CM3110
Transport I
Part II: Heat Transfer

One-Dimensional Heat Transfer *(continued)*

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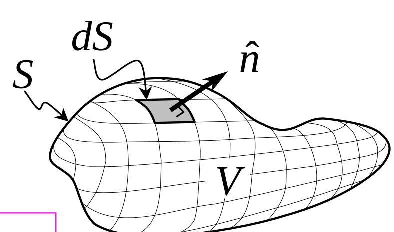
General Energy Transport Equation

(microscopic energy balance)

As for the derivation of the microscopic momentum balance, the microscopic energy balance is derived on an arbitrary volume, *V*, enclosed by a surface, *S*.



Gibbs notation:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

see handout for component notation

General Energy Transport Equation

(microscopic energy balance)

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

convection

source (energy generated per unit volume per time)

rate of change

conduction (all directions)

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

The Equation of Energy for systems with constant k

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

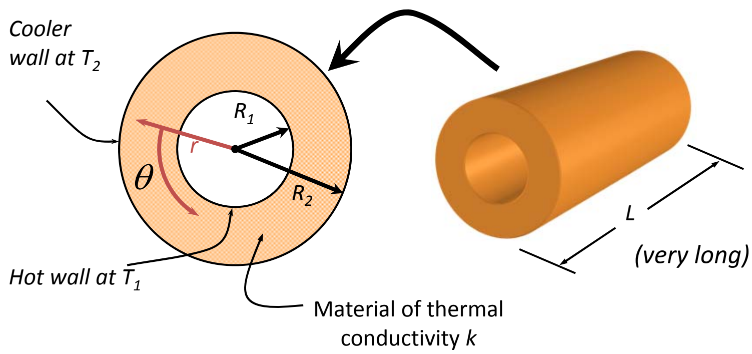
$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S$$

Note: this handout is also on the web

Example 3: Heat flux in a cylindrical shell – Temp BC

Assumptions:
- long pipe
- steady state
- *k* = thermal conductivity of wall

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **inner wall** is at T_1 and the **outer wall** is at T_2? ($T_1 > T_2$)*



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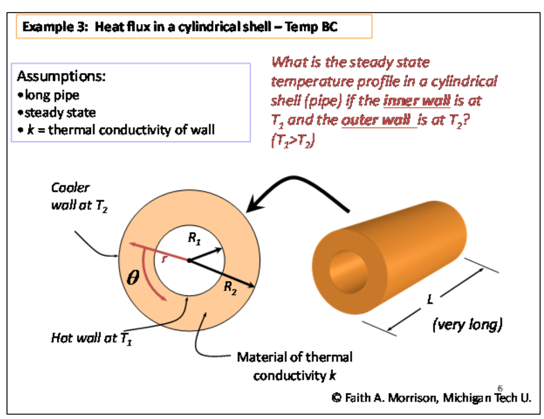
Example 3: Heat flux in a cylindrical shell – Temp BC

Let's try.



Example 3: Heat flux in a cylindrical shell – Temp BC

Solution:

$$\frac{q_r}{A} = \frac{c_1}{r}$$

Not constant

$$T = -\frac{c_1}{k}\ln r + c_2$$

Boundary conditions?

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Example 3: Heat flux in a cylindrical shell – Temp BC

Solution for Cylindrical Shell:

NOT constant

$$\frac{q_r}{A} = \frac{T_1 - T_2}{\ln\frac{R_2}{R_1}}\left(\frac{k}{r}\right)$$

The heat flux $\frac{q_r}{A}$ **DOES** depend on, *k;* also $\frac{q_r}{A}$ decreases as *1/r*

NOT linear

$$\frac{T_2 - T}{T_2 - T_1} = \frac{\ln\frac{R_2}{r}}{\ln\frac{R_2}{R_1}}$$

Note that *T(r)* does not depend on the thermal conductivity, *k* (steady state)

Pipe with temperature BCs

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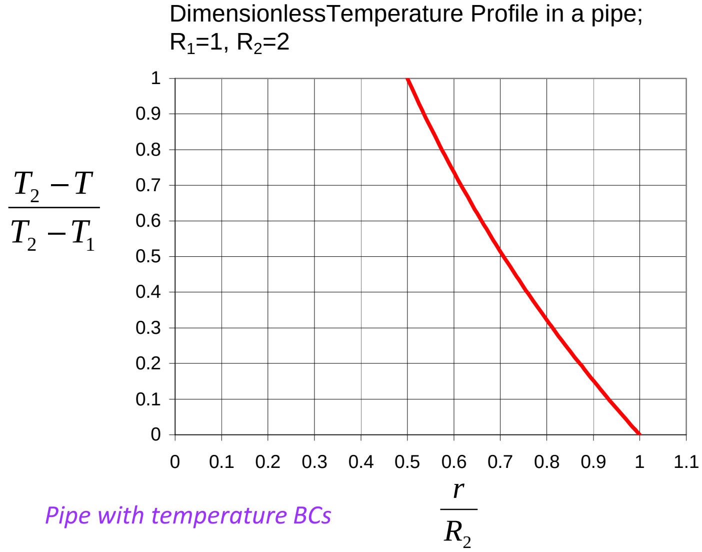
Example 3: Heat flux in a cylindrical shell – Temp BC

