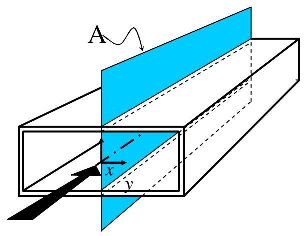
What about more complicated problems?

EXAMPLE IV: Pressure-driven flow of a Newtonian fluid in a rectangular duct:

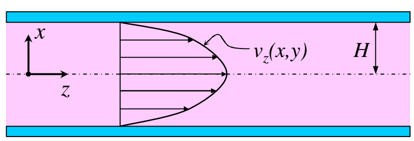
Poiseuille flow

- •steady state
- •well developed
- •long tube
- •$P(0)=P_o$, $P(L)=P_L$



$$\underline{v} = \begin{pmatrix} 0 \\ 0 \\ v_z(x,y) \end{pmatrix}_{xyz}$$

cross-section A:



Velocity varies in two directions

Tricky step: **Solving for v and τ_{ij} can be difficult**

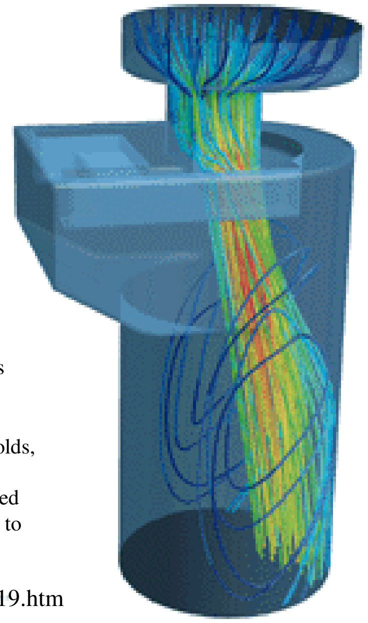
- •partial differential equation in up to three variables
- •boundaries may be complex
- •multiple materials, multiple phases present
- •non-Newtonian fluids

Solution strategies:

- •advanced analytical techniques for solving partial differential equations (PDEs) -- see Bird, et a. *Transport Phenomena*, 1960 or 2001
- •numerical techniques (*Polyflow*)

-- see *http://www.fluent.com/worldwide/polyflow/*

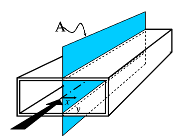
Fluent : Solutions : Example X19
Fluid Handling and Flow
Distribution



Streamlines depict the flow of regenerated catalyst through a slide valve, revealing the source of erosion problems.

"Transport and storage of gases, liquids, or slurries represents a large capital and operating expense in process plants. Fluent's CFD software helps you to design for flow uniformity, balance flows in manifolds, minimize pressure drop, design storage tanks, and accurately size blowers, fans, and pumps. High-speed nozzles and spray systems can be analyzed in order to optimize performance."

http://www.fluent.com/solutions/examples/x19.htm
accessed 4 Feb 2002



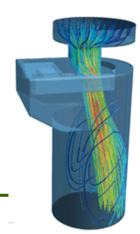
So far:

•We've learned to set-up and sometimes solve flow problems (conservation of mass, momentum)

Question:

•Do we always need to solve the modeling problems that real systems present?

•<u>Can</u> we solve them?



Most industrial flows are not simple:

- •piping
- •pumps
- •mixers
- •flow in an engine
- •fluidized beds
- •flow in a packed bed (catalytic reactor)
- •two-phase flows (extractors)
- •jets (jet engines, ink-jet printing)
- •coating flows
- •evaporators
- •heat exchangers

Most of these flows are impossible to solve in detail

Exception: plastics, high viscosity flows

•Questions:

•Do we always need to solve the modeling problems that real systems present? *-No, not always.*

•Can we solve them? *-No, not always.*

•What do we do instead? *-Experiments, scale-models, and data-correlations*

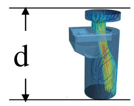
•What experiments do we do? ~~*•Random experiments and hope for the best*~~

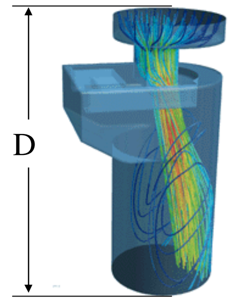
Better choice, but how? *•Small-scale pilot experiments that can scale to the real system*

Designing the Experiments

•Dimensional similarity

= similar proportions





•Dynamic similarity

= similar behavior

?

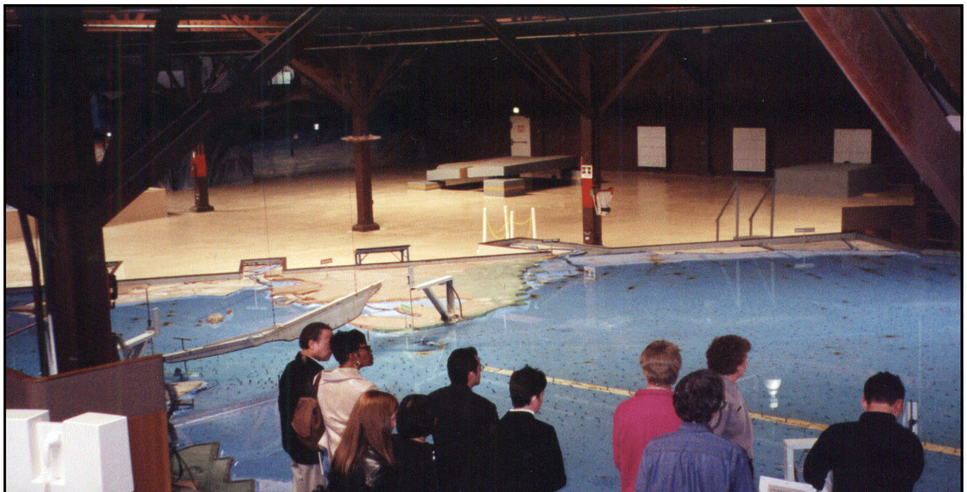
How systems **behave** depends on the laws of physics.

