What we know about Fluid Mechanics

Image from: joeforamerica.com

1.
2.
3.
4.
5.
6.

What we know about Fluid Mechanics

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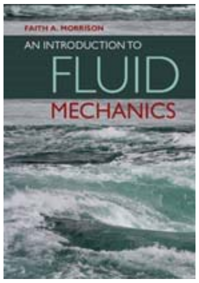
CM3110
Transport Processes and Unit Operations I

How do fluids behave?

1. Viscosity
2. Drag
3. Boundary Layers
4. Laminar versus Turbulent Flow
5. Lift
6. Supersonic
7. Surface Tension
8. Curved Streamlines
9. Magnetohydrodynamics



(Ch2)

www.chem.mtu.edu/~fmorriso/cm310/cm310.html

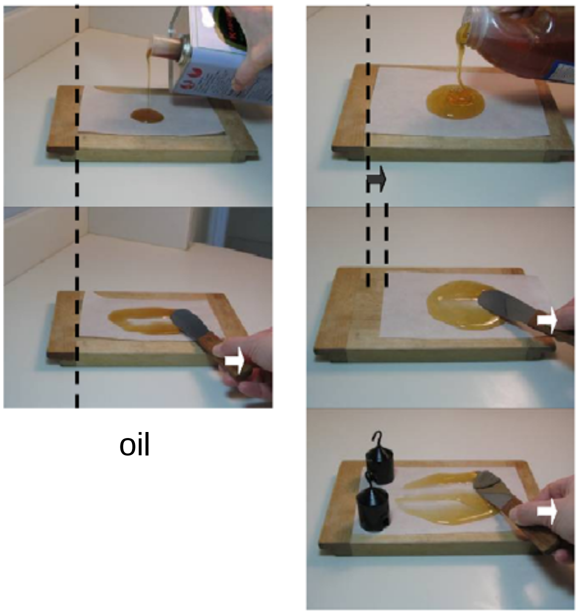
2.1 Viscosity, μ

A measure of a liquid's resistance to flow

water
(modest viscosity)

honey
(high viscosity)

Viscous fluids transmit stress from one location to another.



oil

honey

Momentum Flux

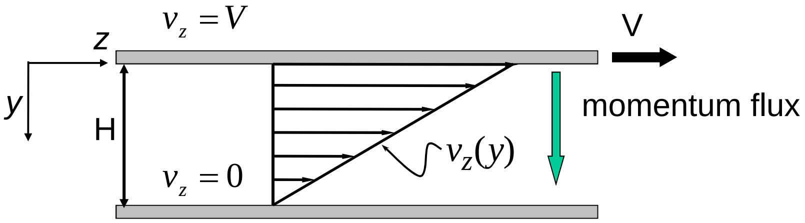
Viscosity determines the magnitude of momentum flux

Momentum ($\underline{p}$) = mass * velocity

$$\boxed{\underline{p} = m\underline{v}}$$

vectors

top plate has momentum, and it transfers this momentum to the top layer of fluid



How is force to move plate related to *V*?

$$\frac{F}{A} = +\mu\frac{V}{H} = \mu\left(\frac{v_z\big|_{y=0} - v_z\big|_{y=H}}{H-0}\right)$$

(Note choice of coordinate system)

Stress on a *y*-surface in the *z*-direction

$$= -\mu\left(\frac{\Delta v_z}{\Delta y}\right)$$

$$\boxed{\tilde{\tau}_{yz} = \mu\left(\frac{dv_z}{dy}\right)}$$

Newton's Law of Viscosity

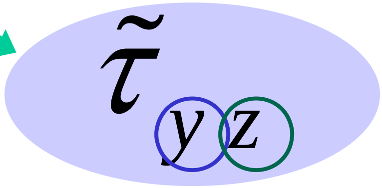
(See discussion of sign convention of stress; we use the tension-positive convention)

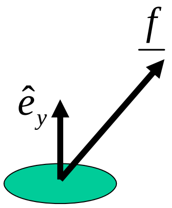
$$-\tilde{\tau}_{yz} = \frac{F}{A}$$

$$\tilde{\tau}_{yz} = \frac{force}{area} = \frac{kg\ m/s^2}{area} = \frac{(kg)(m/s)}{(s)(area)}$$

Momentum Flux

9 stresses at a point in space





$$\underline{f} = A(\tau_{yx}\hat{e}_x + \tau_{yy}\hat{e}_y + \tau_{yz}\hat{e}_z)$$

stress on a *y*-surface → in the *z*-direction

A surface whose unit normal is in the y-direction

$$-\tilde{\tau}_{yz} =$$

in the *y*-direction ← flux of *z*-momentum

(See discussion of sign convention of stress; we use the tension-positive convention)

(p110)

Example 2.1: What are the units of viscosity?

$$\tilde{\tau}_{yz} = \mu \left(\frac{dv_z}{dy} \right)$$

Newton's Law of Viscosity

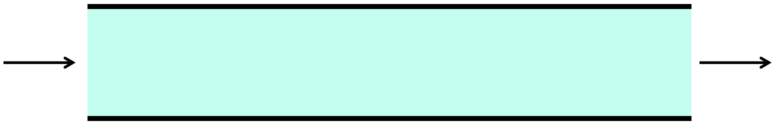
Viscosity, Greek letter "mu"

Viscosity

A measure of a liquid's resistance to flow

Viscous fluids transmit stress from one location to another.

