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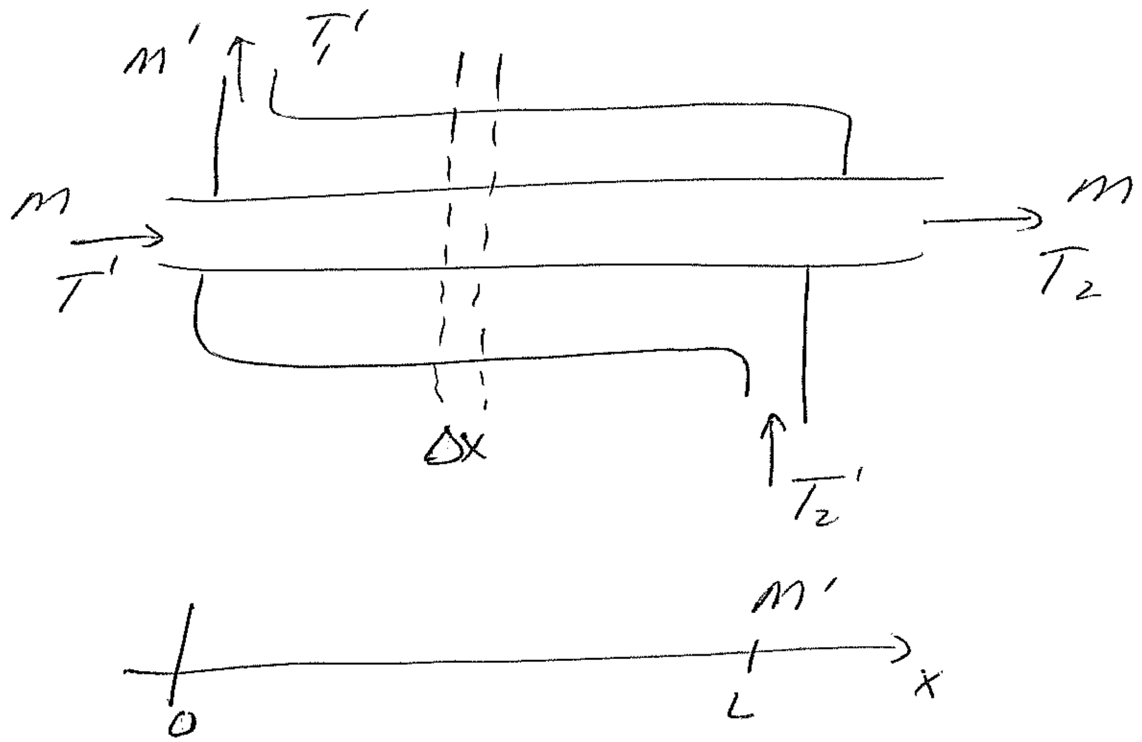
DOUBLE PIPE HEAT EXCHANGER

HOW DO WE RELATE OVERALL HEAT TRANSFER COEF U TO INLET + OUTLET TEMPS?



MACRO E-BAL INSIDE SMALL SYSTEM:

$$\Delta H = Q_{in,\,inner}$$

$$m\hat{H}_{x+\Delta x} - m\hat{H}_x = \Delta Q_{in,\,inner}$$

$$\boxed{\frac{d\hat{H}}{dx}\, m = \frac{dQ_{in,\,inner}}{dx}}$$

$$\boxed{m\hat{C}_p \frac{dT}{dx} = \frac{dQ_{in,\,inner}}{dx}}$$

MACRO E-BAL OUTSIDE

$$\Delta H = Q_{in,outer}$$

$$m'\hat{H}_x - m'\hat{H}_{x+\Delta x} = \Delta Q_{in,outer}$$

$$-m'\frac{d\hat{H}}{dx} = \frac{dQ_{in,outer}}{dx}$$

$$\boxed{-m'\hat{C}_p'\frac{dT'}{dx} = \frac{dQ_{in,outer}}{dx}}$$

OVERALL E-BAL

$$\Delta H = 0 = \Delta H_{inner} + \Delta H_{outer}$$

$$= \Delta Q_{in,inner} + \Delta Q_{in,outer}$$

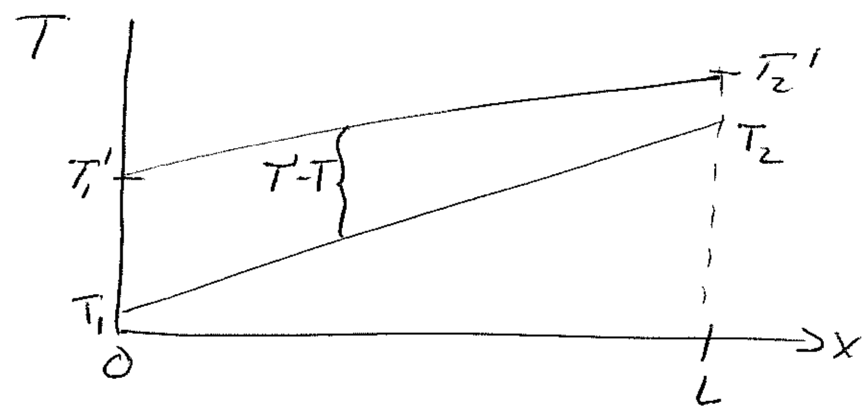
$$\boxed{\frac{dQ_{in}}{dx} \equiv \frac{dQ_{in,inner}}{dx} = -\frac{dQ_{in,outer}}{dx}}$$

