

CM3110
Transport I
Part II: Heat Transfer

MichiganTech

Complex Heat Transfer – Dimensional Analysis



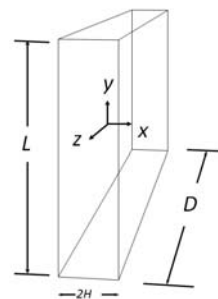
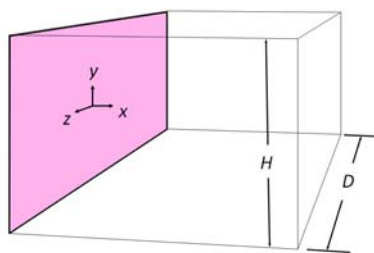
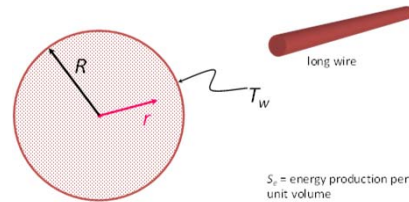
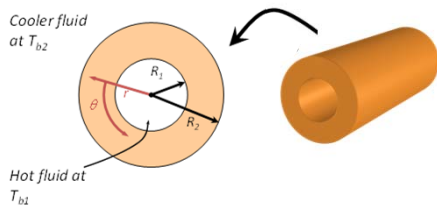
Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

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(what have we been up to?)

Examples of (simple, 1D) Heat Conduction



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Examples of (simple, 1D) Heat Conduction

But these are highly simplified geometries

S_v = energy production per unit volume

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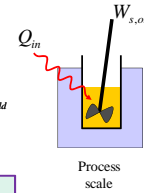
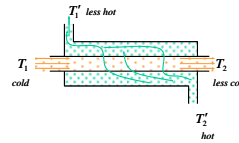
How do we handle complex geometries, complex flows, complex machinery?

Process scale

4
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Complex Heat Transfer – Dimensional Analysis

(Answer: Use the same techniques we have been using in fluid mechanics)



Engineering Modeling

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

5

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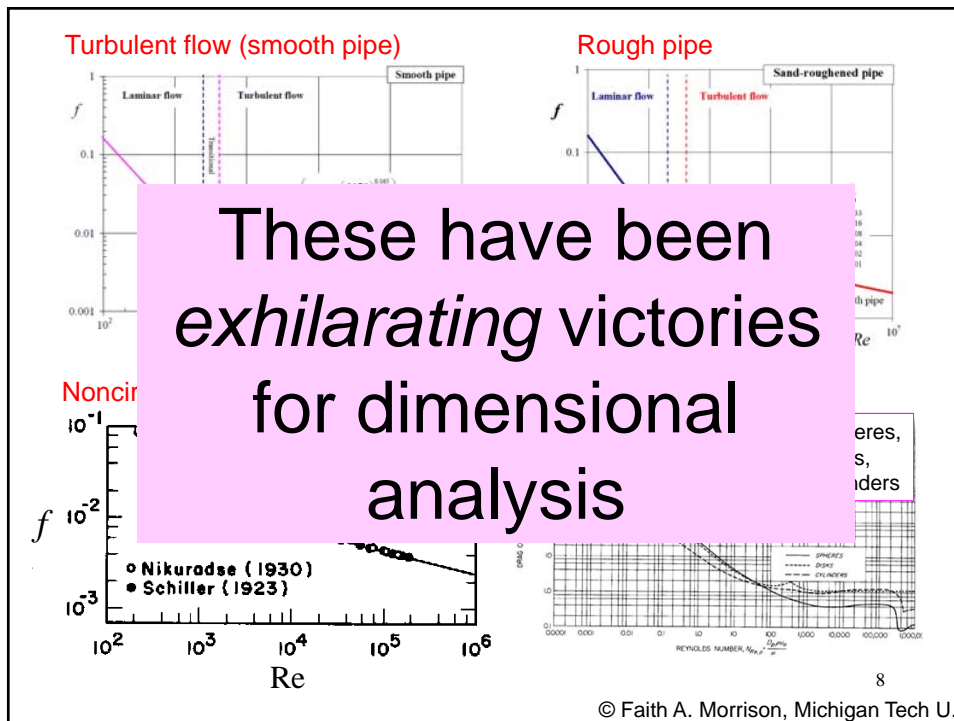
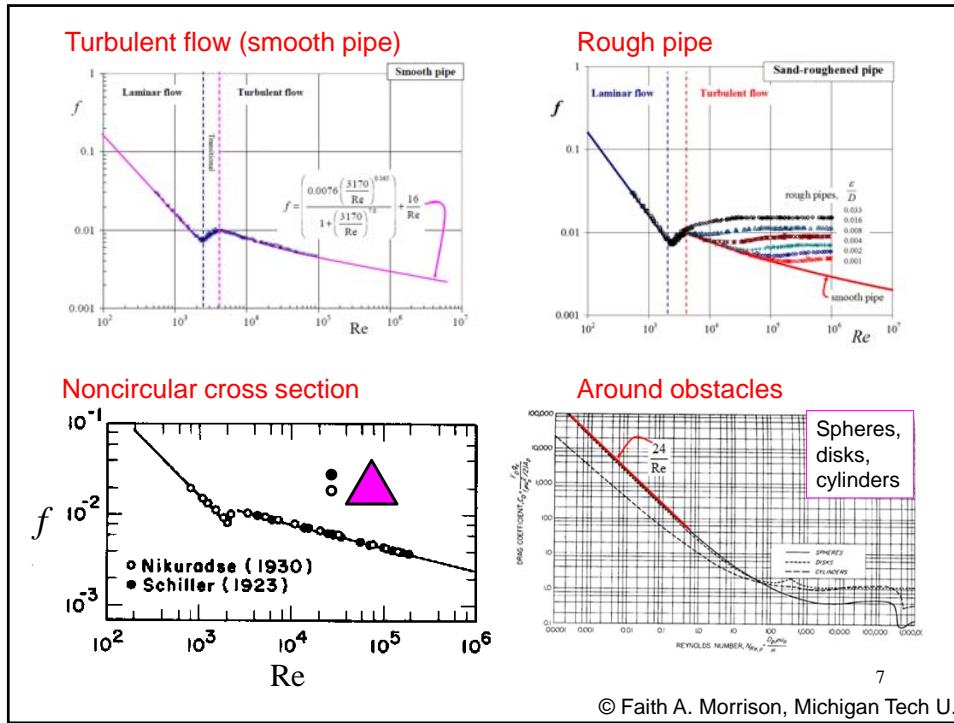
Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis thus far:

- Flow in pipes at all flow rates (laminar and turbulent)
Solution: Navier-Stokes, Re, Fr, L/D , dimensionless wall force = f ; $f = f(\text{Re}, L/D)$
- Rough pipes
Solution: add additional length scale; then nondimensionalize
- Non-circular conduits
Solution: Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes)
Solution: Navier-Stokes, Re, dimensionless drag = CD ; $CD = CD(\text{Re})$
- Boundary layers
Solution: Two components of velocity need independent lengthscales

6

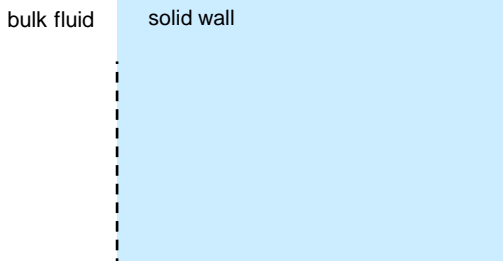
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Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

- Forced convection heat transfer from fluid to wall
Solution: ?
- Natural convection heat transfer from fluid to wall
Solution: ?
- Radiation heat transfer from solid to fluid
Solution: ?



bulk fluid solid wall

9

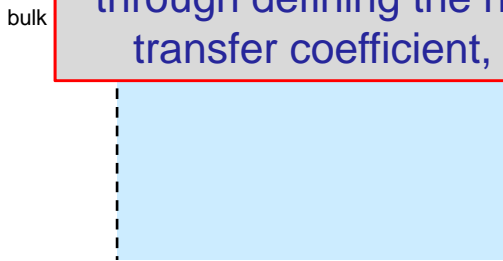
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Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

- Forced convection heat transfer from fluid to wall
Solution: ?
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Solution: ?
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Solution: ?