Viscosity is responsible for the development of pressure distributions in laminar flow.



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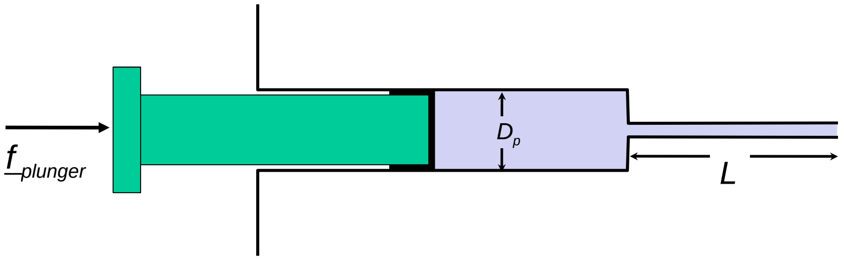
Example 2.2: How much force does it take to inject a water-like solution through a 16-gauge needle (inner diameter=1.194 mm, L=40mm)?



Need to know: $\Delta p(Q)$

From the methods of this course, we shall see that for Newtonian fluids:

Hagen-Poiseuille equation
(slow flow through tubes)

$$\boxed{Q = \frac{\pi\left(p_0 - p_L\right)R^4}{8\mu L}}$$

In the momentum balance, viscosity produces a force:

$$\underbrace{\sum_{all\ \ forces} \underline{f}}_{\substack{\textbf{Forces} \\ \text{(including} \\ \text{viscous} \\ \text{forces)}}} = \underbrace{m\underline{a} = \begin{array}{l}\text{Rate of} \\ \text{change of} \\ \text{momentum}\end{array}}_{\textbf{Inertia}}$$

2.2 Drag F_{drag}

The retarding force on an object due to a fluid (*retarding* implies opposite in direction to the fluid velocity)

Image from: www.g4tv.com

We study how to:

Calculate drag,

Drag is a consequence of viscosity

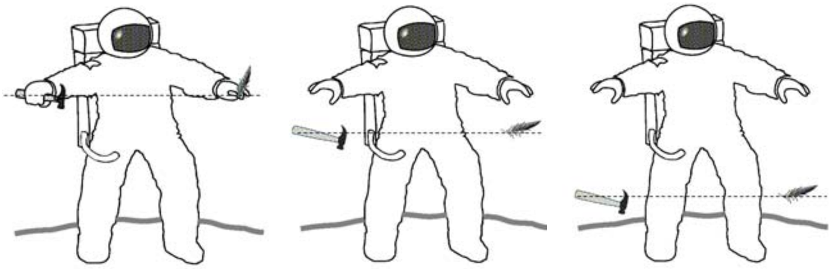
1. Calculate velocity
2. Calculate force on the object surface
3. Calculate the component of that force in the direction of the flow

} Microscopic-momentum balance

When impossible to calculate,

1. Measure force on model in a wind/water tunnel
2. Correlate using dimensional analysis
3. Scale up to system of interest

Without drag, objects of different weights, shapes, fall at the same speed:



In 1971, astronaut David Scott conducted Galileo's experiment on the moon as part of Apollo 15.

Drag Coefficient, C_D

True at high speeds

$$\boxed{C_D \equiv \frac{F_{drag}}{\frac{1}{2}\rho\langle v\rangle^2 A_p}}$$

reference area

Image from: www.seriouswheels.com

Image from: www.autoevolution.com

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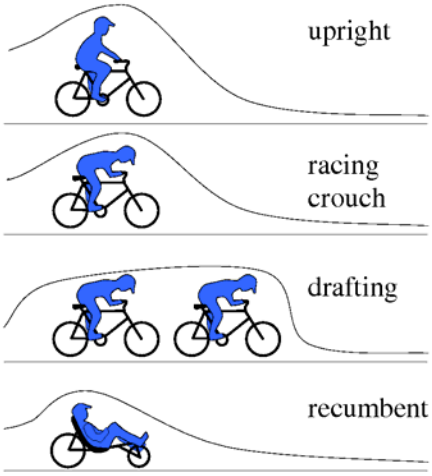
Example 2.4: How much faster will a bicycle racer traveling at 40 mph go if she adopts a racing crouch rather than riding upright?



