consequences to the final results? Discuss various scenarios. Is this distinction important in all cases?
4. What is the difference between solving for the vector force on a surface with the macroscopic momentum balance and solving for the vector force on a surface with the following equation?

$$\underline{F} = \iint_S [\hat{n} \cdot \underline{\underline{\tilde{\Pi}}}]_{surface} \, dS$$

5. Show that the energy velocity-profile parameter α is approximately equal to 0.5 for laminar tube flow.
6. Show that the energy velocity-profile parameter α is equal to 0.99 for turbulent flow. Assume that the velocity profile in turbulent flow is given by Equation 9.43.
7. For a slit flow with the velocity profile given here, what are the correct values of the momentum velocity-profile parameter β and the energy velocity-profile parameter α?

$$\underline{v} = v_x \hat{e}_x$$

$$v_x = v_{max} \left(1 - \frac{y}{H}\right)^{0.24}$$

8. For laminar flow of water (25°C) in a pipe that is 200.0 km long, what are the frictional losses? The pipe inner diameter is 40.0 cm and the flow rate is the highest it can be and still be laminar flow.
9. Derive the rule of thumb that the losses in turbulent flow in a pipe that is 50 diameters long are approximately equal to one velocity head [43].
10. Three 10-foot horizontal sections of pipe are connected and water at room temperature is pushed through by an upstream pressure of 55 psig. The three sections are 1/2-inch, 3/8-inch, and 1/4-inch, nominal type L, copper tubing. What is the flow rate through this series of tubes? At the exit, the 1/4-inch tubing is open to the atmosphere. Do not neglect velocity head changes.
11. Apply the appropriate balances to the fittings shown in Figure 9.46 and show that the expressions obtained are consistent with Equation 1.120.
12. For the flow loop in Figure 1.17, a colleague states that it is not necessary to do the detailed calculation. She suggests that you consider only the straight-pipe friction and neglect all of the fittings. What error would be associated with adopting her suggestion? Is it a good idea?
13. A vertical manometer tube is attached to the wall of a closed-channel water flow as shown in Figure 9.12. The flowing liquid rises in the manometer tube to a height of 12 cm. What is the pressure at that location in the flow? Give your answer in psig and psia.
14. A Pitot-static tube is installed through the wall of a water flow (27°C) such that the curved end (i.e., the Pitot tube) directly faces the oncoming flow at the center of the pipe. The liquid rises in the Pitot tube to a height of 34 cm. In the static tube, the fluid height is 12 cm. What is the fluid velocity in the pipe?

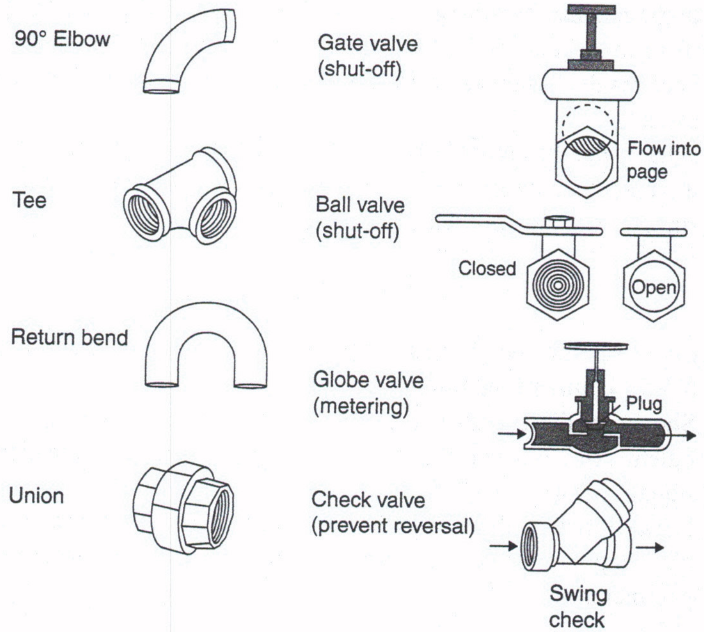

Figure 9.46 For these fittings, Problem 11 asks for the frictional losses.