We have already started using the results/techniques of dimensional analysis through defining the heat transfer coefficient, h



bulk solid wall

10

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Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

- Forced convection heat transfer from fluid to wall
Solution: ?
- Natural convection heat transfer from fluid to wall
Solution: ?
- Radiation heat transfer from fluid to wall
Solution: ?

bulk

We have already started using the results/techniques of dimensional analysis through defining the heat transfer coefficient, h

(recall that we did this in fluids too: we used the $f(Re)$ correlation (Moody chart) long before we knew where that all came from)

11

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Handy tool:
Heat Transfer Coefficient

$T(x)$

bulk fluid solid wall

T_{bulk}

T_{wall}

$T(x)$ in liquid

$T(x)$ in solid

x_{wall} x

$T_{bulk} - T_{wall}$

→ The temperature variation near-wall region is caused by complex phenomena that are lumped together into the heat transfer coefficient, h

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Complex Heat Transfer – Dimensional Analysis

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

This expression serves as the definition of the **heat transfer coefficient**.

$$\left| \frac{q_x}{A} \right| \equiv h |T_{bulk} - T_{wall}|$$

h depends on:

- geometry
- fluid velocity
- fluid properties
- temperature difference

13

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Complex Heat Transfer – Dimensional Analysis

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

This expression serves as the definition of the **heat transfer coefficient**.

$$\left| \frac{q_x}{A} \right| \equiv h |T_{bulk} - T_{wall}|$$

To get values of **h** for various situations, we need to measure data and create data correlations (**dimensional analysis**)

h depends on:

- geometry
- fluid velocity
- fluid properties
- temperature difference

14

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Complex Heat Transfer – Dimensional Analysis

Complex Heat transfer Problems to Solve:

- Forced convection heat transfer from fluid to wall

Solution: ?

- Natural convection heat transfer from fluid to wall

Solution: ?

- Radiation heat transfer from solid to fluid

Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

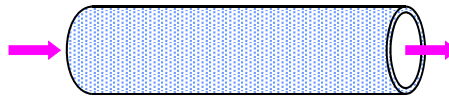
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Complex Heat Transfer – Dimensional Analysis

Chosen problem: Forced Convection Heat Transfer

Solution: Dimensional Analysis



Following procedure familiar from pipe flow,

- **What are governing equations?**
- **Scale factors (dimensionless numbers)?**
- **Quantity of interest?**

Heat flux at the wall

16

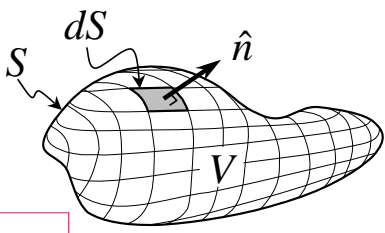
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Complex Heat Transfer – Dimensional Analysis

General Energy Transport Equation

(microscopic energy balance)

As for the derivation of the microscopic momentum balance, the microscopic energy balance is derived on an arbitrary volume, V , enclosed by a surface, S .



Gibbs notation:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

see handout for component notation

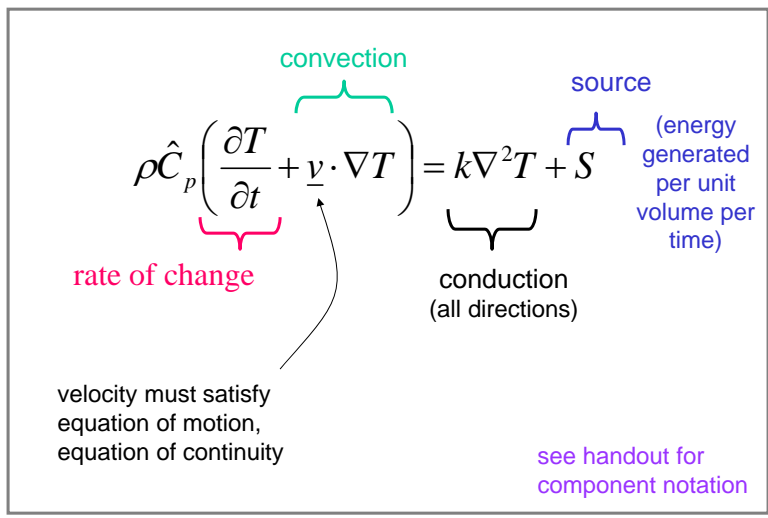
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Complex Heat Transfer – Dimensional Analysis

General Energy Transport Equation

(microscopic energy balance)



rate of change

convection

source (energy generated per unit volume per time)

conduction (all directions)

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

18

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Equation of energy for Newtonian fluids of constant density, ρ , and thermal conductivity, k , with source term (source could be viscous dissipation, electrical energy, chemical energy, etc., with units of energy/(volume time)).