SUBSTITUTING,

$$\begin{cases} m\hat{C}_p \dfrac{dT}{dx} = \dfrac{dQ}{dx} \\ m'\hat{C}_p' \dfrac{dT'}{dx} = \dfrac{dQ}{dx} \end{cases}$$

$$\left.\begin{aligned} \frac{dT}{dx} &= \frac{1}{m\hat{C}_p} \frac{dQ}{dx} \\ \frac{dT'}{dx} &= \frac{1}{m'\hat{C}_p'} \frac{dQ}{dx} \end{aligned}\right\} \text{SUBTRACT TO GET } T'-T$$

$$\frac{d}{dx}(T'-T) = \underbrace{\left(\frac{1}{m'\hat{C}_p'} - \frac{1}{m\hat{C}_p}\right)}_{\text{constants}} \underbrace{\frac{dQ}{dx}}_{\text{depends on } T'-T}$$



HOW CAN WE WRITE $\frac{dQ}{dx}$ IN TERMS OF $T' - T$?

ANSWER: OVERALL HEAT XFER COEF

$$dQ_{in} = UA\Delta T$$
$$= U(2\pi R\,dx)(T' - T)$$

$$\boxed{\frac{dQ_{in}}{dx} = U\,2\pi R\,(T' - T)}$$

$$\frac{d\overbrace{(T'-T)}^{Y}}{dx} = \underbrace{\left(\frac{1}{\dot{m}' C_p'} - \frac{1}{\dot{m} C_p}\right) U\,2\pi R}_{\equiv \alpha_0} (T' - T)$$

DIFFERENTIAL EQN FOR $Y = T' - T$

$$\boxed{\frac{dY}{dx} = \alpha_0\,Y}$$

$$\frac{dY}{Y} = \alpha_0 \, dx$$

$$\ln Y = \alpha_0 X + C_1$$

BOUNDARY CONDITIONS :

$$X = 0$$
$$T' = T_1'$$
$$T = T_1$$
$$\Longrightarrow Y = T_1' - T_1$$

$$\boxed{\ln(T_1' - T_1) = C_1}$$

$$\ln(T' - T) = \alpha_0 X + \ln(T_1' - T_1)$$

$$\ln\left(\frac{T' - T}{T_1' - T_1}\right) = \alpha_0 X$$

$$\boxed{\frac{T' - T}{T_1' - T} = e^{\alpha_0 X}}$$

$$\alpha_0 = \left(\frac{1}{\dot{m}' c_p'} - \frac{1}{\dot{m} c_p}\right) U 2\pi R$$

OUR GOAL IS TO RELATE U TO T_1, T_1', T_2, T_2'. WE CAN EVALUATE THE TEMP PROFILE AT $T' = T_2'$; $T = T_2$ FOR STARTERS.

$$\ln\left(\frac{T_2' - T_2}{T_1' - T_1}\right) = U \overbrace{2\pi R L}^{\text{HEAT XFER AREA}} \underbrace{\left(\frac{1}{m'\hat{C}_p'} - \frac{1}{m\hat{C}_p}\right)}$$

WE NEED TO RELATE THESE TO HEAT FLOW Q; THEN EQN WILL LOOK LIKE $Q = uA\, f(\Delta T)$

To RELATE $m\hat{C}_p$, $M'\hat{C}_p'$ to Q WE USE INNER/OUTER (TOTAL SYSTEM) MACROSCOPIC E-BAL.

E-BAL DOUBLE PIPE - INSIDE

$$\cancel{\Delta E_p} + \cancel{\Delta E_K} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

$$Q_{in} = \Delta H$$

$$\boxed{Q_{in,\,inside} = m \hat{C}_p (T_2 - T_1)}$$

E-BAL OUTSIDE

$$Q_{in} = \Delta H$$

$$\boxed{Q_{in,\,outside} = m' \hat{C}_p' (T_1' - T_2') = -Q_{in,\,inside}}$$

SOLVE FOR $m\hat{C}_p$, $m'\hat{C}_p'$

$$\boxed{\begin{aligned} m \hat{C}_p &= \frac{Q_{in}}{T_2 - T_1} \\ m'\hat{C}_p' &= \frac{Q_{in}}{T_2' - T_1'} \end{aligned}}$$

SUBSTITUTING,

$$\ln\left(\frac{T_2'-T_2}{T_1'-T_1}\right) = U(2\pi RL)\,\frac{\left(T_2'-T_1' - (T_2-T_1)\right)}{Q_{in}}$$

$$Q_{in} = U(2\pi RL)\,\frac{(T_2-T_1)-(T_2'-T_1')}{\ln\left(\dfrac{T_1'-T_1}{T_2'-T_2}\right)}$$

$$\boxed{\begin{aligned} Q_{in} &= U\,\underbrace{(2\pi RL)}_{A}\,\underbrace{\left(\frac{(T_1'-T_1)-(T_2'-T_2)}{\ln\left(\dfrac{T_1'-T_1}{T_2'-T_2}\right)}\right)}_{\equiv\, \Delta T_{lm}} \\ Q &= U A \,\Delta T_{lm} \end{aligned}}$$

LOG MEAN
TEMP
DIFFERENCE.