GOAL of Dimensional Analysis:

To use our knowledge of physical laws (mass, momentum, energy conservation) to guide our studies, modeling, and experimentation on complex (real engineering) flows
(i.e. to save us trial-and-error work)

Specifically:

- •To be able to design devices in which the flow is expected to be complex
- •To scale-up (relate) any experiments to similar flows that are not (yet) available for experimentation
- •To guide the use and production of data correlations (i.e. the plotting and reporting of experimental data)



San Francisco Bay Model, Sausalito, CA.

•distorted scale: the dimension in the vertical direction is 1/10th the scale in the horizontal direction.
•US Army Corps of Engineers
•used to evaluate proposed changes to the bay such as dams and other types of development.

Dimensional Analysis

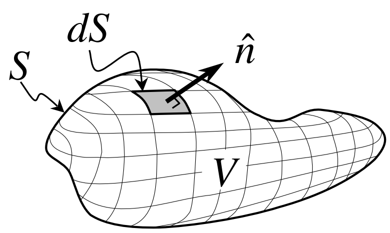
principle: even in complex systems, the same equations still apply:

governing equations
- *continuity equation* (mass conservation)
- *equation of motion* (momentum conservation)

strategy: render the governing equations dimensionless to identify the important parameters that apply in every situation.

$\Longrightarrow$ rely on experiments and data correlations

Continuity Equation

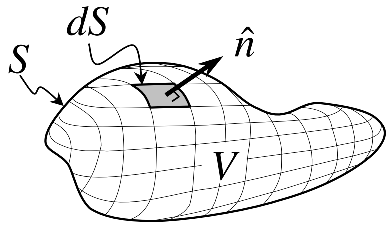


microscopic **mass** balance written on an arbitrarily shaped volume, V, enclosed by a surface, S

$$\boxed{\frac{\partial \rho}{\partial t}+v_x\frac{\partial \rho}{\partial x}+v_y\frac{\partial \rho}{\partial y}+v_z\frac{\partial \rho}{\partial z}=-\rho\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)}$$

Gibbs notation: $\frac{\partial \rho}{\partial t}+\underline{v}\cdot\nabla\rho=-\rho\left(\nabla\cdot\underline{v}\right)$

Equation of Motion



microscopic **momentum** balance written on an arbitrarily shaped volume, V, enclosed by a surface, S

Gibbs notation: $\boxed{\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla P-\nabla\cdot\underline{\underline{\tau}}+\rho\underline{g}}$ **general fluid**

Gibbs notation: $\boxed{\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla P+\mu\nabla^2\underline{v}+\rho\underline{g}}$ **Newtonian fluid**

Navier-Stokes Equation

Dimensional Analysis Procedure:

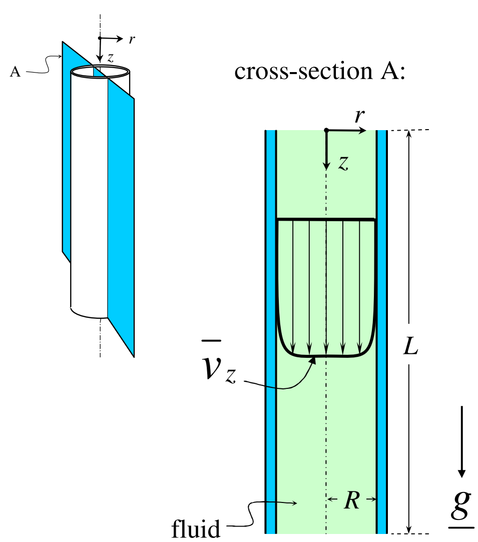
1. select appropriate differential equations and boundary conditions
2. select characteristic quantities with which to scale the variables, e.g. $\underline{v}$, x, P
   - characteristic quantities must be constant
   - must be representative of the system
3. scale all variables in the governing equations; yields dimensionless equation as a function of ***dimensionless groups***

The values of the dimensionless groups determine the properties of the differential equations.

4. design scaled-down experiments to develop ***data correlations*** for the system of interest
5. use data correlations to design and evaluate systems

OR

4. perform experiments on an existing system and ***correlate*** results using dimensionless groups



EXAMPLE I: **Pressure-driven flow of a Newtonian fluid in a tube:**

NOT Laminar

- steady state
- well developed
- long tube

locally the flow is 3D:

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z}$$

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$
$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

z-component of the **nondimensional** Navier-Stokes Equation:

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{\mu}{\rho V D}\left(\nabla^2 v_z\right)^* + \frac{gD}{V^2} g^*$$

$$\left(\nabla^2 v_z\right)^* \equiv \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial v_z^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^* \frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial v_z^*}{\partial \theta} + v_z \frac{\partial v_z^*}{\partial z^*}\right)$$

Two dimensionless groups:

$$\boxed{\mathrm{Re} = \frac{\rho V D}{\mu}}$$

Reynolds number = ratio of inertial to viscous forces

$$\boxed{Fr = \frac{V^2}{gD}}$$

Froude number = ratio of inertial to gravity forces

If for two systems Re and Fr are the same, the two systems are governed by the same momentum balance.

If the dimensionless boundary conditions are also the same, the two systems are mathematically identical

= Dynamic Similarity