CM3110
Transport I
Part II: Heat Transfer

Applied Heat Transfer:

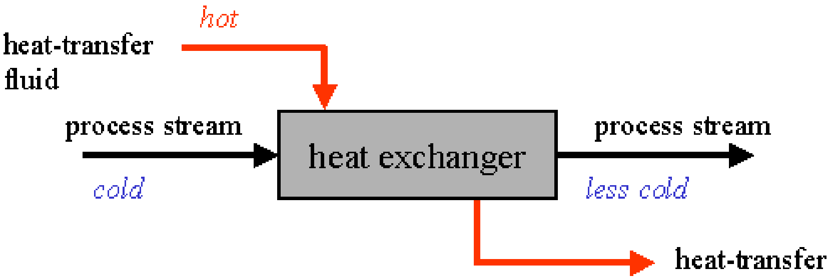
Heat Exchanger Modeling, Sizing, and Design

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

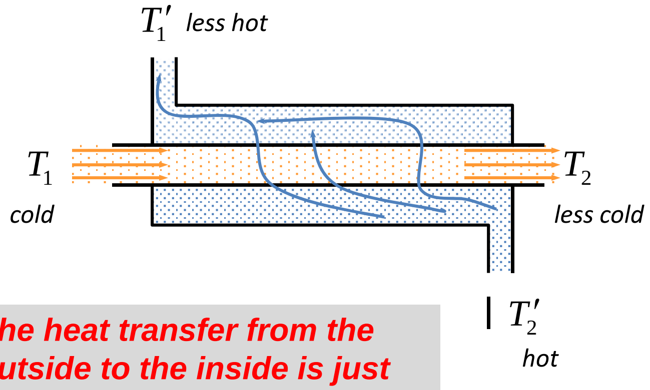
Before turning to radiation (last topic) we will discuss a few practical applications

How can we use Fundamental Heat Transfer to understand real devices like heat exchangers?



The Simplest Heat Exchanger:

Double-Pipe Heat exchanger - counter current



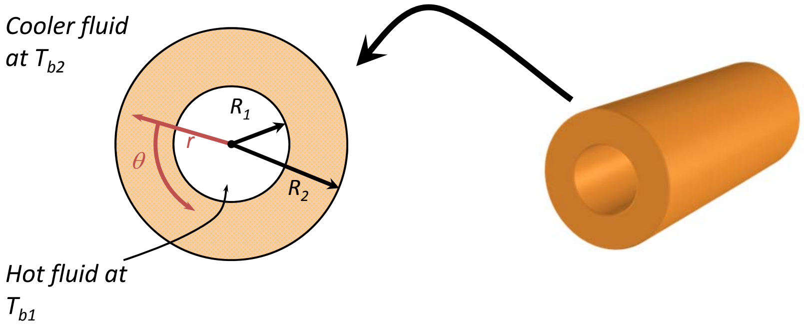
The heat transfer from the outside to the inside is just heat flux in an annular shell

---

Example 4: Heat flux in a cylindrical shell

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

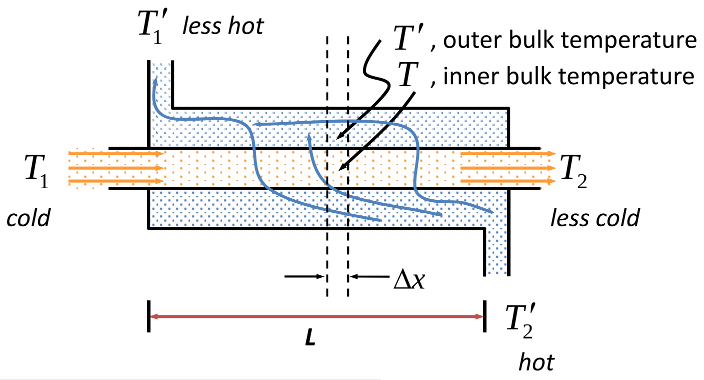
Maybe we can use <u>heat transfer coefficient</u> to understand forced-convection heat exchangers. . .
BUT . . .



BUT: *The temperature difference between the fluid and the wall* ***varies*** *along the length of the heat exchanger.*

The Simplest Heat Exchanger:

Double-Pipe Heat exchanger - counter current



How can we adapt *h* so that we can use the concept to characterize heat exchangers?

Let's look at the solution for radial conduction in an annulus

Example 4: Heat flux in a cylindrical shell

Solution:

$$\frac{q_r}{A} = \frac{c_1}{r}$$

Not constant

$$T = -\frac{c_1}{k}\ln r + c_2$$

Boundary conditions?

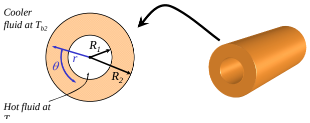
Example 4: Heat flux in a cylindrical shell, Newton's law of cooling boundary Conditions

Results: Radial Heat Flux in an Annulus

$$T - T_{b2} = \frac{(T_{b1} - T_{b2})\left(\ln\left(\frac{R_2}{r}\right) + \frac{k}{h_2 R_2}\right)}{\frac{k}{h_2 R_2} + \ln\left(\frac{R_2}{R_1}\right) + \frac{k}{h_1 R_1}}$$

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

Example 4: Heat flux in a cylindrical shell



Solution for Heat Flux:

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

Calculate Total Heat flow:

$$Q = \frac{q_r}{A}(2\pi r L) = \frac{(T_{b1} - T_{b2})(2\pi L)}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}$$

Note that total heat flow is proportional to bulk ΔT and (almost) area of heat transfer

Overall Heat Transfer Coefficient, U

$$\boxed{Q = UA\Delta T}$$
$$= UA(T_{b1} - T_{b2})$$

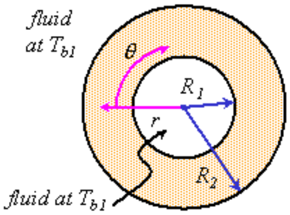
this equation serves as the definition of U

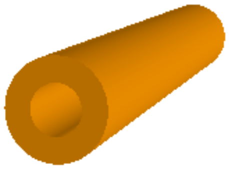
A = *area of heat transfer (not always unambiguous)*

ΔT = *driving temperature difference*

Example: in a pipe

do we use inner or outer area?





---

overall heat xfer coeffs in pipe

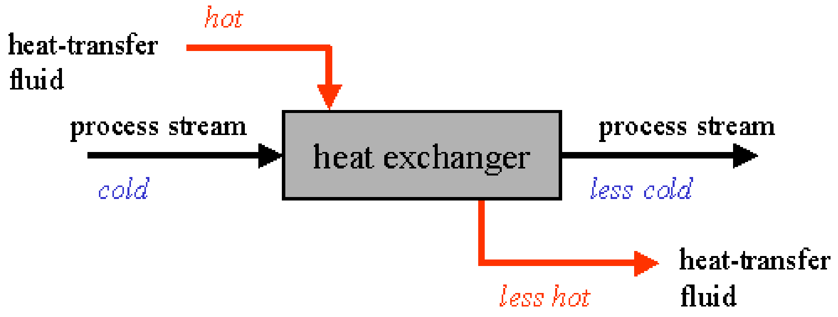
Area must be specified when *U* is reported

$$Q = U_1 A_1 \Delta T$$
$$= \left(\frac{\frac{1}{R_1}}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}} \right)(2\pi R_1 L)(T_{b1} - T_{b2})$$

$$Q = U_2 A_2 \Delta T$$
$$= \left(\frac{\frac{1}{R_2}}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}} \right)(2\pi R_2 L)(T_{b1} - T_{b2})$$

Heat flux in a cylindrical shell: $Q = UA(T_{b1} - T_{b2})$

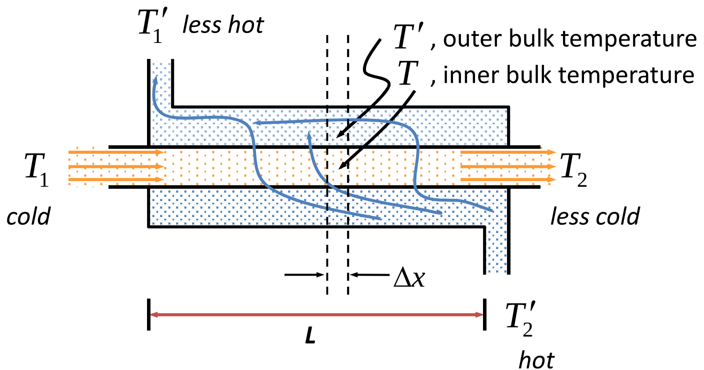
But, in an actual heat exchanger, T_{b1} and T_{b2} vary along the length of the heat exchanger