CM310 Fall 1999 Faith Morrison

Source: R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Processes*, Wiley, NY, 1960, page 319.

Gibbs notation (vector notation) **Note: this handout is on the web:**
www.chem.mtu.edu/~fmorriso/cm310/energy2013.pdf

$$\left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T\right) = \frac{k}{\rho \hat{C}_p} \nabla^2 T + \frac{S}{\rho \hat{C}_p}$$

Cartesian (xyz) coordinates:

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{S}{\rho \hat{C}_p}$$

Cylindrical (r θ z) coordinates:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{S}{\rho \hat{C}_p}$$

Spherical (r θ ϕ) coordinates:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} = \frac{k}{\rho \hat{C}_p} \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2}\right) + \frac{S}{\rho \hat{C}_p}$$

19
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**** REVIEW ** REVIEW ****

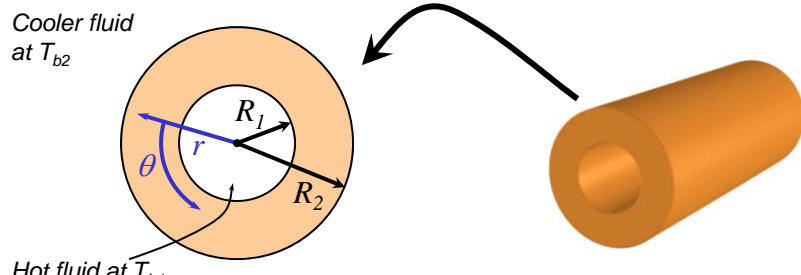
Example: Heat flux in a cylindrical shell

Assumptions:

- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **fluid on the inside** is at T_{b1} and the **fluid on the outside** is at T_{b2} ? ($T_{b1} > T_{b2}$)*

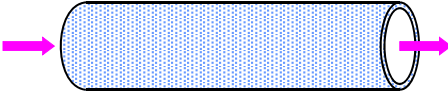
Forced-convection heat transfer



20
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Now: How do we develop correlations for h ?

Consider: Heat-transfer to from flowing fluid inside of a tube – forced-convection heat transfer



T_1 = core bulk temperature
 T_o = wall temperature
 $T(r, \theta, z)$ = temp distribution in the fluid

In principle, with the right math/computer tools, we could calculate the complete temperature and velocity profiles in the moving fluid.

21
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Complex Heat Transfer – Dimensional Analysis

What are governing equations?
 Microscopic energy balance plus Navier-Stokes, continuity

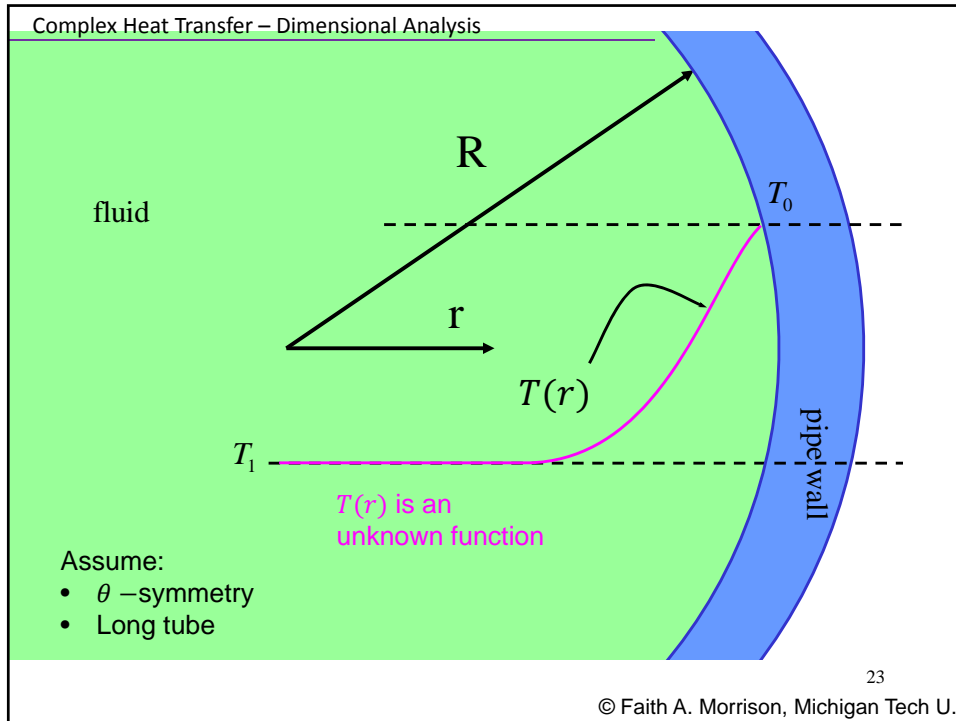
Scale factors?
 Re, Fr, L/D plus whatever comes from the rest of the analysis