$C_D = 1.1$

$C_D = 0.88$

$C_D = 0.50$

$C_D = 0.12$

Under what conditions is drag a simple matter of knowing C_D?

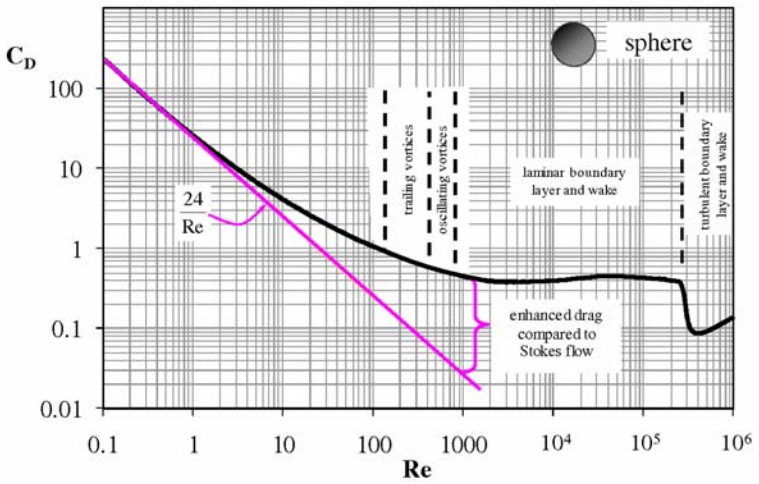
Could vary with:
- Flow speed
- Shape
- Density
- Viscosity
- Temperature
- . . .

$$\boxed{C_D \equiv \frac{F_{drag}}{\frac{1}{2}\rho\langle v\rangle^2 A_p}}$$

(i.e., Why is this so?
When is this so?)

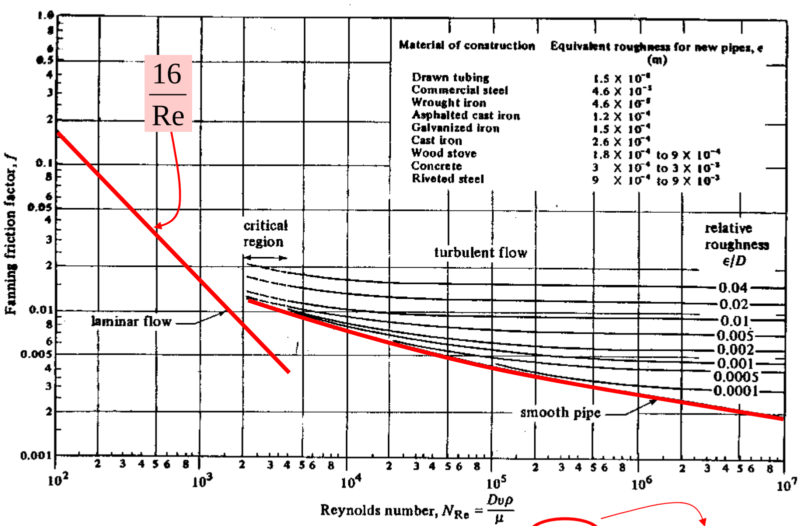
Drag behavior of a sphere

(Similar to using the Moody Chart when interested in wall drag in pipes)



Moody Chart: Data Correlation for Friction in Straight Pipes

(Review)



Material of construction	Equivalent roughness for new pipes, ϵ (m)
Drawn tubing	1.5×10^{-6}
Commercial steel	4.6×10^{-5}
Wrought iron	4.6×10^{-5}
Asphalted cast iron	1.2×10^{-4}
Galvanized iron	1.5×10^{-4}
Cast iron	2.6×10^{-4}
Wood stove	1.8×10^{-4} to 9×10^{-4}
Concrete	3×10^{-4} to 3×10^{-3}
Riveted steel	9×10^{-4} to 9×10^{-3}

FIGURE 2.10-3. *Friction factors for fluids inside pipes. [Based on L. F. Moody, Trans. A.S.M.E.,* **66**, *671, (1944); Mech. Eng. 69, 1005 (1947). With permission.]*