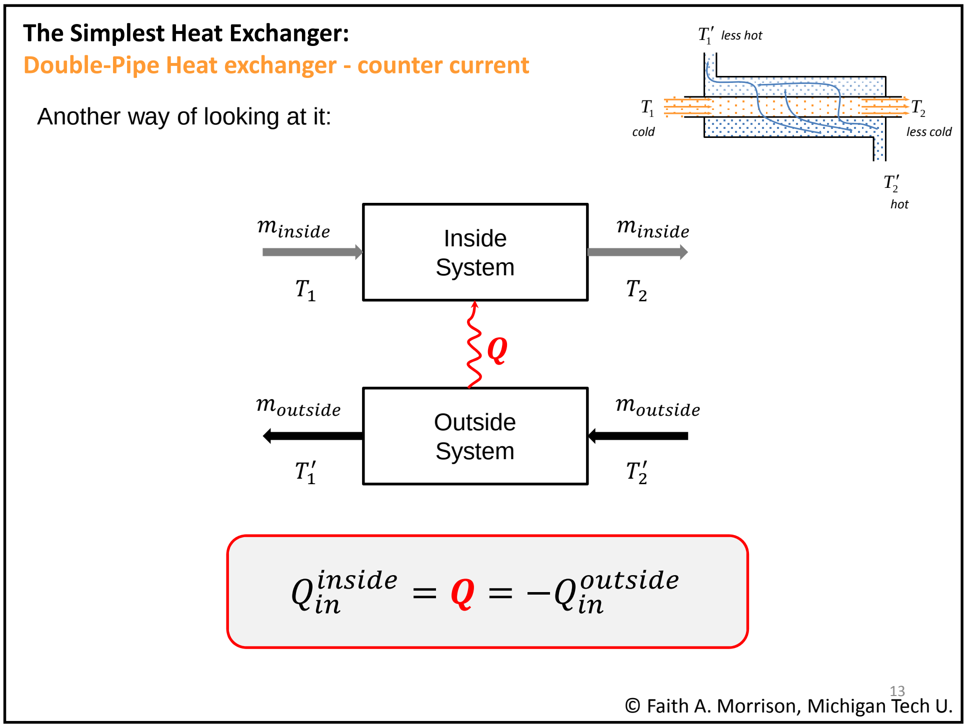
What kind of average ΔT do we use?

The Simplest Heat Exchanger:

Double-Pipe Heat exchanger - counter current



We will do an open-system energy balance on a differential section to determine the correct average temperature difference to use.

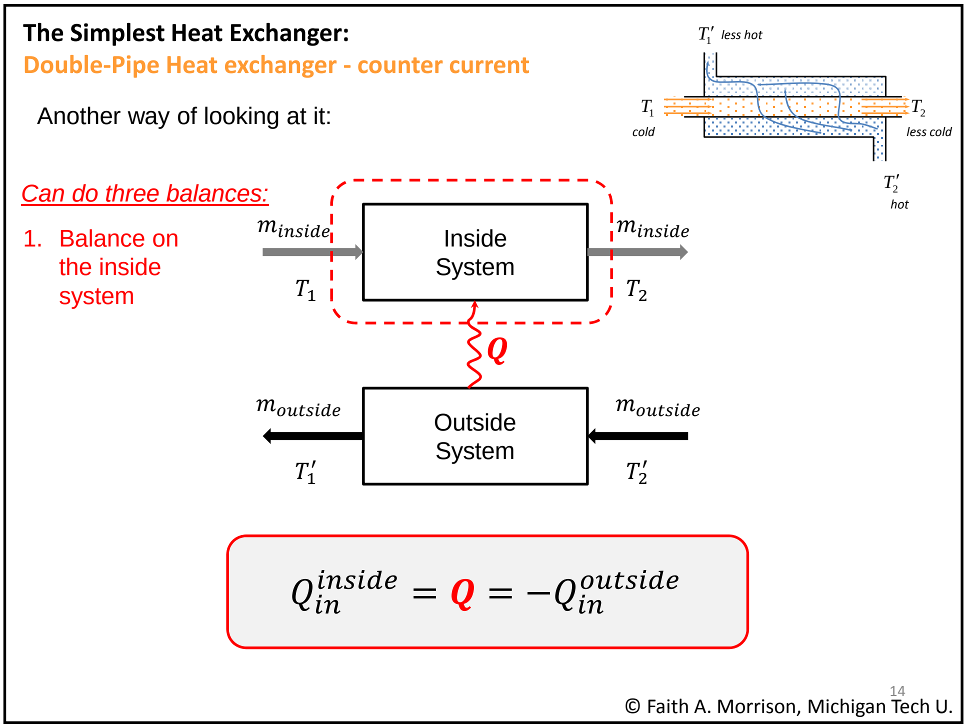


The Simplest Heat Exchanger:

Double-Pipe Heat exchanger - counter current

Another way of looking at it:

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



The Simplest Heat Exchanger:

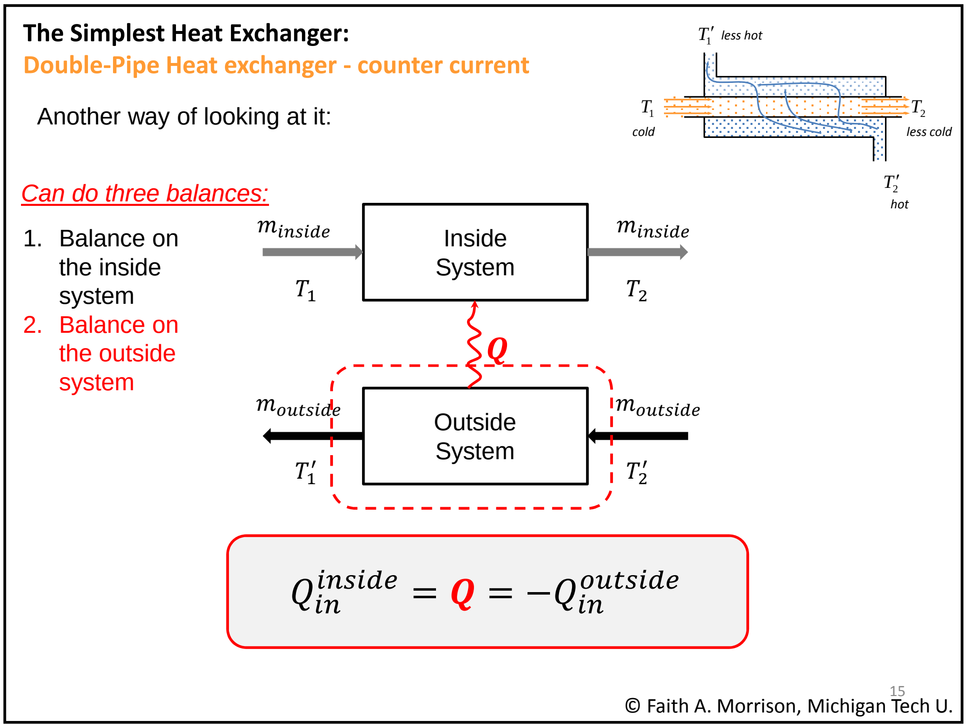
Double-Pipe Heat exchanger - counter current

Another way of looking at it:

Can do three balances:

1. Balance on the inside system

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



The Simplest Heat Exchanger:

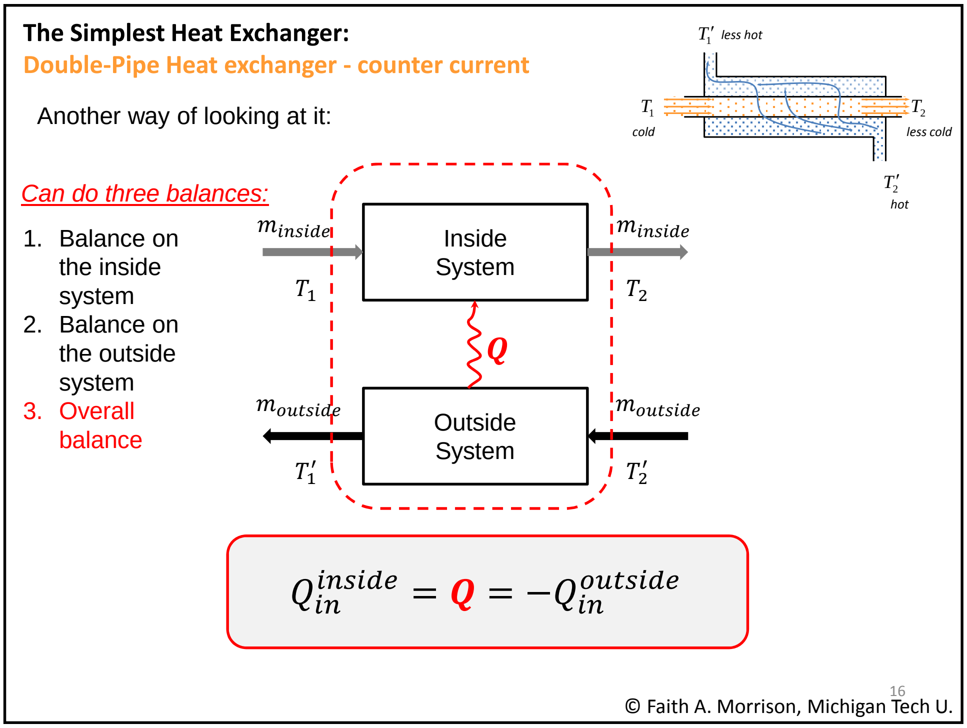
Double-Pipe Heat exchanger - counter current

Another way of looking at it:

Can do three balances:

1. Balance on the inside system
2. Balance on the outside system

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



The Simplest Heat Exchanger:

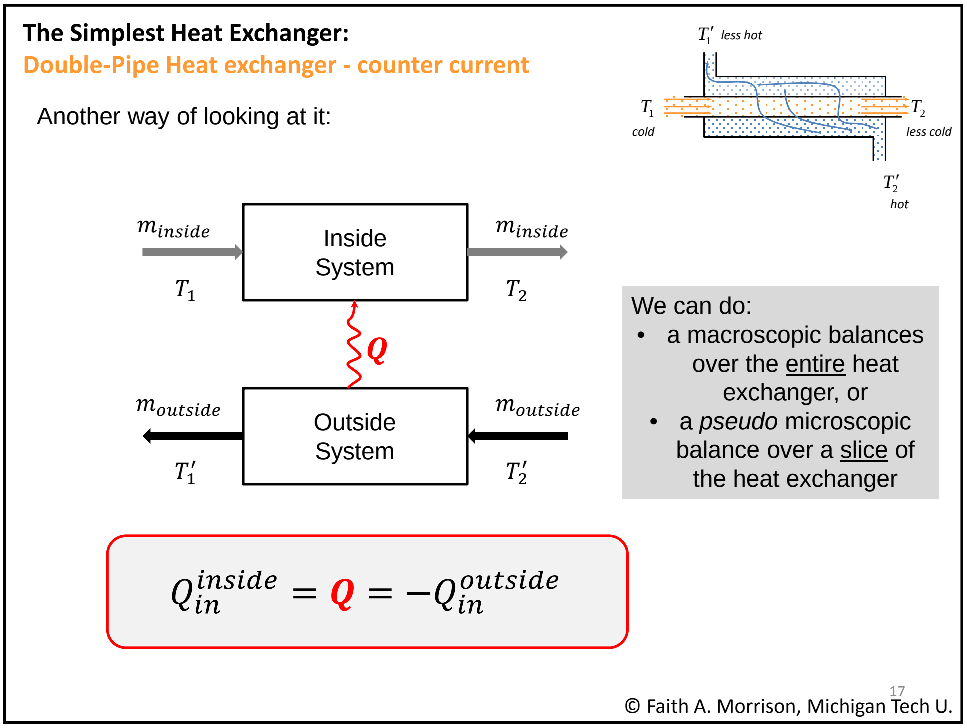
Double-Pipe Heat exchanger - counter current

Another way of looking at it:

Can do three balances:

1. Balance on the inside system
2. Balance on the outside system
3. Overall balance

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



The Simplest Heat Exchanger:

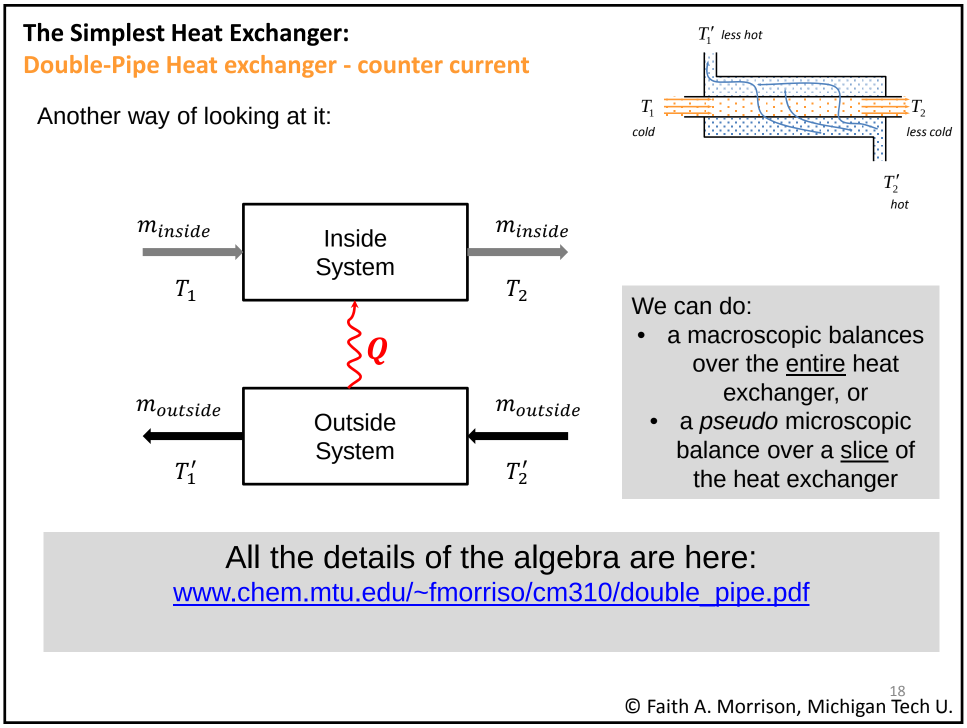
Double-Pipe Heat exchanger - counter current

Another way of looking at it:

We can do:

- a macroscopic balances over the <u>entire</u> heat exchanger, or
- a *pseudo* microscopic balance over a <u>slice</u> of the heat exchanger

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



The Simplest Heat Exchanger:

Double-Pipe Heat exchanger - counter current

Another way of looking at it:

We can do:

- a macroscopic balances over the <u>entire</u> heat exchanger, or
- a *pseudo* microscopic balance over a <u>slice</u> of the heat exchanger

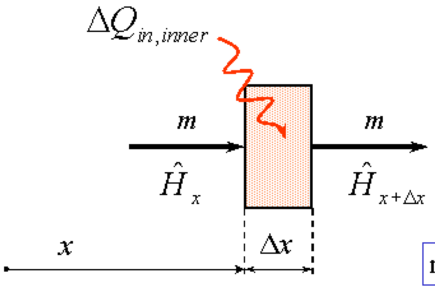
All the details of the algebra are here:
www.chem.mtu.edu/~fmorriso/cm310/double_pipe.pdf

Pseudo Microscopic Energy Balance on a slice of the heat exchanger

Open system energy balance on a differential control volume

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

$$\boxed{\Delta H = Q_{in}}$$



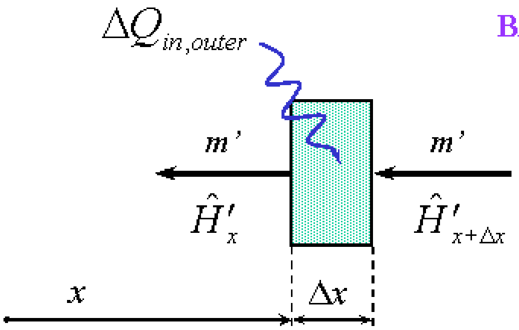
INSIDE BALANCE

recall: Δ is out-in

Pseudo Microscopic Energy Balance on a slice of the heat exchanger

$$\boxed{\Delta H = Q_{in}}$$

OUTSIDE BALANCE



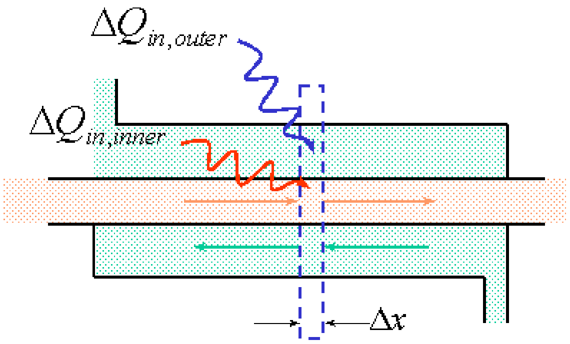
recall: Δ is out-in

Pseudo Microscopic Energy Balance on a slice of the heat exchanger

Adiabatic Heat Exchanger –> $Q_{in} = 0$

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = \cancel{Q_{in}} + \cancel{W_{s,on}}$$

OVERALL BALANCE



$$\boxed{\Delta H = 0}$$

---

energy balance on <u>overall</u> differential system

$$\boxed{\Delta H = 0}$$

$$= \Delta H_{\substack{inner\\system}} + \Delta H_{\substack{outer\\system}}$$

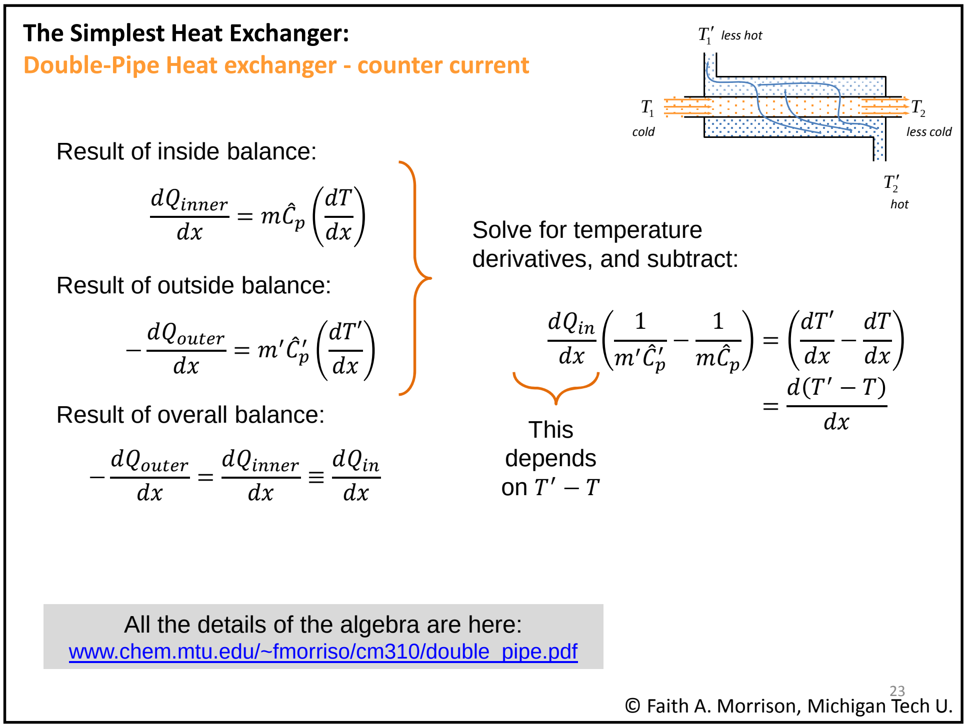
$$= \underbrace{\Delta Q_{in,inner}}_{\text{heat into inner differential system}} + \underbrace{\Delta Q_{in,outer}}_{\text{heat into outer differential system}} = 0$$

divide by Δx and take the limit as Δx goes to zero:

$$\left(\frac{dQ_{in,inner}}{dx}\right) = -\left(\frac{dQ_{in,outer}}{dx}\right)$$

$$\equiv \frac{dQ_{in}}{dx}$$

This expression characterizes the rate of change of heat transferred with respect to distance down the heat exchanger