Quantity of interest (like wall force, drag)?
Heat transfer coefficient

The quantity of interest in forced-convection heat transfer is h

How is the heat transfer coefficient related to the full solution for $T(r, \theta, z)$ in the fluid?

22
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Complex Heat Transfer – Dimensional Analysis

At the boundary, (Newton's Law of Cooling is the **boundary condition**)

Total heat flow through the wall in terms of h

$$\frac{q_r}{A} = h|T_1 - T_0|$$

$$Q = (2\pi RL)(h)(T_1 - T_0)$$

We can calculate the total heat transferred from $T(r)$ in the fluid:

Total heat conducted to the wall from the fluid

$$Q = \iint_S [\hat{n} \cdot \tilde{q}]_{surface} dS$$

$\tilde{q} = \frac{q_r}{A} = -k \frac{\partial T}{\partial r}$ **We need $T(r)$ in the fluid**

24

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Complex Heat Transfer – Dimensional Analysis

Equate these two: Total heat flow through the wall

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S [\hat{e}_r \cdot \underline{\tilde{q}}]_{surface} dS$$

Total heat flow through
the wall in terms of h

$$(2\pi RL)(h)(T_1 - T_0) = Q = \int_0^{2\pi} \int_0^L -k \frac{\partial T}{\partial r} \Big|_{r=R} R dz d\theta$$

Total heat conducted to the
wall from the fluid

25

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Complex Heat Transfer – Dimensional Analysis

Equate these two: Total heat flow through the wall

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S [\hat{e}_r \cdot \underline{\tilde{q}}]_{surface} dS$$

$$(2\pi RL)(h)(T_1 - T_0) = Q = \int_0^{2\pi} \int_0^L -k \frac{\partial T}{\partial r} \Big|_{r=R} R dz d\theta$$

Now, non-dimensionalize
this expression

26

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Complex Heat Transfer – Dimensional Analysis

Non-dimensionalize

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$$

27
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Complex Heat Transfer – Dimensional Analysis

$$h(\cancel{\pi DL})(\cancel{T_1 - T_o}) = \int_0^{2\pi} \int_0^{L/D} -k \frac{\partial T^*}{\partial r^*} \Big|_{r^*=1/2} \frac{(\cancel{T_1 - T_o}) \cancel{D^2}}{\cancel{D}} dz^* d\theta$$

$$2\pi \left(\frac{hD}{k} \right) \left(\frac{L}{D} \right) = \int_0^{2\pi} \int_0^{L/D} - \frac{\partial T^*}{\partial r^*} \Big|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$Nu = Nu\left(T^*, \frac{L}{D}\right)$$

one additional
dimensionless group

28
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Complex Heat Transfer – Dimensional Analysis

$$h(\pi DL)(T_1 - T_o) = \int_0^{2\pi} \int_0^{L/D} -k \frac{\partial T^*}{\partial r^*} \Big|_{r^*=1/2} \frac{(T_1 - T_o) D^2}{D} dz^* d\theta$$

This is a function of Re through ν

$$2\pi \left(\frac{hD}{k} \right) \left(\frac{L}{D} \right) = \int_0^{2\pi} \int_0^{L/D} \frac{\partial T^*}{\partial r^*} \Big|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$Nu = Nu \left(T^*, \frac{L}{D} \right)$$

one additional dimensionless group

29
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Complex Heat Transfer – Dimensional Analysis

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^* \frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*} \frac{\partial T^*}{\partial \theta} + v_z^* \frac{\partial T^*}{\partial z^*} \right) = \frac{1}{\text{Pe}} \left(\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) + S^*$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\text{Re}} (\nabla^2 v_z^*) + \frac{1}{\text{Fr}} g^*$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