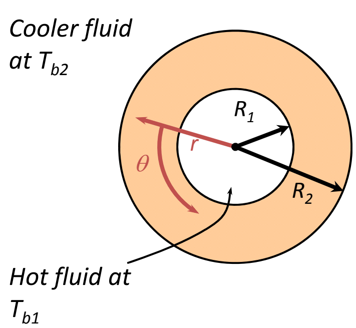


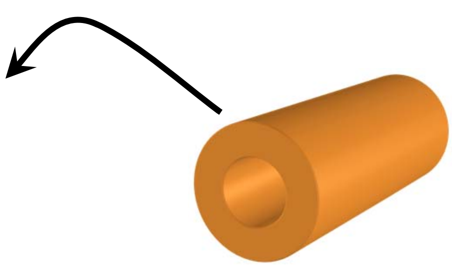
Pipe with temperature BCs

Example 4: Heat flux in a cylindrical shell – Newton's law of cooling

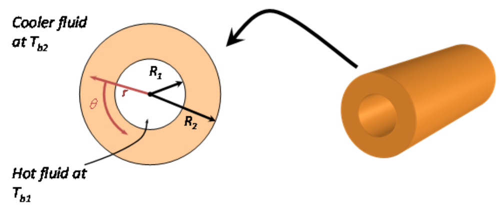
Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **fluid on the inside** is at T_{b1} and the **fluid on the outside** is at T_{b2}? (T_{b1}>T_{b2})*





Example 4: Heat flux in a cylindrical shell



You try.

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Example 4: Heat flux in a cylindrical shell

Solution:

$$\frac{q_r}{A} = \frac{c_1}{r}$$

Not constant

$$T = -\frac{c_1}{k} \ln r + c_2$$

Boundary conditions?

Example 4: Heat flux in a cylindrical shell

$$\frac{c_1}{R_1} = h_1\left(T_{b1} - T_{w1}\right)$$

$$\frac{c_1}{R_2} = h_2\left(T_{w2} - T_{b2}\right)$$

$$T_{w1} = -\frac{c_1}{k}\ln R_1 + c_2$$

$$T_{w2} = -\frac{c_1}{k}\ln R_2 + c_2$$

4 equations

4 unknowns; c_1, T_{w1}, c_2, T_{w2}

SOLVE

Example 4: Heat flux in a cylindrical shell

Newton's law of cooling boundary conditions

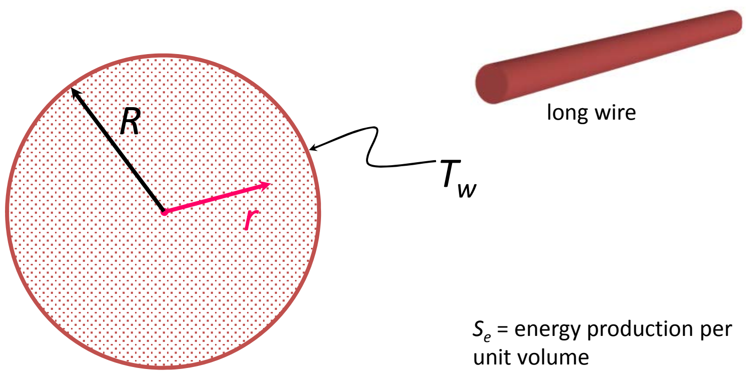
Solution: Radial Heat Flux in an Annulus

$$T - T_{b2} = \frac{(T_{b1} - T_{b2})\left(\ln\left(\frac{R_2}{r}\right) + \frac{k}{h_2 R_2}\right)}{\frac{k}{h_2 R_2} + \ln\left(\frac{R_2}{R_1}\right) + \frac{k}{h_1 R_1}}$$

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

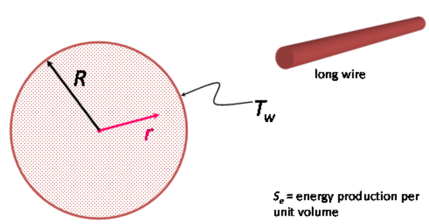
Example 5: Heat Conduction with Generation

What is the steady state temperature profile in a wire if heat is generated uniformly throughout the wire at a rate of S_e *W/m³ and the outer radius is held at* T_w*?*



S_e = energy production per unit volume

Example 5: Heat conduction with generation



You try.