The Simplest Heat Exchanger:

Double-Pipe Heat exchanger - counter current

Result of inside balance:

$$\frac{dQ_{inner}}{dx} = m\hat{C}_p\left(\frac{dT}{dx}\right)$$

Result of outside balance:

$$-\frac{dQ_{outer}}{dx} = m'\hat{C}_p'\left(\frac{dT'}{dx}\right)$$

Result of overall balance:

$$-\frac{dQ_{outer}}{dx} = \frac{dQ_{inner}}{dx} \equiv \frac{dQ_{in}}{dx}$$

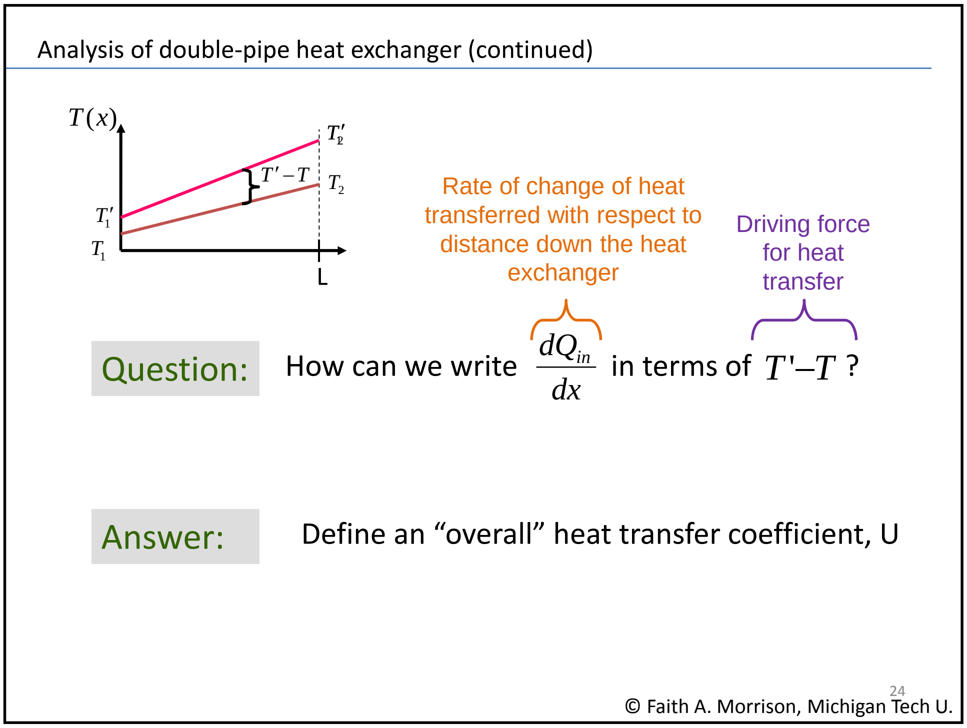
Solve for temperature derivatives, and subtract:

$$\underbrace{\frac{dQ_{in}}{dx}}_{\text{This depends on } T'-T}\left(\frac{1}{m'\hat{C}_p'} - \frac{1}{m\hat{C}_p}\right) = \left(\frac{dT'}{dx} - \frac{dT}{dx}\right)$$
$$= \frac{d(T'-T)}{dx}$$

All the details of the algebra are here:
www.chem.mtu.edu/~fmorriso/cm310/double_pipe.pdf

23

© Faith A. Morrison, Michigan Tech U.



Analysis of double-pipe heat exchanger (continued)

Rate of change of heat transferred with respect to distance down the heat exchanger

Driving force for heat transfer

Question: How can we write $\frac{dQ_{in}}{dx}$ in terms of $T'-T$?

Answer: Define an "overall" heat transfer coefficient, U

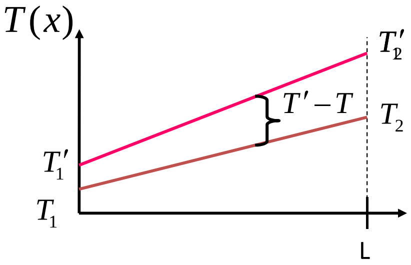
24

© Faith A. Morrison, Michigan Tech U.

$$\underbrace{\frac{d(T'-T)}{dx}} = \frac{dQ_{in}}{dx}\left(\frac{1}{m'\hat{C}'_p} - \frac{1}{m\hat{C}_p}\right)$$

Want to integrate to solve for $T'-T$,

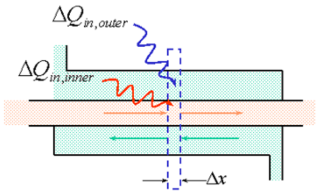
but this is a function of $T'-T$



For the **differential slice of the heat exchanger** that we are considering (modeling our ideas on Newton's law of cooling),

$$\frac{dQ_{in}}{dA} = (?)(T'-T)$$

For the **differential slice of the heat exchanger** that we are considering (modeling our ideas on Newton's law of cooling),



$$\frac{dQ_{in}}{dA} = (?)(T'-T)$$

$$\begin{aligned} dQ_{in} &= (U)dA(T'-T) \\ &= U(2\pi R dx(T'-T) \end{aligned}$$

$$\frac{dQ_{in}}{dx} = U(2\pi R)(T'-T)$$

This is the missing piece that we needed.

We can write $\frac{dQ_{in}}{dx}$ in terms of $T'-T$ if we define an "overall" heat transfer coefficient, U

$$\frac{d(T'-T)}{dx} = \frac{dQ_{in}}{dx}\left(\frac{1}{m'\hat{C}_p'} - \frac{1}{m\hat{C}_p}\right)$$

$$\frac{dQ_{in}}{dx} = 2\pi RU(T'-T)$$

$$\frac{d(T'-T)}{dx} = 2\pi RU(T'-T)\left(\frac{1}{\hat{C}_p'm'} - \frac{1}{\hat{C}_p m}\right)$$

$$\frac{d(T'-T)}{(T'-T)} = \left[2\pi RU\left(\frac{1}{\hat{C}_p'm'} - \frac{1}{\hat{C}_p m}\right)\right]dx$$

$$\frac{d(T'-T)}{(T'-T)} = \left[2\pi RU\left(\frac{1}{\hat{C}_p'm'} - \frac{1}{\hat{C}_p m}\right)\right]dx$$

$$\Phi \equiv T'-T$$

$$\alpha_0 \equiv 2\pi RU\left(\frac{1}{\hat{C}_p'm'} - \frac{1}{\hat{C}_p m}\right)$$

(we'll *assume* *U* is constant)

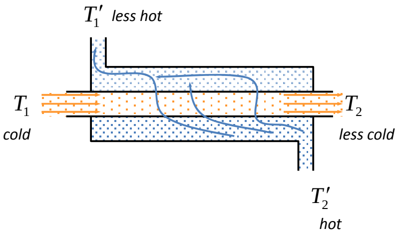
$$\frac{d\Phi}{\Phi} = \alpha_0 dx$$

$$\int \frac{d\Phi}{\Phi} = \alpha_0 \int dx$$

$$\ln\Phi = \alpha_0 x + \text{constant}$$

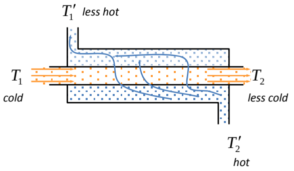
$$\Phi = \Phi_0 e^{\alpha_0 x}$$

B.C:
$x = 0, T - T' = T_1 - T_1'$



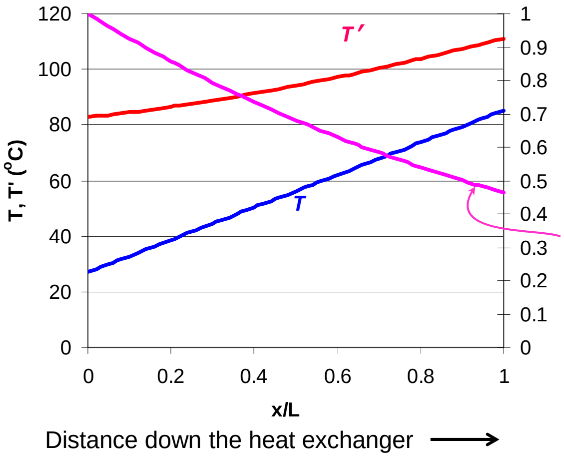
Temperature profile in a double-pipe heat exchanger:

$$\frac{T'-T}{T_1'-T_1}=e^{\alpha_0 x} \qquad \alpha_0=2\pi RU\left(\frac{1}{\hat{C}_p'm'}-\frac{1}{\hat{C}_p m}\right)$$



$$\frac{T'-T}{T_1'-T_1}=e^{\alpha_0 L\left(\frac{x}{L}\right)}$$

$$\alpha_0 L = 2\pi RLU\left(\frac{1}{\hat{C}_p'm'}-\frac{1}{\hat{C}_p m}\right)$$



$\frac{T'-T}{T_1'-T_1}$

$U = 300\,W/mK$
$2\pi RL = 15.4 m^2$
$m'C_p' = 6\,kW/K$
$mC_p = 3\,kW/K$

Note that the temperature curves are not linear.

