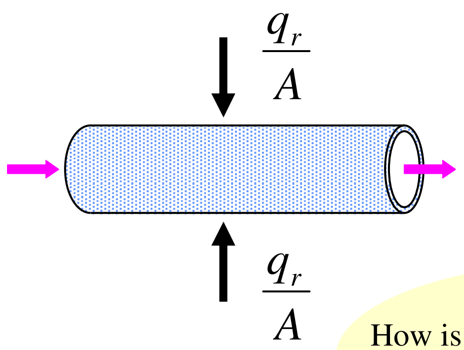
Now: How do we use Dimensional Analysis to develop correlations for *h*?

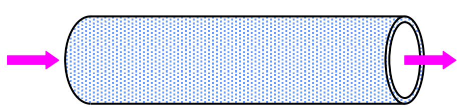
Our problem we are considering is heat-transfer to fluid inside of a heated tube – forced-convection heat transfer plus radial conduction



T_1= core temperature
T_o= wall temperature
$T(r,\theta,z)$ = fluid temp distribution

How is the heat transfer coefficient related to the full solution for $T(r,\theta,z)$?

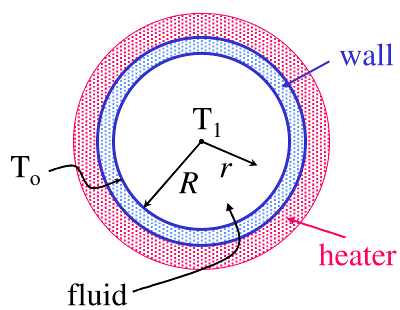
Heat-transfer coefficient on the inside of a heated tube

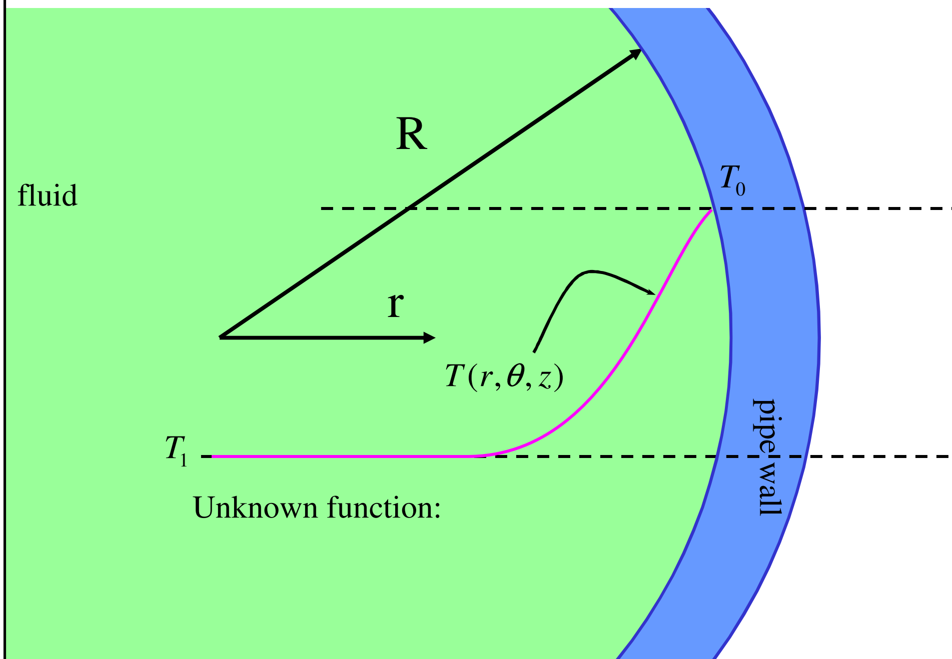


T_1= core temperature
T_o= wall temperature
$T(r,\theta,z)$ = fluid temp distribution

$T_o > T_1$

$\frac{q_r}{A} < 0$





At the wall (r = R), we can relate T profile to h by equating total energy transferred:

may be a function of θ, z

Total heat into the wall in terms of h (also negative)

$$\int_0^{2\pi}\int_0^{L} -k\left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta = q_r = h(2\pi RL)(T_1 - T_o)$$

Total heat conducted to the wall from the fluid (radial conduction only; $\underline{v} = 0$; negative)

Now, non-dimensionalize this expression

<u>Non-dimensionalize</u>

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$$

$$h(\pi \cancel{D} L)\cancel{(T_1 - T_o)} = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{(T_1 - T_o)}}{\cancel{D}} \frac{\cancel{D^2}}{2} dz^* d\theta$$

$$2\pi \underbrace{\left(\frac{hD}{k}\right)} \left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} - \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$\boxed{Nu = Nu\left(T^*, \frac{L}{D}\right)}$$

one additional dimensionless group

Now: How do we use Dimensional Analysis to develop correlations for h?