15. The friction generated by straight piping can be written in terms of head. How many meters of 20 mm inner-diameter smooth tubing does it take to generate 1 meter of friction head? The Reynolds number of the flow is 20,000 and the fluid is water.
16. How does friction in a sudden contraction depend on fluid velocity? Can you derive your answer from fundamental relationships (i.e., mass, energy, and momentum balances)?
17. Devise a way to redefine the valve flow coefficient that is independent of the units of pressure drop and flow rate.
18. Show that for a noncircular conduit, the drag is given by:

$$\mathcal{F}_{\text{drag}} = \Delta p \; A_{xs}$$

where A_{xs} is the cross-sectional area of the conduit.
19. What is the direction and magnitude of the force needed to support the 90-degree expanding pipe bend shown in Figure 9.47? The water flow is steady and turbulent, the cross section of the inlet of the pipe bend is πR_1^2, and the cross section of the outlet of the bend is πR_2^2, where $R_2 > R_1$. Evaluate your answer for $R_1 = 0.545$ inch and $R_2 = 0.834$ inch and various values of flow parameters.
20. What is the direction and magnitude of the force needed to support the 60-degree expanding pipe bend shown in Figure 9.48? The water flow is steady and turbulent, the cross section of the inlet of the pipe bend is πR_1^2, and the cross section of the outlet of the bend is πR_2^2, where $R_2 > R_1$. Evaluate your answer for $R_1 = 0.545$ inch and $R_2 = 0.834$ inch.
21. What is the direction and magnitude of the force needed to support the 90-degreee contracting pipe bend shown in Figure 9.49? The water flow is steady and turbulent, the cross section of the outlet of the pipe bend is πR_1^2, and the

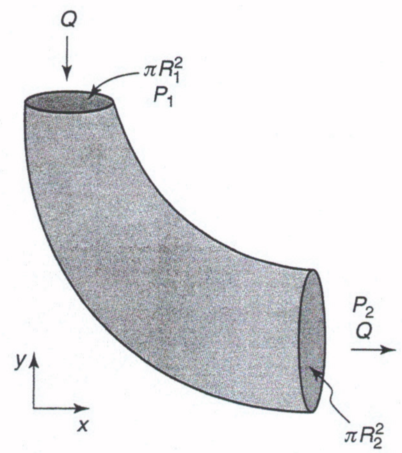

Figure 9.47 Force on a 90-degree expanding bend (Problem 19).

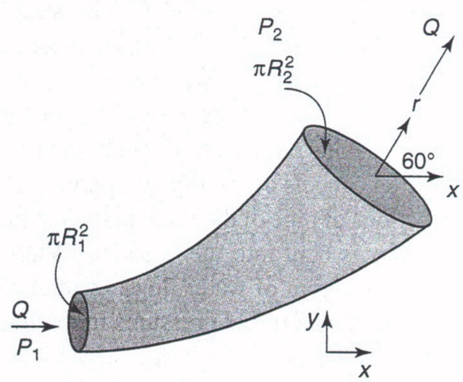

Figure 9.48 Force on a 60-degree expanding bend (Problem 20).

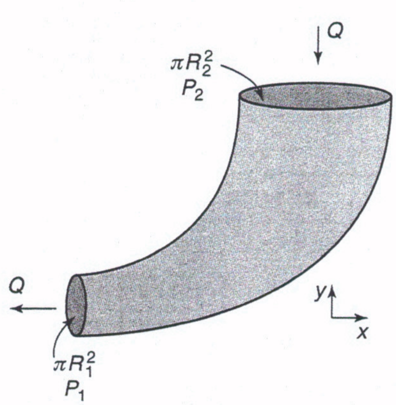

Figure 9.49 Force on a 90-degree contracting bend (Problem 21).

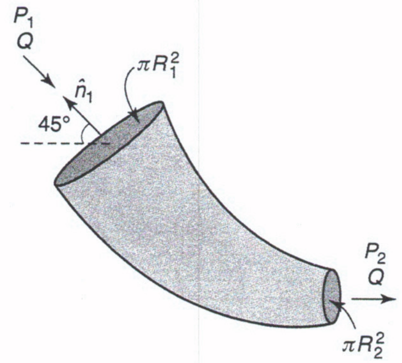

Figure 9.50 Force on a 45-degree contracting bend (Problem 22).

cross section of the inlet of the bend is πR_2^2, where $R_2 > R_1$. Evaluate your answer for $R_2 = 0.834$ inch and $R_1 = 0.430$ inch.

22. What is the direction and magnitude of the force needed to support the 45-degree contracting pipe bend shown in Figure 9.50? The water flow is steady and turbulent, the cross section of the inlet of the pipe bend is πR_1^2, and the cross section of the outlet of the bend is πR_2^2, where $R_2 < R_1$.
23. For the flow in a U-tube bend, carry out the integration in Equations 9.258 and 9.260 to show that the viscous contribution due to the rate-of-change of velocity in the flow direction cancels out in this problem.
24. Consider two different sections of pipe of the same diameter and same length. One is straight and the other is bent into the U shape. What are the forces on these two sections when 3.0 gpm of water flows through them (installed horizontally)? Assume the inlet and outlet pressures in the two cases are the same. Discuss the effect of the shape of the fitting on the force to which the fitting is subjected.
25. A Y-shaped piping installation for water flow is extended as shown in Figure 9.51 by the addition of a 70-foot section of the same type of pipe. The flow rate into the piping is a constant value of 4.0 gpm. What is the flow-rate split before the modification? What is the flow-rate split after the modification? All of the piping is 1/2-inch nominal Schedule 40 steel pipe. You may neglect the losses in the fittings.