Temperature profile in a double-pipe heat exchanger:

$$\boxed{\frac{T'-T}{T_1'-T_1}=e^{\alpha_0 x}}$$

$$\alpha_0 = 2\pi RU\left(\frac{1}{\hat{C}_p' m'}-\frac{1}{\hat{C}_p m}\right)$$

Useful result, but what we **REALLY** want is an easy way to relate $Q_{in,\,overall}$ to inlet and outlet temperatures.

At the exit: $x = L,\ \ (T-T') = (T_2 - T_2')$

$$\boxed{\ln\left(\frac{T_2'-T_2}{T_1'-T_1}\right)=U(2\pi RL)\left(\frac{1}{\hat{C}_p' m'}-\frac{1}{\hat{C}_p m}\right)}$$

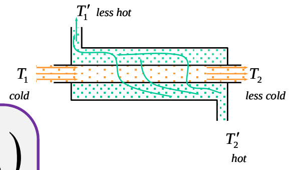
$$\ln\left(\frac{T_2'-T_2}{T_1'-T_1}\right)=U\underbrace{(2\pi RL)}\left(\frac{1}{\hat{C}_p' m'}-\frac{1}{\hat{C}_p m}\right)$$

The $m\hat{C}_p$ terms appear in the overall macroscopic energy balances. We can therefore rearrange this equation by replacing the $m\hat{C}_p$ terms with Q_{in}:

$$Q_{in} = m\hat{C}_p(T_2 - T_1)$$
$$\Rightarrow \frac{1}{m\hat{C}_p} = \frac{T_2 - T_1}{Q_{in}}$$

$$-Q_{in} = m\hat{C}_p'(T_1' - T_2')$$
$$\Rightarrow \frac{1}{m\hat{C}_p'} = \frac{-(T_1' - T_2')}{Q_{in}}$$

$$\underbrace{Q_{in}}_{\text{total heat transferred in exchanger}} = UA\underbrace{\frac{(T_2'-T_2)-(T_1'-T_1)}{\ln\left(\frac{T_2'-T_2}{T_1'-T_1}\right)}}_{\textbf{average temperature driving force}}$$



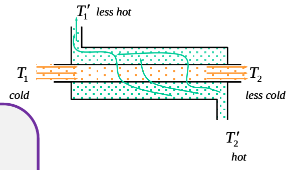
FINAL RESULT:

$$\boxed{Q = U\underbrace{\left(2\pi RL\right)}_{A}\underbrace{\frac{\left(T_1'-T_1\right)-\left(T_2'-T_2\right)}{\ln\dfrac{\left(T_1'-T_1\right)}{\left(T_2'-T_2\right)}}}_{\equiv \Delta T_{lm}}}$$

$$\boxed{Q = UA\Delta T_{lm}}$$

=log-mean temperature difference

ΔT_{lm} is the correct average temperature to use for the overall heat-transfer coefficients in a double-pipe heat exchanger.



FINAL RESULT:

$$\boxed{Q = UA\underbrace{\left[\frac{\left(\Delta T_{left}-\Delta T_{right}\right)}{\ln\left(\dfrac{\Delta T_{left}}{\Delta T_{right}}\right)}\right]}_{\equiv \Delta T_{lm}}}$$

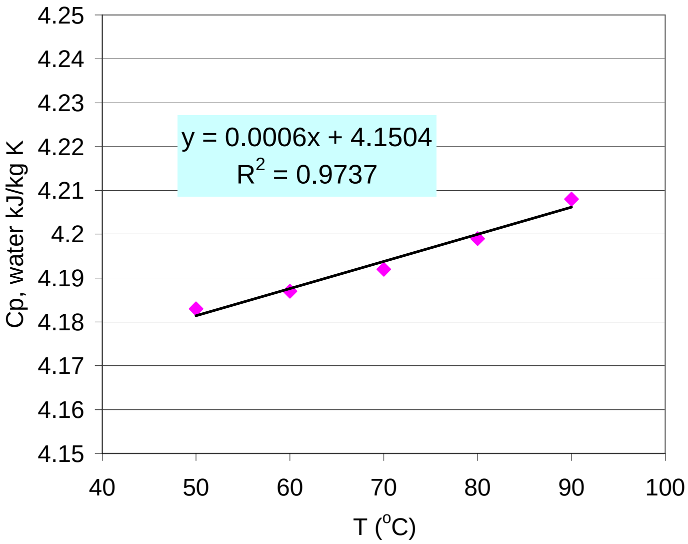
$$\boxed{Q = UA\Delta T_{lm}}$$

=log-mean temperature difference

ΔT_{lm} is the correct average temperature to use for the overall heat-transfer coefficients in a double-pipe heat exchanger.

Example: Heat Transfer in a Double-Pipe Heat Exchanger: *Geankoplis 4th ed. 4.5-4*

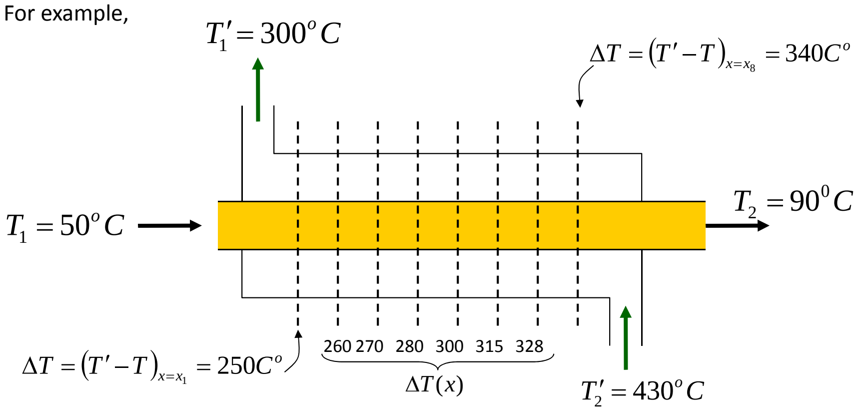
Water flowing at a rate of 13.85 kg/s is to be heated from 54.5 to 87.8°C in a double-pipe heat exchanger by 54,430 kg/h of hot gas flowing counterflow and entering at 427°C ($\hat{C}_{pm} = 1.005\ kJ/kg\ K$). The overall heat-transfer coefficient based on the outer surface is U_o =69.1 W/m^2 K. Calculate the exit-gas temperature and the heat transfer area needed.



Summary:

Double-Pipe Heat Exchanger – the driving force for heat transfer changes along the length of the heat xchanger

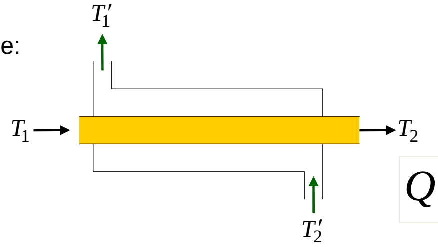
For example,



The correct *average* driving force for the whole xchanger is the log-mean temperature difference $= \Delta T_{lm}$

Optimizing Heat Exchangers

double-pipe:



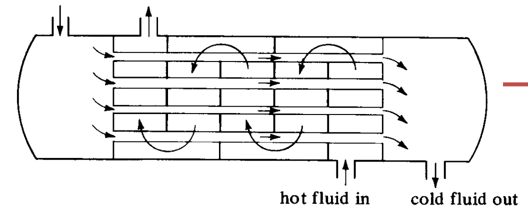
$$Q = UA\Delta T_{lm}$$

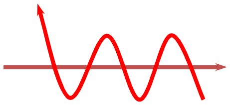
To increase Q appreciably, we must increase A, i.e. R_i

But:
- •only small increases possible
- •increasing R_i decreases h

1-1 Shell and Tube Heat Exchanger

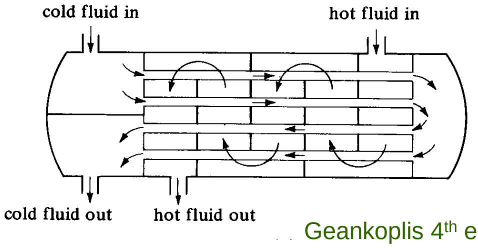
(Same as double pipe H.E.)