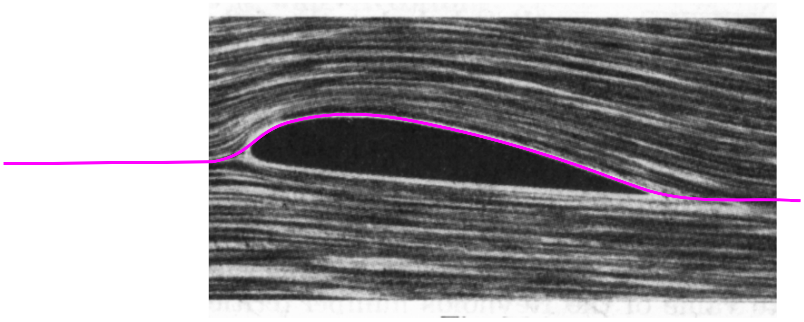
Moody Chart

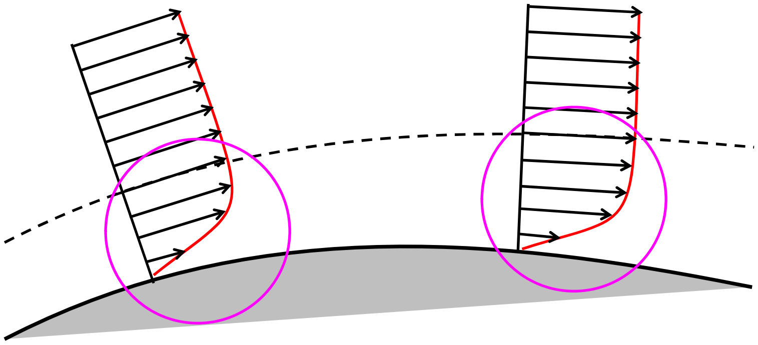
(image from: Geankoplis)

2.3 Boundary Layers

Form at high speeds

- Regions near solid surfaces in which viscosity dominates the flow behavior, especially at high speeds





Newton's Law of Viscosity

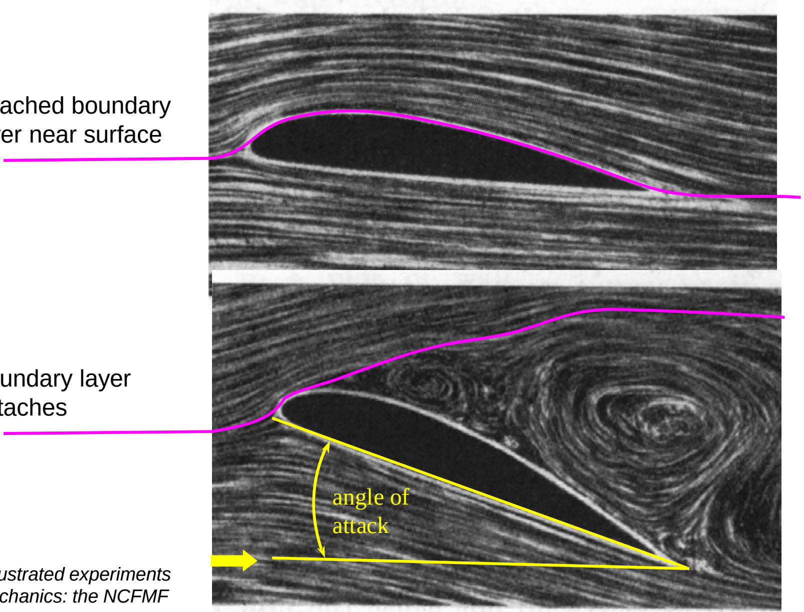
$$\tilde{\tau}_{yz} = \mu\left(\frac{dv_z}{dy}\right)$$

There is relative motion near the surfaces (viscosity is important);

Away from surfaces, the flow is uniform (viscosity is not important; inertia dominates)

Attached boundary layer near surface

Boundary layer detaches

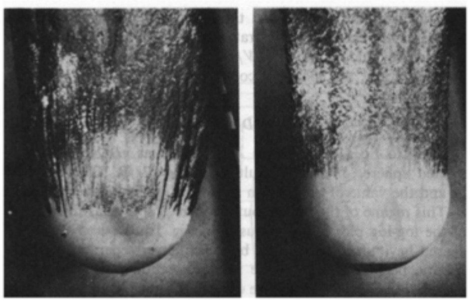


Source: *Illustrated experiments in fluid mechanics: the NCFMF book of film notes*, MIT Press, 1972

Why is the surface of a golf ball designed the way it is?

Manipulate boundary-layer separation

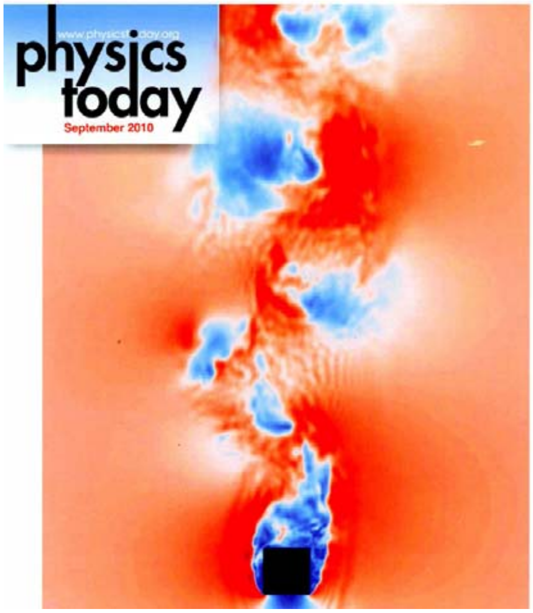
When the boundary layer is turbulent, it detaches farther back (yielding lower drag)