Quantity of interest

$$Nu = \frac{1}{2\pi L/D} \int_0^{2\pi} \int_0^{L/D} -k \frac{\partial T^*}{\partial r^*} \Big|_{r^*=1/2} dz^* d\theta$$

$\text{Pe} = \text{Pr Re} = \frac{\hat{C}_p \mu}{k} \frac{\rho V D}{\mu}$

$\text{Pr} = \frac{\hat{C}_p \mu}{k}$

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Complex Heat Transfer – Dimensional Analysis

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of four dimensionless groups:

no free surfaces

Peclet number

$$Pe \equiv \frac{\rho \hat{c}_p V D}{k} = \frac{\hat{c}_p \mu \rho V D}{k \mu}$$

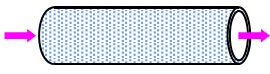
Prandtl number

$$Pr \equiv \frac{\hat{c}_p \mu}{k}$$

$$Nu = Nu \left(Re, Pr, \cancel{Fr}, \frac{L}{D} \right)$$

Now, do the experiments.

31
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Complex Heat Transfer – Dimensional Analysis 

Now, do the experiments.

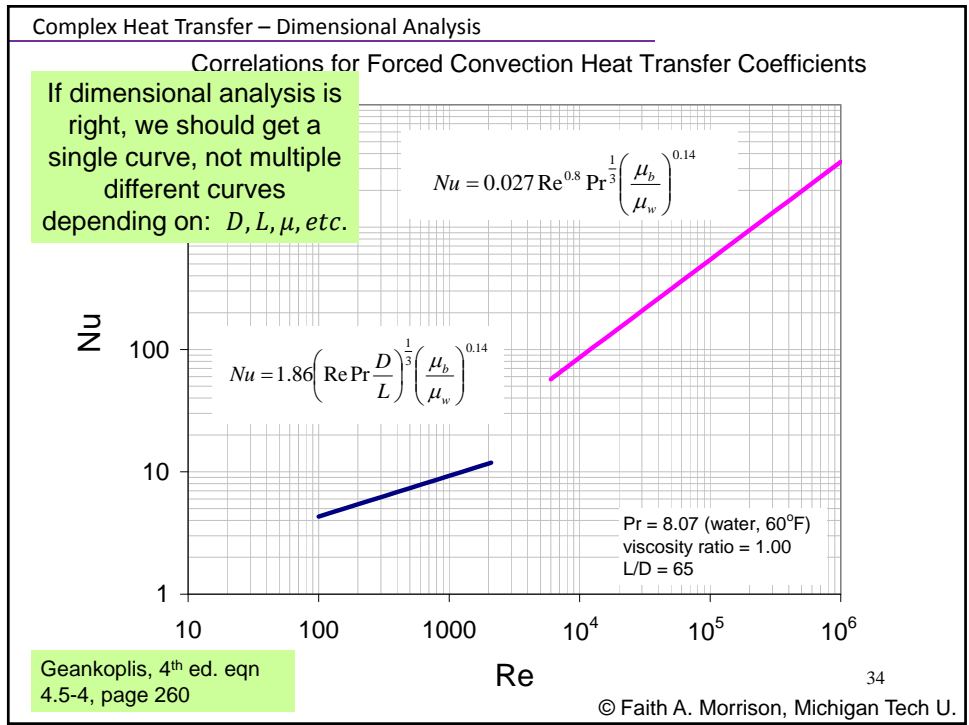
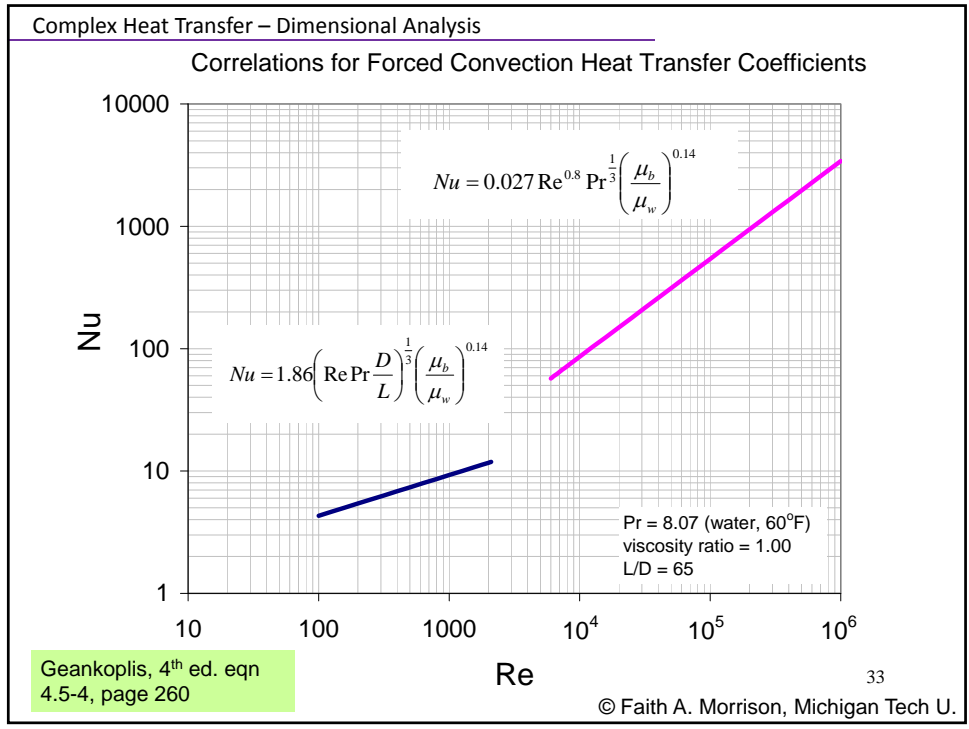
Forced Convection Heat Transfer