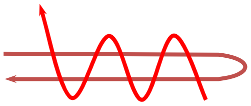




1 shell
1 tube

1-2 Shell and Tube Heat Exchanger

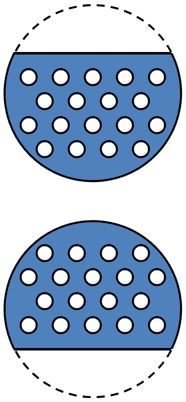




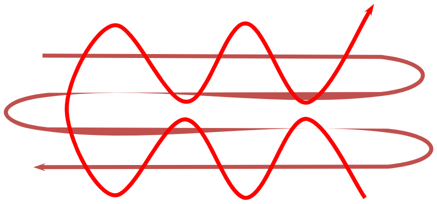
1 shell
2 tube

Geankoplis 4th ed., p292

Cross Baffles in Shell-and-Tube Heat Exchangers



And other more complex arrangements:



For double-pipe heat exchanger:

$$Q = UA\Delta T_{lm}$$

For shell-and-tube heat exchangers:

$$Q = UA\underbrace{\left[\Delta T_{lm}\left(F_T\right)\right]}_{\equiv \Delta T_m}$$

calculated correction factor (obtain from experimentally determined charts)

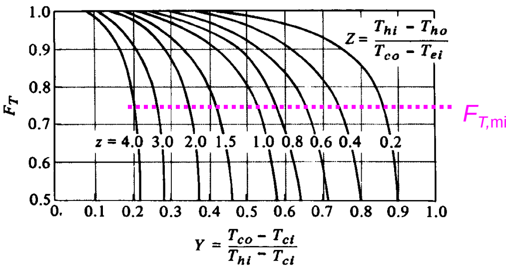
correct mean temperature difference for shell-and-tube heat exchangers

F_T

Shell-and-Tube Heat Exchangers

(1-1 exchanger, $F_T = 1$)

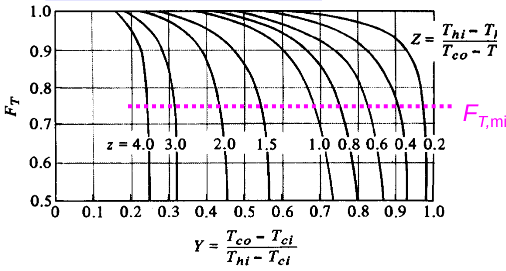
1-2 Shell and Tube H.E.



$F_{T,\min}$=0.75

Efficiency is low when F_T is below $F_{T,min}$

2-4 Shell and Tube H.E.



$F_{T,\min}$=0.75

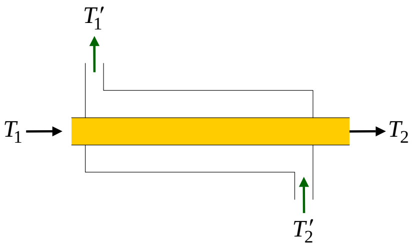
T_{hi} = hot, in
T_{ho} = hot, out
T_{ci} = cold, in
T_{co} = cold, out

Geankoplis 4th ed., p295

Heat Exchanger Design

To calculate Q, we need both inlet and outlet temperatures:

$$Q = UA\Delta T_m = UA(F_T \Delta T_{lm})$$

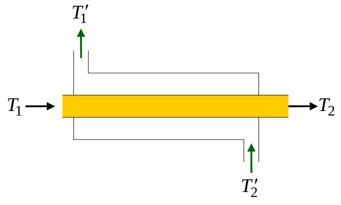


$$Q = UA\left[\frac{(\Delta T_{left} - \Delta T_{right})}{\ln\left(\frac{\Delta T_{left}}{\Delta T_{right}}\right)}\right]$$

What if the outlet temperatures are unknown?
i.e. the design/spec problem.

Example Problem:
How will this heat exchanger perform?

Water flowing at a rate of $0.723\ kg/s$ enters the inside of a countercurrent, double-pipe heat exchanger at 300.K and is heated by an oil stream that enters at $385\ K$ at a rate of $3.2\ kg/s$. The heat capacity of the oil is $1.89\ kJ/kgK$, and the average heat capacity of the water of the temperature range of interest is $4.192\ kJ/kgK$. The overall heat-transfer coefficient of the exchanger is $300.\ W/m^2K$, and the area for heat transfer is $15.4\ m^2$. What is the total amount of heat transferred?



Example Problem:
How will this heat exchanger perform?

Water flowing at a rate of $0.723\ kg/s$ enters the inside of a countercurrent, double-pipe heat exchanger at 300.K and is heated by an oil stream that enters at $385\ K$ at a rate of $3.2\ kg/s$. The heat capacity of the oil is $1.89\ kJ/kgK$, and the average heat capacity of the water of the temperature range of interest is $4.192\ kJ/kgK$. The overall heat-transfer coefficient of the exchanger is $300.\ W/m^2K$, and the area for heat transfer is $15.4\ m^2$. What is the total amount of heat transferred?

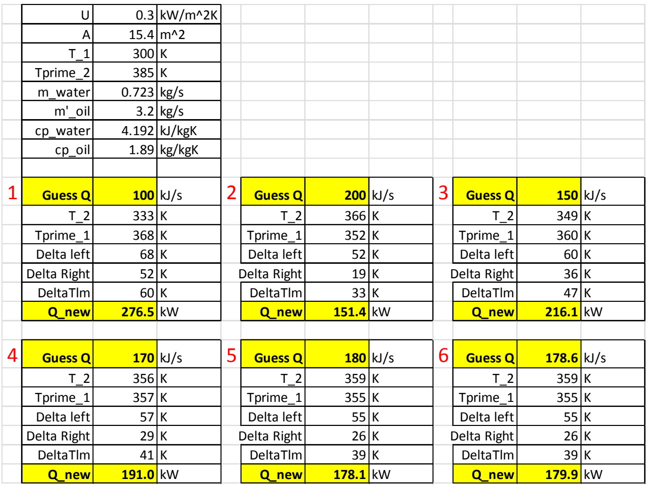
You try.

Example Problem:
How will this heat exchanger perform?

To calculate unknown outlet temperatures:

Procedure:
1. Guess Q
2. Calculate outlet temperatures
3. Calculate ΔT_{lm}
4. Calculate Q
5. Compare, adjust, repeat

Example Problem:
How will this heat exchanger perform?

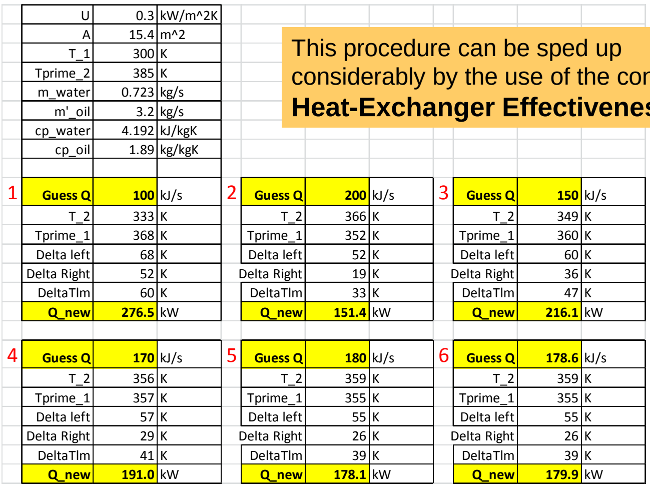


U	0.3	kW/m^2K
A	15.4	m^2
T_1	300	K
Tprime_2	385	K
m_water	0.723	kg/s
m'_oil	3.2	kg/s
cp_water	4.192	kJ/kgK
cp_oil	1.89	kg/kgK

1	**Guess Q**	**100**	kJ/s	2	**Guess Q**	**200**	kJ/s	3	**Guess Q**	**150**	kJ/s
	T_2	333	K		T_2	366	K		T_2	349	K
	Tprime_1	368	K		Tprime_1	352	K		Tprime_1	360	K
	Delta left	68	K		Delta left	52	K		Delta left	60	K
	Delta Right	52	K		Delta Right	19	K		Delta Right	36	K
	DeltaTlm	60	K		DeltaTlm	33	K		DeltaTlm	47	K
	Q_new	**276.5**	kW		**Q_new**	**151.4**	kW		**Q_new**	**216.1**	kW
4	**Guess Q**	**170**	kJ/s	5	**Guess Q**	**180**	kJ/s	6	**Guess Q**	**178.6**	kJ/s
	T_2	356	K		T_2	359	K		T_2	359	K
	Tprime_1	357	K		Tprime_1	355	K		Tprime_1	355	K
	Delta left	57	K		Delta left	55	K		Delta left	55	K
	Delta Right	29	K		Delta Right	26	K		Delta Right	26	K
	DeltaTlm	41	K		DeltaTlm	39	K		DeltaTlm	39	K
	Q_new	**191.0**	kW		**Q_new**	**178.1**	kW		**Q_new**	**179.9**	kW

2013HeatExchEffecExample.xlsx

Example Problem:
How will this heat exchanger perform?



