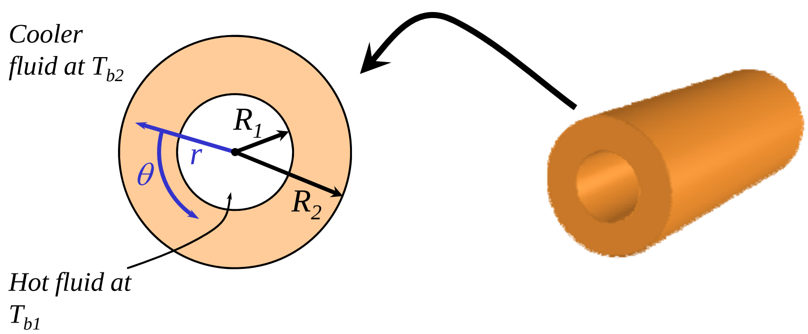
Example 4: Heat flux in a cylindrical shell

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

What is the steady state temperature profile in a cylindrical shell (pipe) if the ***fluid on the inside*** *is at T_{b1} and the* ***fluid on the outside*** *is at T_{b2}? ($T_{b1}>T_{b2}$)*



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Solution:

$$\frac{q_r}{A} = \frac{c_1}{r}$$

← Not constant

$$T = -\frac{c_1}{k}\ln r + c_2$$

Boundary conditions?

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$$\frac{c_1}{R_1} = h_1\left(T_{b1} - T_{w1}\right)$$

$$\frac{c_1}{R_2} = h_2\left(T_{w2} - T_{b2}\right)$$

$$T_{w1} = -\frac{c_1}{k}\ln R_1 + c_2$$

$$T_{w2} = -\frac{c_1}{k}\ln R_2 + c_2$$

4 equations

4 unknowns; c_1, T_{w1}, c_2, T_{w2}

SOLVE

Example 4: Heat flux in a cylindrical shell, Newton's law of cooling boundary Conditions

Results: Radial Heat flux in an Annulus

$$T - T_{b2} = \frac{\left(T_{b1} - T_{b2}\right)\left(\ln\left(\frac{R_2}{r}\right) + \frac{k}{h_2 R_2}\right)}{\frac{k}{h_2 R_2} + \ln\left(\frac{R_2}{R_1}\right) + \frac{k}{h_1 R_1}}$$

$$\frac{q_r}{A} = \left(\frac{\left(T_{b1} - T_{b2}\right)}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\right)\left(\frac{1}{r}\right)$$