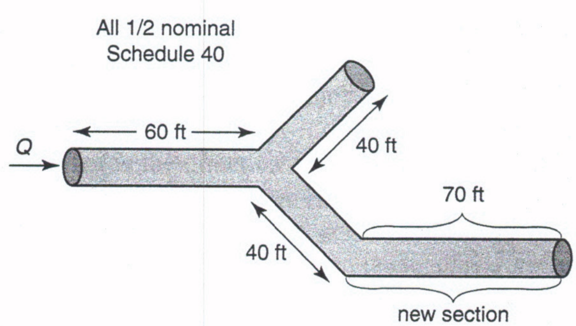

Figure 9.51 A modified Y-shaped piping installation with a split (Problem 25).

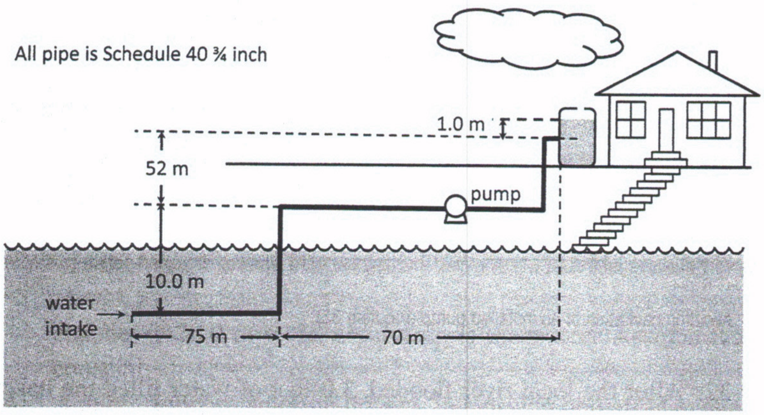

Figure 9.52 Designing a lawn-irrigation system (Problem 30).

26. For the installation described in Problem 25, if the inlet flow pressure is held constant (instead of the flow rate) at 60 psig, what is the flow-rate split before and after the modification?
27. What is the difference between a centrifugal pump and a positive-displacement pump?
28. What is net positive suction head? What is the danger involved in ignoring NPSH when installing a pump?
29. What are the signs and implications of cavitation?
30. A lawn-irrigation system is to be built next to a natural pond. The installers plan to obtain water for irrigation directly from the lake. To bring the water from the lake, they plan to install the piping system shown schematically in Figure 9.52. When the pump is running to fill the storage tank, they desire a flow rate of at least 14 liters/min. Which pump should they install? If possible choose your suggestion from among those with pumping-head curves in Figure 9.29.
31. For the pump installation shown in Figure 9.53, calculate and plot the system curve. The tank water levels are 6 feet apart in elevation. What is the minimum head needed to pump water in this loop at low flow rates? All pipe is Schedule 40.

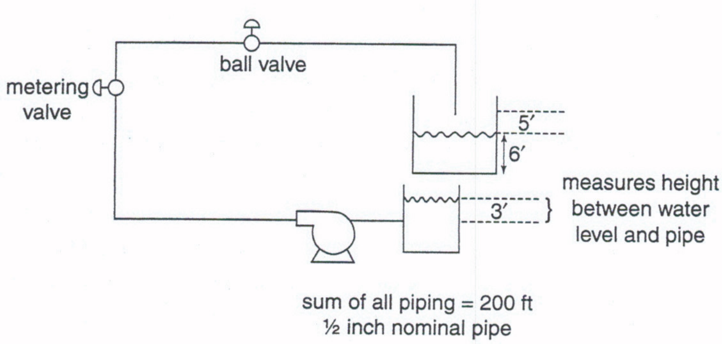

Figure 9.53 To choose a pump for the system shown (Problem 31), a system-head curve is constructed.

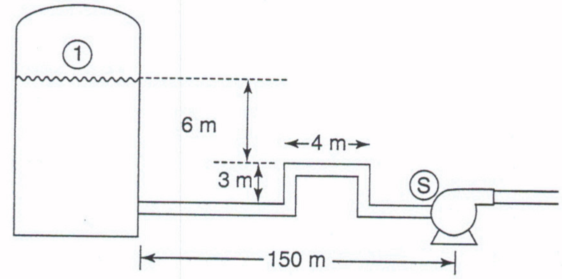

Figure 9.54 Avoiding cavitation in an installed pump (Problem 35).