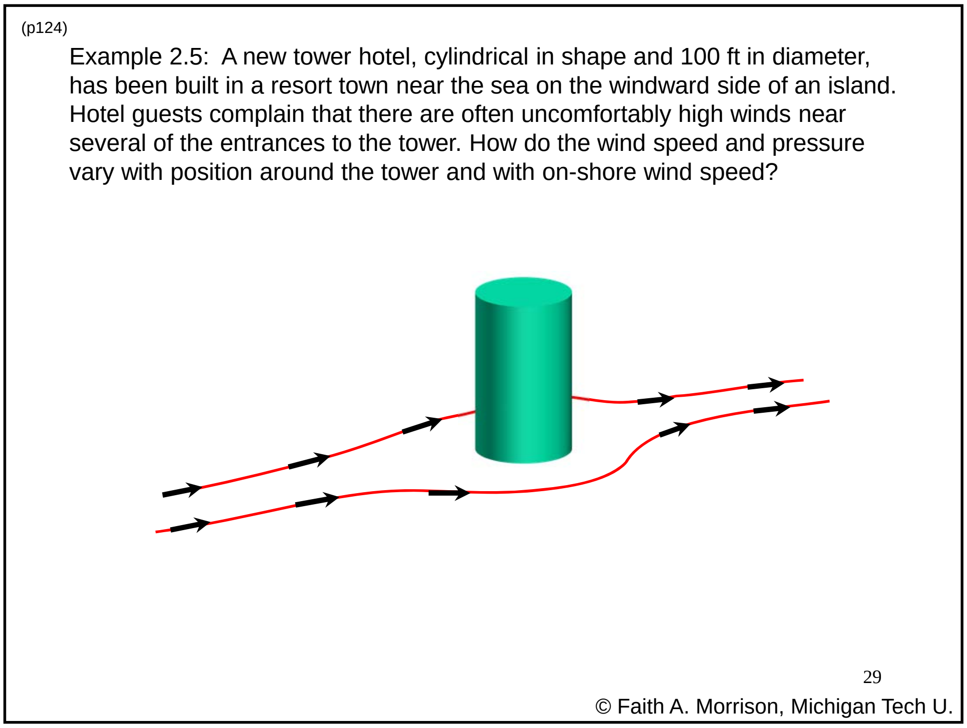


smooth ball rough ball

H. Schlichting, Boundary Layer Theory (McGraw-Hill, NY 1955.

cover: This image from a simulation of wind blowing past a building (black square) reveals the vortices that are shed downwind of the building; dark orange represents the highest air speeds, dark blue the lowest. As a result of such vortex formation and shedding, tall buildings can experience large, potentially catastrophic forces. (Courtesy of the computational fluid dynamics group at Rowan William Davies and Irwin Inc.)





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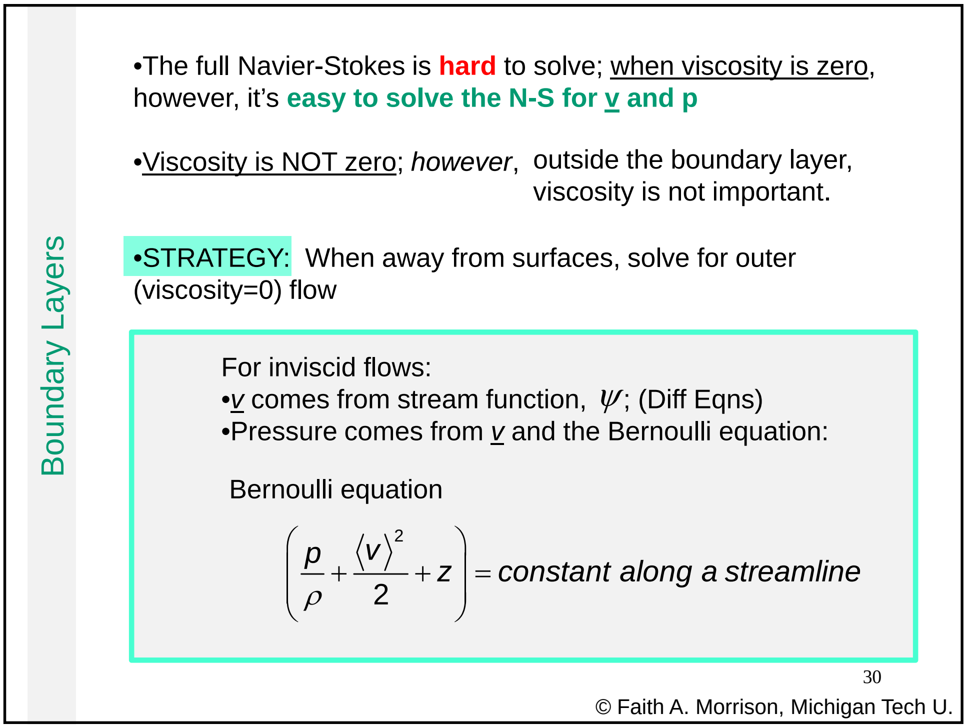
Example 2.5: A new tower hotel, cylindrical in shape and 100 ft in diameter, has been built in a resort town near the sea on the windward side of an island. Hotel guests complain that there are often uncomfortably high winds near several of the entrances to the tower. How do the wind speed and pressure vary with position around the tower and with on-shore wind speed?