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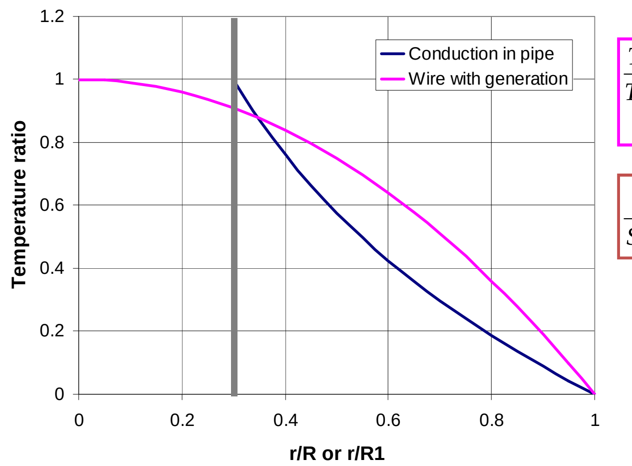
Compare solutions

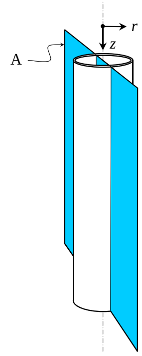


$$\frac{T-T_1}{T_1-T_2}\frac{1}{\ln\frac{R_1}{R_2}}=\ln\frac{r}{R_1}$$

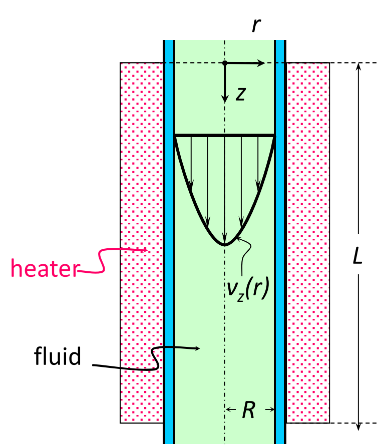
$$\frac{T-T_w}{S_eR^2/4k}=1-\left(\frac{r}{R}\right)^2$$

Example 6: **Wall heating of laminar flow**. *What is the steady state temperature profile in a flowing fluid in a tube if the walls are heated (constant flux, q_1/A) and if the fluid is a Newtonian fluid in laminar flow?*

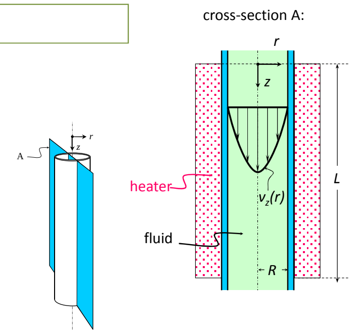
assume:
constant viscosity



cross-section A:

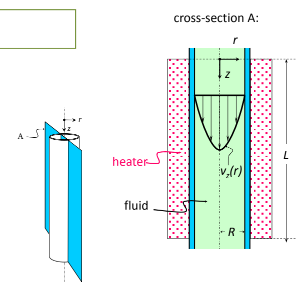


Example 5: Wall heating of laminar flow



You try.

Example 5: Wall heating of laminar flow



We need to solve this partial differential equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)-\frac{\rho\hat{C}_p}{k}v_z(r)\frac{\partial T}{\partial t}=0$$

with the appropriate boundary conditions. To see the solution see:

- R. Siegel, E. M. Sparrow, T. M. Hallman, *Appl. Science Research* A7, 386-392 (1958)
- R. B. Bird, W. Stewart, and E. Lightfoot, *Transport Phenomena*, Wiley, 1960, p295.

SUMMARY

Steady State Heat Transfer

Example 1: Heat flux in a rectangular solid – Temperature BC

Example 2: Heat flux in a rectangular solid – Newton's law of cooling

Example 3: Heat flux in a cylindrical shell – Temperature BC

Example 4: Heat flux in a cylindrical shell – Newton's law of cooling

Example 5: Heat conduction with generation

Example 6: Wall heating of laminar flow

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Example 6: Wall heating of laminar flow

Conclusion: When we can simplify geometry, assume steady state, assume symmetry, the solutions are easily obtained

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Unsteady State Heat Transfer

???

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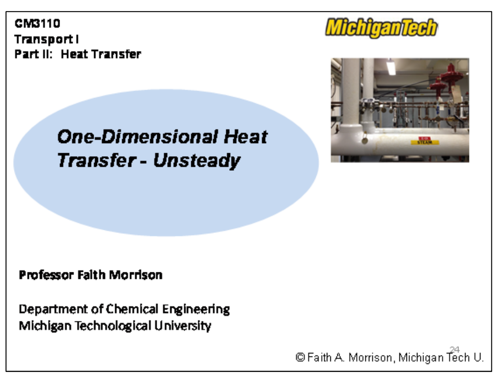

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