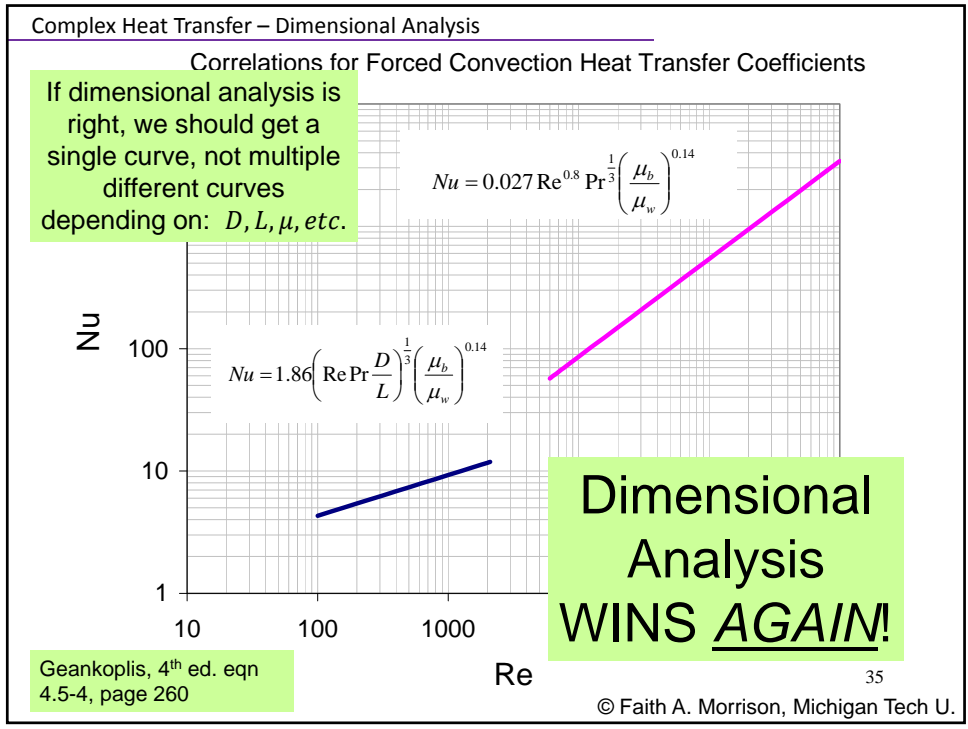
- Build apparatus (*several* actually, with different D, L)
- Run fluid through the inside (at different V; for different fluids ρ, μ, \hat{c}_p, k)
- Measure T_{bulk} on inside; T_{wall} on inside
- Measure rate of heat transfer, Q
- Calculate h : $|Q| = hA|T_{bulk} - T_{wall}|$
- Report h values in terms of dimensionless correlation:

$$Nu = \frac{hD}{k} = f \left(Re, Pr, \frac{L}{D} \right)$$

It should only be a function of these dimensionless numbers (**if** our Dimensional Analysis is correct.....)