Boundary Layers

- The full Navier-Stokes is **hard** to solve; when viscosity is zero, however, it's **easy to solve the N-S for $\underline{v}$ and p**
- Viscosity is NOT zero; *however*, outside the boundary layer, viscosity is not important.
- STRATEGY: When away from surfaces, solve for outer (viscosity=0) flow

For inviscid flows:
- $\underline{v}$ comes from stream function, ψ; (Diff Eqns)
- Pressure comes from $\underline{v}$ and the Bernoulli equation:

Bernoulli equation

$$\left(\frac{p}{\rho}+\frac{\langle v\rangle^2}{2}+z\right)=\textit{constant along a streamline}$$

Boundary Layers

Velocity outside boundary layer

cylinder in uniform flow:

(potential flow solution)

$$\underline{v} = \begin{pmatrix} U\left(1-\frac{R^2}{r^2}\right)\cos\theta \\ -U\left(1+\frac{R^2}{r^2}\right)\sin\theta \\ 0 \end{pmatrix}_{r\theta z}$$

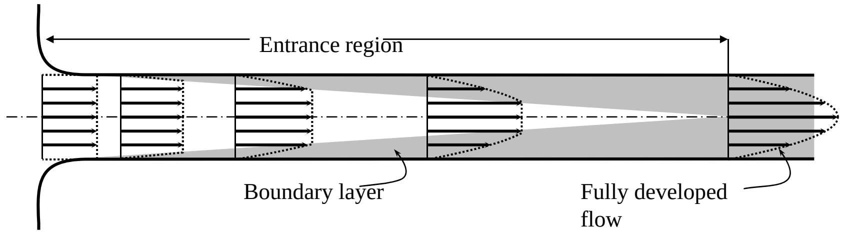
Pressure outside boundary layer

Bernoulli equation

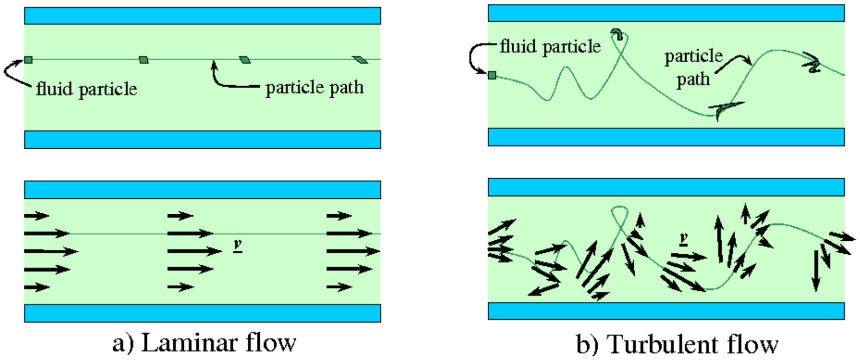
$$\left(\frac{p}{\rho}+\frac{v^2}{2}+z\right) = \textit{constant along a streamline}$$

Boundary Layers in Internal Flow:

Entrance flow field in pipe flow



2.4 Laminar versus Turbulent Flow



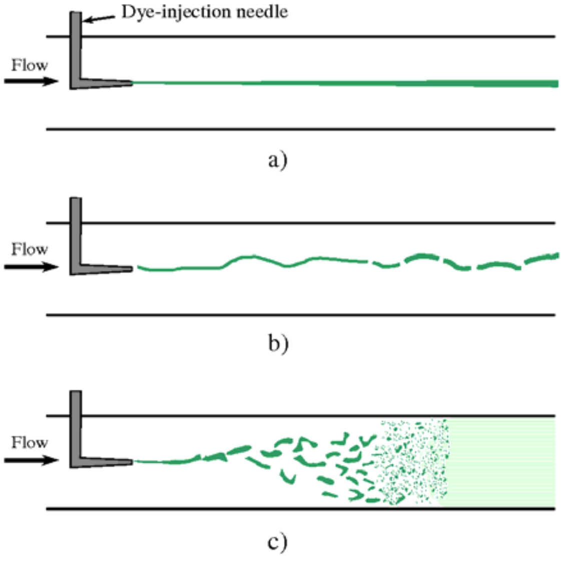
a) Laminar flow

b) Turbulent flow

Viscosity dominates

Inertia dominates

Reynolds' Experiment



Viscosity dominates

Inertia dominates

CM3110
Transport Processes and Unit Operations I

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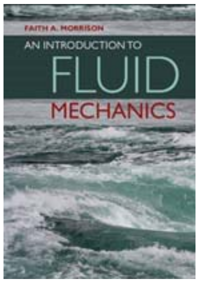
Advanced



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www.chem.mtu.edu/~fmorriso/cm310/cm310.html

Take-Away from Today:

How do fluids behave?