Non-dimensional Energy Equation

$$\frac{\partial T^*}{\partial t^*} + v_x^* \frac{\partial T^*}{\partial x^*} + v_y^* \frac{\partial T^*}{\partial y^*} + v_z^* \frac{\partial T^*}{\partial z^*} = \frac{1}{\mathrm{Pe}} \left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) + S^*$$

$$\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}} \left(\nabla^2 v_z \right)^* + \frac{1}{Fr} g^*$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

Definition of h

$$Nu = \frac{1}{2\pi L / D} \int_0^{2\pi} \int_0^{L/D} -\left. \frac{\partial T^*}{\partial r^*} \right|_{r^*=1/2} dz^* d\theta$$

Now: How do we use Dimensional Analysis to develop correlations for h?

According to our dimensional analysis calculations, the dimensionless heat transfer coefficient should be found to be a function of four dimensionless groups:

no free surfaces

$$\boxed{Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D} \right)}$$

Laminar flow in pipes: data correlation

$$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

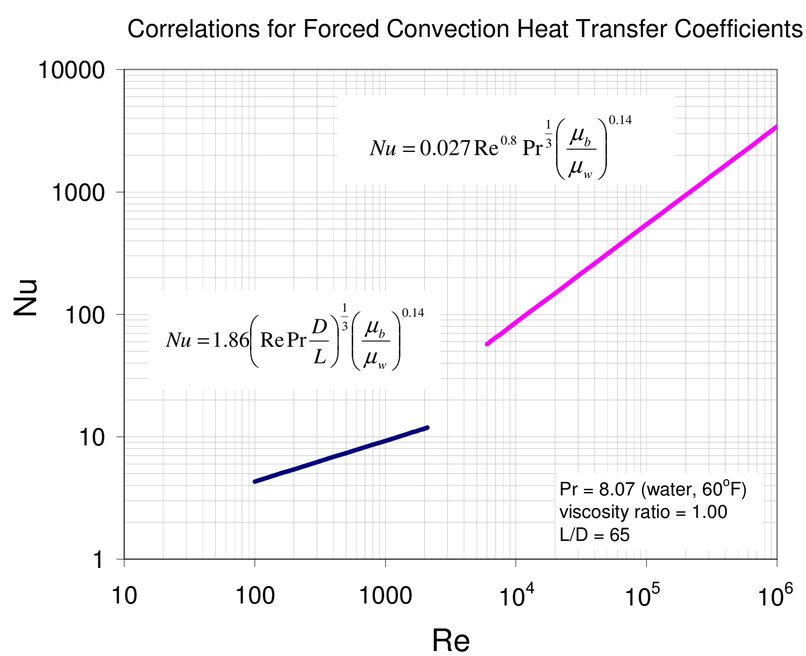
the subscript "*a*" refers to the type of average temperature used in reporting the correlation

$$q = h_a A \Delta T_a$$

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

Geankoplis, 4th ed. eqn 4.5-4, page 260

Re<2100, (RePrD/L)>100, horizontal pipes; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.



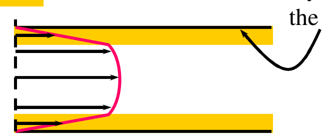
? We assumed constant ρ, k, μ, etc. Therefore we did not predict a viscosity-temperature dependence. If viscosity is not assumed constant, the dimensionless group shown below is predicted to appear in correlations.

$$\boxed{Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}}$$

for more information, see Bird, Steward, Lightfoot, *Transport Phenomena*, Wiley, 1960, page398

Viscous fluids with large ΔT

heating



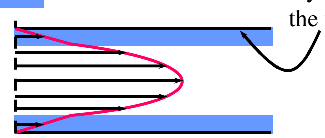
lower viscosity fluid layer speeds flow near the wall ==> higher h

$\mu_b > \mu_w$

empirical result:

$$\boxed{\left(\frac{\mu_b}{\mu_w}\right)^{0.14}}$$

cooling



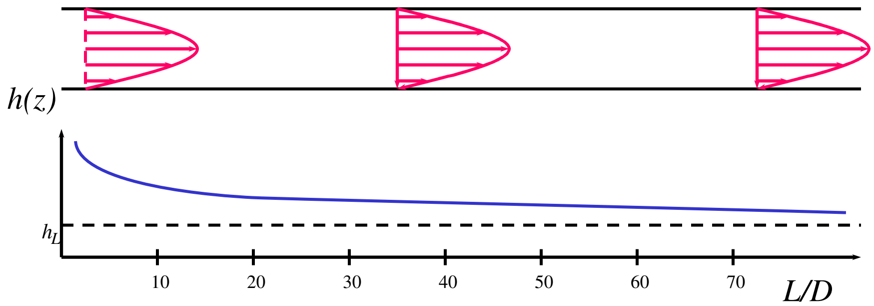
higher viscosity fluid layer retards flow near the wall ==> lower h