U	0.3	kW/m^2K
A	15.4	m^2
T_1	300	K
Tprime_2	385	K
m_water	0.723	kg/s
m'_oil	3.2	kg/s
cp_water	4.192	kJ/kgK
cp_oil	1.89	kg/kgK

1

Guess Q	**100**	kJ/s
T_2	333	K
Tprime_1	368	K
Delta left	68	K
Delta Right	52	K
DeltaTlm	60	K
Q_new	**276.5**	kW

2

Guess Q	**200**	kJ/s
T_2	366	K
Tprime_1	352	K
Delta left	52	K
Delta Right	19	K
DeltaTlm	33	K
Q_new	**151.4**	kW

3

Guess Q	**150**	kJ/s
T_2	349	K
Tprime_1	360	K
Delta left	60	K
Delta Right	36	K
DeltaTlm	47	K
Q_new	**216.1**	kW

4

Guess Q	**170**	kJ/s
T_2	356	K
Tprime_1	357	K
Delta left	57	K
Delta Right	29	K
DeltaTlm	41	K
Q_new	**191.0**	kW

5

Guess Q	**180**	kJ/s
T_2	359	K
Tprime_1	355	K
Delta left	55	K
Delta Right	26	K
DeltaTlm	39	K
Q_new	**178.1**	kW

6

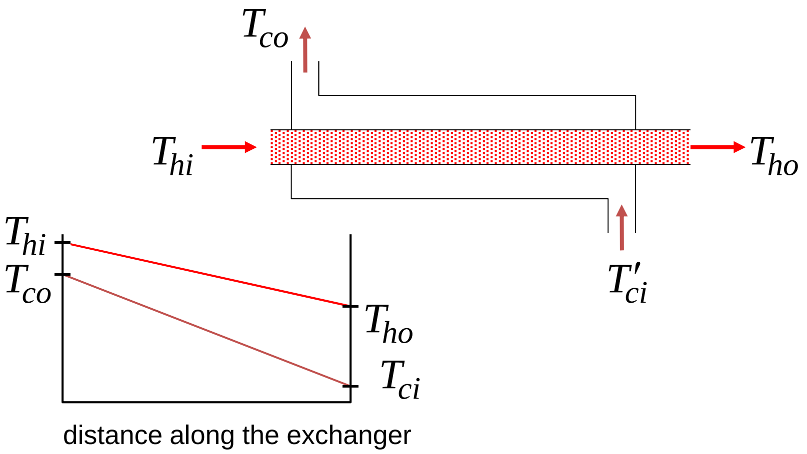
Guess Q	**178.6**	kJ/s
T_2	359	K
Tprime_1	355	K
Delta left	55	K
Delta Right	26	K
DeltaTlm	39	K
Q_new	**179.9**	kW

This procedure can be sped up considerably by the use of the concept of **Heat-Exchanger Effectiveness, ε.**

2013HeatExchEffecExample.xlsx

Heat Exchanger Effectiveness

Consider a *counter-current* double-pipe heat exchanger:



Energy balance cold side:

$$Q_{in,cold} = Q = \left(mC_p\right)_{cold}\left(T_{co} - T_{ci}\right)$$

Energy balance hot side:

$$Q_{in,hot} = -Q = \left(mC_p\right)_{hot}\left(T_{ho} - T_{hi}\right)$$

Equate:

$$\left(m\hat{C}_p\right)_{cold}\left(T_{co} - T_{ci}\right) = -\left(m\hat{C}_p\right)_{hot}\left(T_{ho} - T_{hi}\right)$$

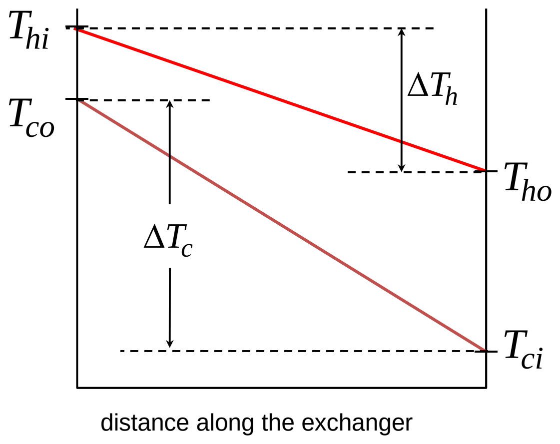
$$\boxed{\frac{\left(mC_p\right)_{hot}}{\left(mC_p\right)_{cold}} = \frac{\left(T_{co} - T_{ci}\right)}{-\left(T_{ho} - T_{hi}\right)} = \frac{\Delta T_c}{\Delta T_h}}$$

$$\boxed{\frac{\left(m\hat{C}_p\right)_{hot}}{\left(m\hat{C}_p\right)_{cold}} = \frac{\Delta T_{cold}}{\Delta T_{hot}}}$$

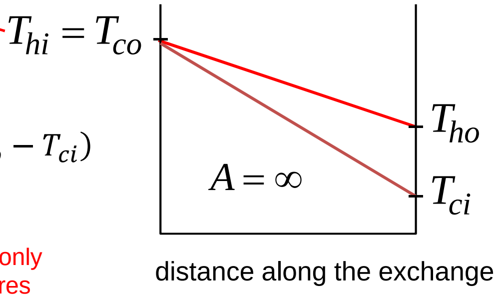
Case 1: $\begin{cases} \left(mC_p\right)_{hot} > \left(mC_p\right)_{cold} \\ \Delta T_c > \Delta T_h \end{cases}$

cold fluid = minimum fluid

We want to compare the amount of heat transferred in this case to the amount of heat transferred in a PERFECT heat exchanger.



If the heat exchanger were *perfect*, $T_{hi} = T_{co}$ (no heat left un-transferred)



cold side:

$$Q_{A=\infty} = (m\hat{C}_p)_{cold}(T_{co} - T_{ci})$$

this temperature difference only depends on inlet temperatures

$$Q_{A=\infty} = (m\hat{C}_p)_{cold}\overbrace{(T_{hi} - T_{ci})}$$

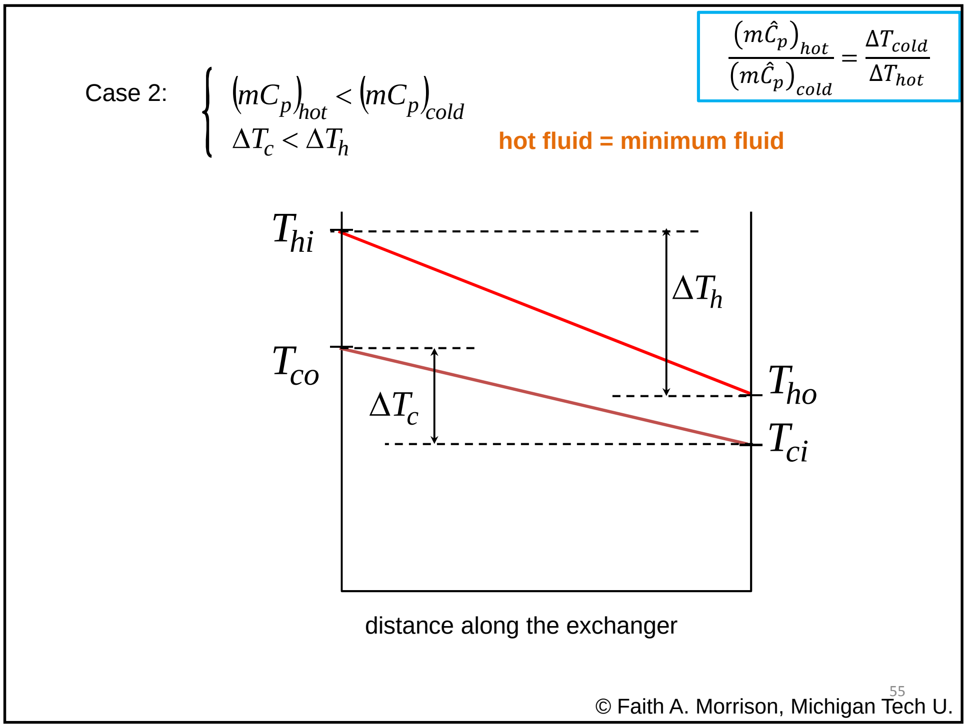
Heat Exchanger Effectiveness, ε

$$\boxed{\varepsilon \equiv \frac{Q}{Q_{A=\infty}}}$$

$$\boxed{\Rightarrow Q = \varepsilon\,(mC_p)_{cold}(T_{hi} - T_{ci})}$$

cold fluid = minimum fluid

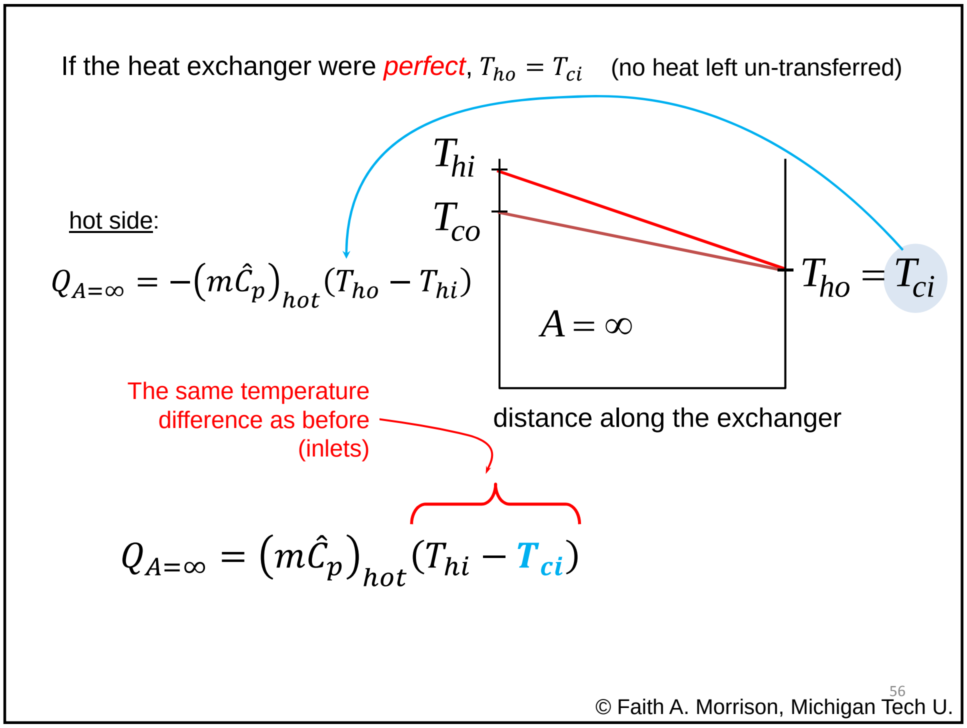
if ε is known, we can calculate Q without iterations



$$\frac{(m\hat{C}_p)_{hot}}{(m\hat{C}_p)_{cold}} = \frac{\Delta T_{cold}}{\Delta T_{hot}}$$