1. Viscosity

$$\tilde{\tau}_{yz} = \mu \left(\frac{dv_z}{dy} \right)$$

2. Drag

$$C_D \equiv \frac{F_{drag}}{\frac{1}{2}\rho \langle v \rangle^2 A_p}$$

3. Boundary Layers

Viscous effects within BL; no viscous effects in main stream;
Bernoulli equation (like MEB) assumes no viscous effects

(outside the boundary layer)

$$\left(\frac{p}{\rho} + \frac{v^2}{2} + z \right) = \textit{constant along a streamline}$$

4. Laminar versus Turbulent Flow

(we know about this already)

Take-Away from Today:

How do fluids behave?

Viscous stress τ

1. Viscosity

$$\tilde{\tau}_{yz} = \mu\left(\frac{dv_z}{dy}\right)$$

2. Drag

$$C_D \equiv \frac{F_{drag}}{\frac{1}{2}\rho\langle v\rangle^2 A_p}$$

Viscous effects dominate near walls

3. Boundary Layers

Viscous effects within BL; no viscous effects in main stream;
Bernoulli equation (like MEB) assumes no viscous effects

(outside the boundary layer)

$$\left(\frac{p}{\rho}+\frac{v^2}{2}+z\right) = \textit{constant along a streamline}$$

Inertial effects dominate

4. Laminar versus Turbulent Flow

(we know about this already)

What we know about Fluid Mechanics

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What we know about Fluid Mechanics

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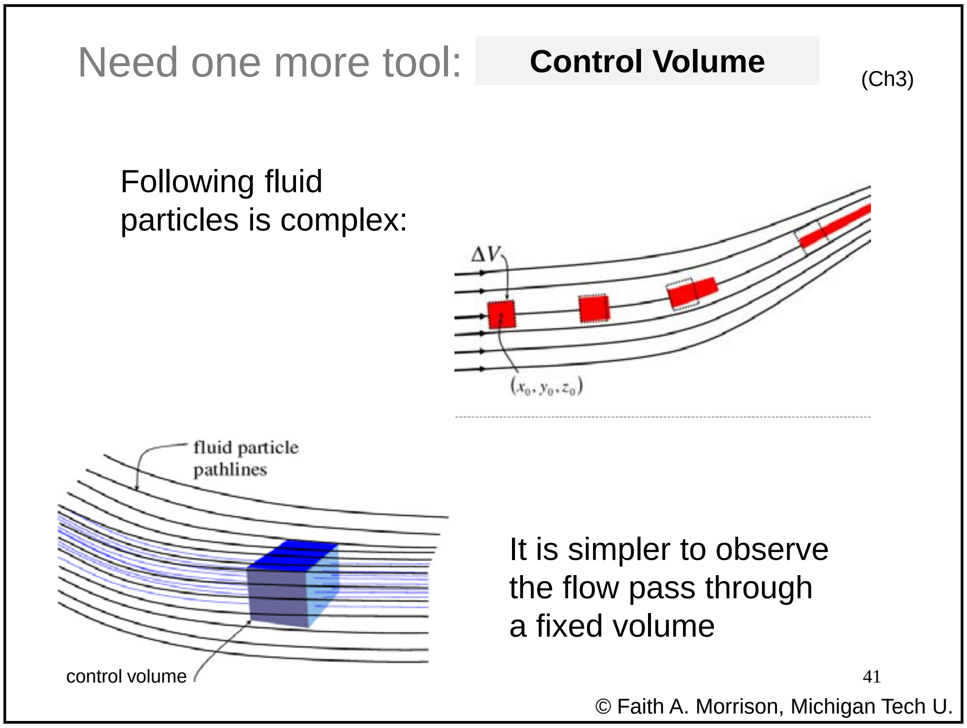
2. Fluid Statics ($p_{bot} = p_{top} + \rho g h$); same elevation, same pressure; good for manometers, water in tanks)
3. **Newton's Law of Viscosity** (fluids transmit forces through momentum flux)
4. **Momentum flux** (=stress) has 9 components
5. **Drag** is a consequence of viscosity
6. **Boundary layers form** (viscous effects are confined near surfaces at high speeds)