32. After the local river flooded, 3.0 feet of water filled the basement of a university building. Several pumps are available to help empty the basement (see Figure 9.29). Which pump do you recommend for this operation? Justify your answer. Make reasonable assumptions.
33. Careful calculation of frictional losses in a system led to the following equation for system head in feet as a function of flow rate Q in gpm. At the last minute, the final tank in the installation was raised 5.0 feet. What is the new curve for system head?

$$H_{\text{system}} = 0.023Q^2 + 35.2Q + 34$$

34. The frictional losses in a system are represented accurately by the following equation for system head in feet as a function of flow rate Q in gpm.

$$H_{\text{system}} = 0.023Q^2 + 34$$

Included in the frictional losses are those for a metering valve. The equation was calculated for the valve fullopen. Plot the system-head equation. Sketch qualitatively how the curve will shift to if the metering valve is closed halfway.

35. The pump with characteristic curves given in Figure 9.38 is installed as shown in Figure 9.54. The pressure in the tank headspace is 1,660 mmHg, and the desired flow rate is 2.2×10^4 gpm. If frictional head loss $h_f = F_{s,1}/g$ from the reservoir to the pump inlet is 11.0 feet, at what elevation should the pump suction be placed relative to the feed-tank fluid level to avoid cavitation for water at 122°F? All piping is 6.0 inches ID.
36. What data must be collected to determine the efficiency of a pump?
37. What is a hydraulic jump?
38. When heavy rains cause flooding, the water moves downstream under the pull of gravity. For a given flow rate, if the water moved very fast, the depth of the moving water could be shallow. For the same flow rate if the water moved less rapidly, the moving water would be very deep. What physics determines which of these two states nature chooses?
39. A rough cement rectangular channel (roughness $\varepsilon = 0.0085$ feet) slopes downward at an angle of 1.2 degrees carrying water. The channel is 20.0 m wide and the water depth is 1.2 m. What is the flow rate in the channel? Assume that the depth is constant.
40. A rough cement rectangular channel (roughness $\varepsilon = 0.0080$ feet) slopes downward at an angle of 3.2 degrees carrying water at 2.0 million gpm. The

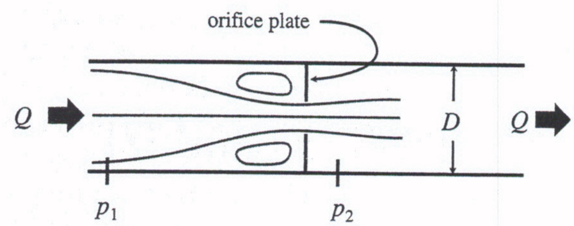

Figure 9.55 An orifice meter measures flow rate from the pressure drop across a plate with a hole in the center (Problem 43).

channel is 20.0 m wide. How deep is the water? Assume that the depth is constant.

41. A dilute aqueous solution flowing in a drying operation over a tilted surface encounters a slope change in the middle of its passage down the surface. The initial slope of the surface is 20 degrees relative to horizontal, and the new slope is 10 degrees relative to horizontal. The slope change produces a hydraulic jump. The flow is 15 m wide and the fluid velocity in the upstream region is 0.52 m/s. The fluid depth in the upstream region is 1.0 cm. What is the fluid depth after the hydraulic jump?
42. Calculate the flow rate for the circumstances described in Example 9.19. Show all of your work.
43. An orifice meter (see Figure 9.55 and Glossary) is a device that is used to measure flow rates of liquids and gases. The flow in a pipe of inner diameter D is obstructed by a plate with an orifice of diameter D_0. The flow streamlines contract from an upstream cross-sectional area of $\pi D^2/4$ to a jet of approximate cross-sectional area $\pi D_0^2/4$. The pressure in the pipe is measured upstream and slightly downstream of the orifice plate as shown in Figure 9.55. Show that the volumetric flow rate may be obtained from the following equation:

$$Q = \frac{\pi D^2}{4}\sqrt{\frac{\frac{2(p_2 - p_1)}{\rho}}{\left(1 - \frac{D^4}{D_0^4}\right)}}$$

You may neglect friction; friction in an orifice meter is accounted for by including a prefactor C_0, which is determined experimentally (For $\frac{D_0}{D} < 0.5$ and $Re = \frac{\rho v_2 D_0}{\mu} > 2 \times 10^4$, $C_0 \approx 0.61$ [132].