$\mu_b < \mu_w$

ref: McCabe, Smith, Harriott, 5th ed, p339

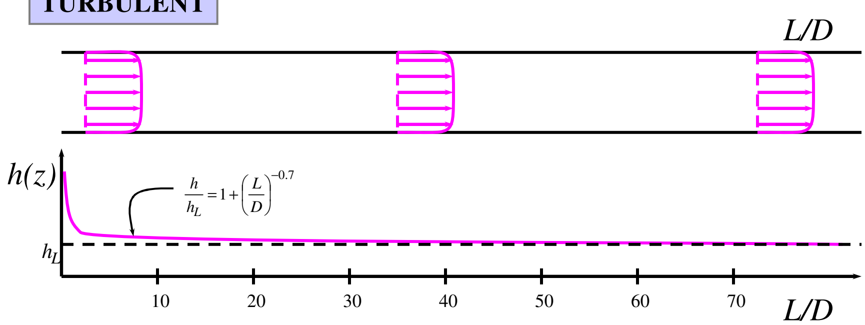
Why does L/D appear in laminar flow correlations and not in the turbulent flow correlations?

LAMINAR



Less lateral mixing in laminar flow means more variation in $h(x)$.

TURBULENT



In turbulent flow, good lateral mixing reduces the variation in h along the pipe length.

Example of *partial* solution to HW7, #3

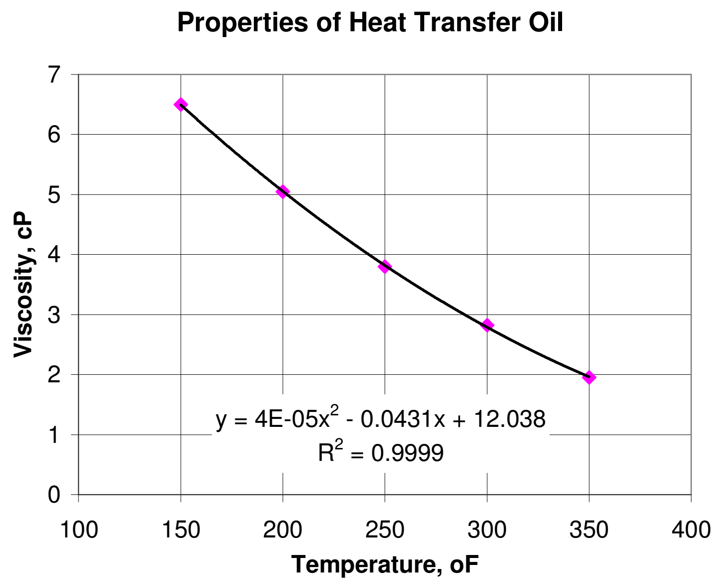
laminar flow in pipes	$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re<2100, (RePrD/L)>100, horizontal pipes, eqn 4.5-4, page 238; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.
turbulent flow in smooth tubes	$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re>6000, 0.7 <Pr <16,000, L/D>60 , eqn 4.5-8, page 239; all properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the wall temperature. The mean is the average of the inlet and outlet bulk temperatures; not valid for liquid metals.
air at 1atm in turbulent flow in pipes	$h_{lm} = \frac{3.52V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = \frac{0.5V(ft/s)^{0.8}}{D(ft)^{0.2}}$	equation 4.5-9, page 239
water in turbulent flow in pipes	$h_{lm} = 1429\left(1+0.0146T\left({}^oC\right)\right)\frac{V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = 150\left(1+0.011T\left({}^oF\right)\right)\frac{V(ft/s)^{0.8}}{D(ft)^{0.2}}$	4 < T(°C)<105, equation 4.5-10, page 239

Homework 8, Problem 1

A hydrocarbon oil (mean heat capacity = 0.50 BTU/(lb_m °F); mean thermal conductivity=0.083 BTU/(h ft °F)) is to be heated by flowing through a hot pipe. The pipe is heated in such a way that the inside surface of the pipe (the surface in contact with the oil) is held at a constant temperature of 325°F. The oil is to be heated to 250°F in the pipe, which is 15 ft long and has an inside diameter of 0.0303 ft. The inlet oil temperature is 175°F. What should the flow rate of the oil be (in units of lb_m/h) such that the oil exits at the desired temperature of 250°C?