What we know about Fluid Mechanics

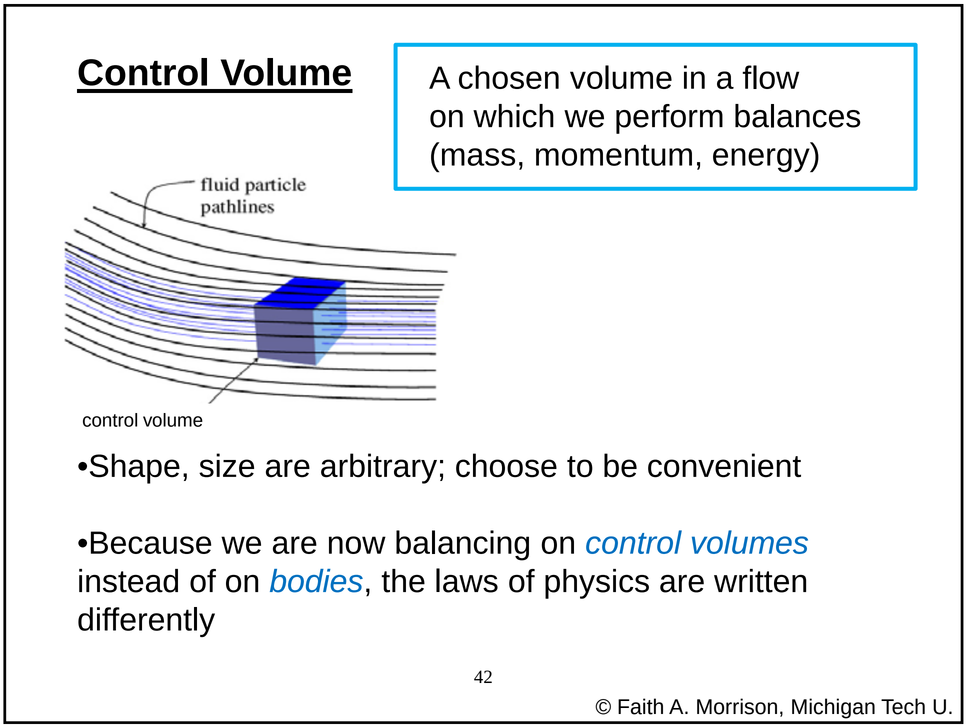
8. **Sometimes viscous effects dominate; sometimes inertial effects dominate**



Need one more tool: **Control Volume** (Ch3)

Following fluid particles is complex:

It is simpler to observe the flow pass through a fixed volume



Control Volume

A chosen volume in a flow on which we perform balances (mass, momentum, energy)

- Shape, size are arbitrary; choose to be convenient

- Because we are now balancing on *control volumes* instead of on *bodies*, the laws of physics are written differently

Mass balance, flowing system (open system; control volume):

$$\underset{\sum_{in} - \sum_{out}}{\left\{\begin{matrix}\text{net mass}\\ \text{flowing in}\end{matrix}\right\}} = \underset{\text{steady state}}{\left\{\cancel{\begin{matrix}\text{rate of}\\ \text{accumulation}\\ \text{of mass}\end{matrix}}\right\}}$$

Momentum balance, flowing system (open system; control volume):

$$\sum_{all\ forces} \underline{f} = m\underline{a} \Rightarrow$$

$$\left\{\begin{matrix}\text{sum of forces}\\ \text{acting }\underline{\text{on}}\text{ control vol}\end{matrix}\right\} + \underset{\sum_{in} - \sum_{out}}{\left\{\begin{matrix}\text{net momentum}\\ \text{flowing in}\end{matrix}\right\}} = \underset{\text{steady state}}{\left\{\cancel{\begin{matrix}\text{rate of}\\ \text{accumulation}\\ \text{of momentum}\end{matrix}}\right\}}$$

$$\sum_i \underline{F}_{on_i} + \sum_i \left\{\begin{matrix}\text{momentum}\\ \text{flowing }\underline{\text{in}}\\ \text{in the streams}\end{matrix}\right\} - \sum_i \left\{\begin{matrix}\text{momentum}\\ \text{flowing }\underline{\text{out}}\\ \text{in the streams}\end{matrix}\right\} = 0$$

note that momentum is a vector quantity

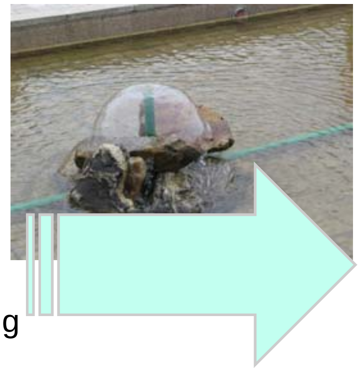
We are ready to try a momentum balance.

Tools:

- Mass balance (mass conserved)
- Newton's 2nd law (momentum conserved)
- Control volume
- Newton's law of viscosity
- Calculus 3 (multivariable calculus)

CM3110
Transport I
Part I: Fluid Mechanics: *Microscopic Balances*

NEXT



Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University