Case 2: $\begin{cases}(mC_p)_{hot} < (mC_p)_{cold} \\ \Delta T_c < \Delta T_h\end{cases}$

hot fluid = minimum fluid



If the heat exchanger were *perfect*, $T_{ho} = T_{ci}$ (no heat left un-transferred)

hot side:

$$Q_{A=\infty} = -(m\hat{C}_p)_{hot}(T_{ho} - T_{hi})$$

The same temperature difference as before (inlets)

$$Q_{A=\infty} = (m\hat{C}_p)_{hot}\overbrace{(T_{hi} - \boldsymbol{T_{ci}})}$$

Heat Exchanger Effectiveness

$$\boxed{\varepsilon \equiv \frac{Q}{Q_{A=\infty}}}$$

$$\boxed{\Rightarrow Q = \varepsilon \left(mC_p\right)_{hot}\left(T_{hi} - T_{ci}\right)}$$

hot fluid = minimum fluid

in general,

$$Q = \varepsilon \left(mC_p\right)_{\min}\left(T_{hi} - T_{ci}\right)$$

if ε is known, we can calculate *Q* without iterations

But where do we get ε ?

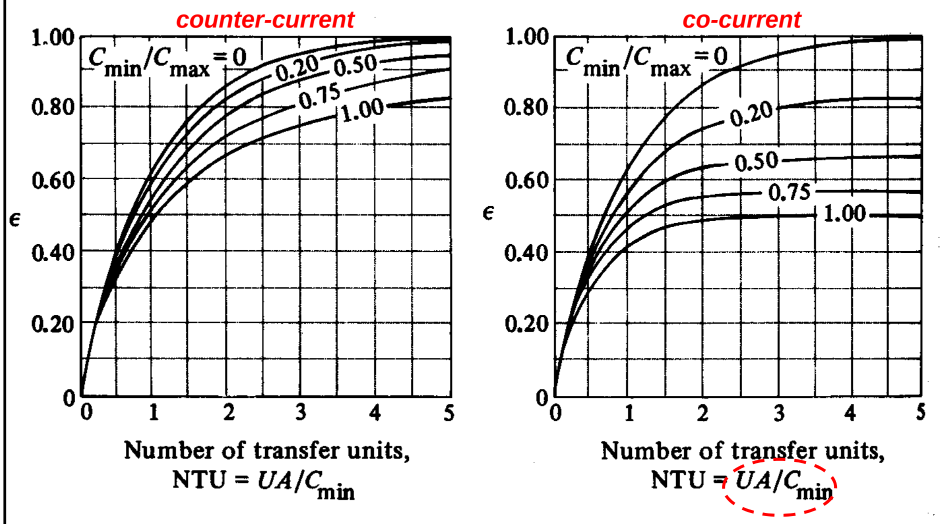
The same equations we use in the trial-and-error solution can be combined algebraically to give ε as a function of *$(mC_p)_{min}$*, *$(mC_p)_{max}$*.

counter-current flow:

$$\boxed{\varepsilon = \left(\frac{1 - e^{\frac{-UA}{\left(mC_p\right)_{\min}}\left(1 - \frac{\left(mC_p\right)_{\min}}{\left(mC_p\right)_{\max}}\right)}}{1 - \frac{\left(mC_p\right)_{\min}}{\left(mC_p\right)_{\max}} e^{\frac{-UA}{\left(mC_p\right)_{\min}}\left(1 - \frac{\left(mC_p\right)_{\min}}{\left(mC_p\right)_{\min}}\right)}} \right)}$$

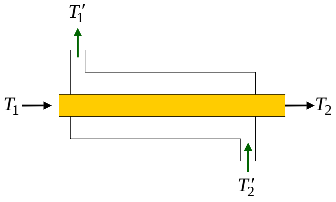
This relation is plotted in Geankoplis, as is the relation for co-current flow.

Heat Exchanger Effectiveness for Double-pipe or 1-1 Shell-and-Tube Heat Exchangers



note: Geankoplis’ $C_{min}=(mC_p)_{min}$

Geankoplis 4th ed., p299



Example Problem:
How will this heat exchanger perform?

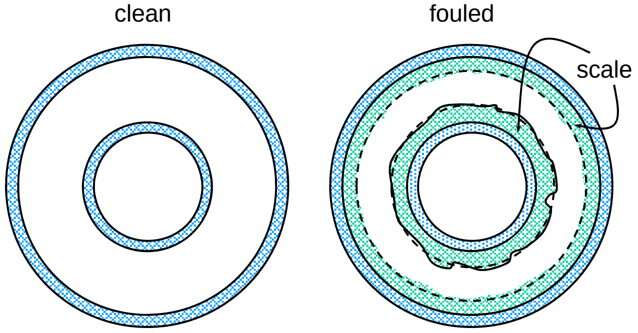
Water flowing at a rate of $0.723\ kg/s$ enters the inside of a countercurrent, double-pipe heat exchanger at 300.K and is heated by an oil stream that enters at $385\ K$ at a rate of $3.2\ kg/s$. The heat capacity of the oil is $1.89\ kJ/kgK$, and the average heat capacity of the water of the temperature range of interest is $4.192\ kJ/kgK$. The overall heat-transfer coefficient of the exchanger is $300.\ W/m^2K$, and the area for heat transfer is $15.4\ m^2$. What is the total amount of heat transferred?

You try.

Heat Exchanger Fouling

- material deposits on hot surfaces
- rust, impurities
- strong effect when boiling occurs

scale adds an additional resistance to heat transfer



Heat transfer resistances

$$U_{i\,or\,o}=\frac{\dfrac{1}{R_{i\,or\,o}}}{\dfrac{1}{h_iR_i}+\dfrac{1}{k}\ln\left(\dfrac{R_o}{R_i}\right)+\dfrac{1}{h_oR_o}}$$

resistance due to interface

resistance due to limited thermal conductivity

add effect of fouling

$$U_{i\,or\,o}=\frac{\dfrac{1}{R_{i\,or\,o}}}{\dfrac{1}{h_iR_i}+\dfrac{1}{h_{di}R_i}+\dfrac{1}{k}\ln\left(\dfrac{R_o}{R_i}\right)+\dfrac{1}{h_{do}R_o}+\dfrac{1}{h_oR_o}}$$

see Perry's Handbook, or Geankoplis 4th ed. Table 4.9-1, page 300 for values of h_d

Heat Exchanger Fouling

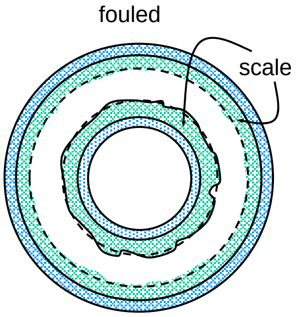


Table 4.9-1. *Typical Fouling Coefficients (P3, N1)*

	h_d ($W/m^2 \cdot K$)
Distilled and seawater	11 350
City water	5680
Muddy water	1990–2840
Gases	2840
Vaporizing liquids	2840
Vegetable and gas oils	1990

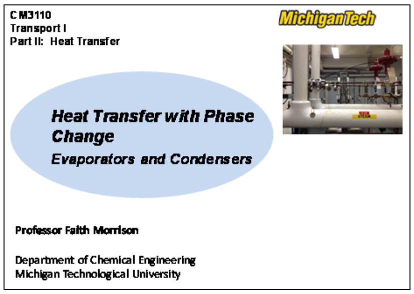
Table 4.9-2. *Typical Values of Overall Heat-Transfer Coefficients in Shell-and-Tube Exchangers (H1, P3, W1)*

	U ($W/m^2 \cdot K$)
Water to water	1140–1700
Water to brine	570–1140
Water to organic liquids	570–1140
Water to condensing steam	1420–2270
Water to gasoline	340–570
Water to gas oil	140–340
Water to vegetable oil	110–285
Gas oil to gas oil	110–285
Steam to boiling water	1420–2270
Water to air (finned tube)	110–230
Light organics to light organics	230–425
Heavy organics to heavy organics	55–230

Geankoplis, 4th edition, p300

Next:

- Heat transfer with phase change
- Evaporators
- Radiation
- *DONE*



CM3110
Transport I
Part II: Heat Transfer

Heat Transfer with Phase Change
Evaporators and Condensers

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University