The viscosity of the oil varies with temperature as follows:

150°F, 6.50 cP
200°F, 5.05 cP
250°F, 3.80 cP
300°F, 2.82 cP
350°F, 1.95 cP



Other types of Heat Transfer

So far:

- •conduction (Fourier's Law)
- •forced convection (due to flow)
- •source terms

Missing, but important:

next two topics

- •free convection (due to density variations brought on by temperature differences)
- •heat transfer with phase change (e.g. condensing fluids)

last subject in the course

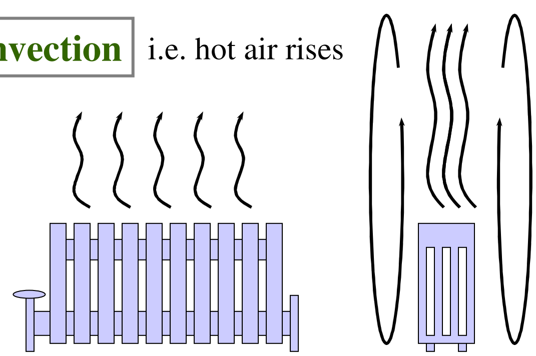
- •radiation

Example: Geankoplis 4.10-3

A horizontal oxidized steel pipe carrying steam and having an OD of 0.1683m has a surface temperature of 374.9 K and is exposed to air at 297.1 K in a large enclosure. Calculate the heat loss for 0.305 m of pipe.

Free Convection

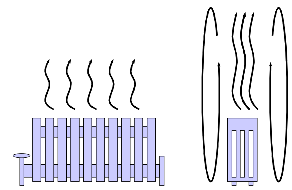
i.e. hot air rises



•heat moves from hot surface to cold air (fluid) by radiation and conduction
•increase in fluid temperature decreases fluid density
•recirculation flow begins
•recirculation adds to the heat-transfer from conduction and radiation

$\Rightarrow$ coupled heat and momentum transport

Free Convection i.e. hot air rises



How can we solve **real** problems involving free (natural) convection?

We'll try this: Let's review how we approached solving real problems in *earlier* cases, i.e. in fluid mechanics.

**** REVIEW OF Fluids Lecture 14 ** REVIEW OF Fluids Lecture 14 ****

Real Flows (*continued*)

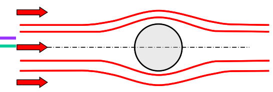
So far we have talked about internal flows

- ideal flows (Poiseuille flow in a tube)
- real flows (turbulent flow in a tube)

Strategy for handling real flows:	Dimensional analysis and data correlations
How did we arrive at correlations?	non-Dimensionalize ideal flow; use to guide expts on similar non-ideal flows; take data; develop empirical correlations from data
What do we do with the correlations?	use in MEB; calculate pressure-drop flow-rate relations

** REVIEW OF Fluids Lecture 14 ** REVIEW OF Fluids Lecture 14 **

Real Flows (*continued*)



Now, we will talk about external flows

- ideal flows (flow around a sphere)
- real flows (turbulent flow around a sphere, other obstacles)

Strategy for handling real flows:	Dimensional analysis and data correlations
How did we arrive at correlations?	non-Dimensionalize ideal flow; use to guide expts on similar non-ideal flows; take data; develop empirical correlations from data
What do we do with the correlations?	calculate drag - superficial velocity relations

Now, apply the method to free convection heat transfer . ..

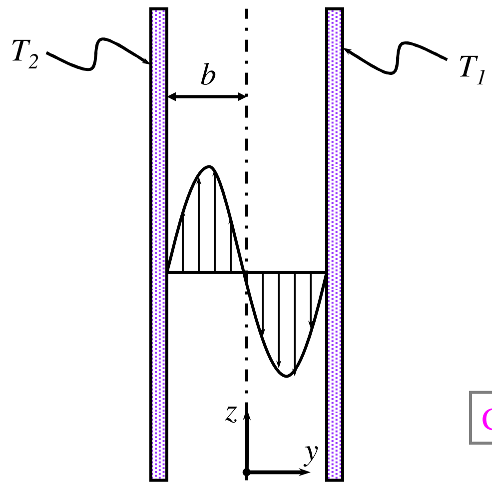
Real Heat Transfer

Now, we will talk about natural convection

- ideal flows (flow between two infinite plates, one hotter)
- real flows (cooling of a hot electronics part)

Strategy for handling real flows:	Dimensional analysis and data correlations
How did we arrive at correlations?	non-Dimensionalize ideal situation; use to guide expts on similar non-ideal situations; take data; develop empirical correlations from data
What do we do with the correlations?	calculate heat transfer rates; size equipment

Example: Free convection between long parallel plates *or* heat transfer through double-pane glass windows



$$T_2 > T_1$$

assumptions:
long, wide slit
steady state
no source terms
viscosity constant
density varies with T

Calculate: T, v profiles