32
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Complex Heat Transfer – Dimensional Analysis

Heat Transfer in Laminar flow in pipes:
[data correlation for forced convection heat transfer coefficients](#)

$$Nu_a = \frac{h_a D}{k} = 1.86 \left(Re Pr \frac{D}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

the subscript “a” refers to **the type of average temperature** used in calculating the heat flow, q

$$q = h_a A \Delta T_a$$

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

Geankoplis, 4th ed. eqn 4.5-4, page 260

$Re < 2100, (Re Pr \frac{D}{L}) > 100$, horizontal pipes; all physical properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the (constant) wall temperature.

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Complex Heat Transfer – Dimensional Analysis

Forced convection
Heat Transfer in Laminar flow in pipes

$$Nu_a = \frac{h_a D}{k} = 1.86 \left(\text{Re Pr} \frac{D}{L} \right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

Physical Properties evaluated at:
 $\frac{T_{b,in} + T_{b,out}}{2}$

Forced convection
Heat Transfer in Turbulent flow in pipes

$$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027 \text{Re}^{0.8} \text{Pr}^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

May have to be estimated
 $\frac{T_{b,in} + T_{b,out}}{2}$

bulk mean temperature

- all physical properties (except μ_w) evaluated at the **bulk mean temperature**
- Laminar or turbulent flow

37
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Complex Heat Transfer – Dimensional Analysis

? In our dimensional analysis, we **assumed** constant ρ , k , μ , etc. Therefore we did not predict a viscosity-temperature dependence. If viscosity is not assumed constant, the dimensionless group shown below is predicted to appear in correlations.

$$Nu_a = \frac{h_a D}{k} = 1.86 \left(\text{Re Pr} \frac{D}{L} \right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

(reminiscent of pipe wall roughness; needed to modify dimensional analysis to correlate on roughness)

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Complex Heat Transfer – Dimensional Analysis

Viscous fluids with large ΔT

heating

cooling

$\mu_b > \mu_w$

empirical result:

$\mu_b < \mu_w$

$\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$

ref: McCabe, Smith, Harriott, 5th ed, p339

39

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Complex Heat Transfer – Dimensional Analysis

Why does $\frac{L}{D}$ appear in laminar flow correlations and not in the turbulent flow correlations?

LAMINAR

$h(z)$

Less lateral mixing in laminar flow means more variation in $h(x)$.

40

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Complex Heat Transfer – Dimensional Analysis

TURBULENT

In turbulent flow, good lateral mixing reduces the variation in h along the pipe length.

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Example of *partial* solution to Homework

laminar flow in pipes	$Nu_a = \frac{h_a D}{k} = 1.86 \left(\text{Re Pr} \frac{D}{L} \right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$	Re < 2100, (RePrD/L) > 100, horizontal pipes, eqn 4.5-4, page 238; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.	$(T_{bulk\ mean})$
turbulent flow in smooth tubes	$Nu_{tm} = \frac{h_{tm} D}{k} = 0.027 \text{Re}^{0.8} \text{Pr}^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$	Re > 6000, 0.7 < Pr < 16,000, L/D > 60, eqn 4.5-8, page 239; all properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the wall temperature. The mean is the average of the inlet and outlet bulk temperatures; not valid for liquid metals.	
air at 1atm in turbulent flow in pipes	$h_{tm} = \frac{3.52V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{tm} = \frac{0.5V(ft/s)^{0.8}}{D(ft)^{0.2}}$	equation 4.5-9, page 239	
water in turbulent flow in pipes	$h_{tm} = 1429(1 + 0.0146T(^{\circ}C))^{\frac{0.8}{3}} \frac{V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{tm} = 150(1 + 0.011T(^{\circ}F))^{\frac{0.8}{3}} \frac{V(ft/s)^{0.8}}{D(ft)^{0.2}}$	4 < T(^{\circ}C) < 105, equation 4.5-10, page 239	

42
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Complex Heat transfer Problems to Solve:

✓ • Forced convection heat transfer from fluid to wall
Solution: ?

➔ • Natural convection heat transfer from fluid to wall
Solution: ?

• Radiation heat transfer from solid to fluid
Solution: ?

We started with a forced-convection pipe problem, did dimensional analysis, and found the dimensionless numbers.

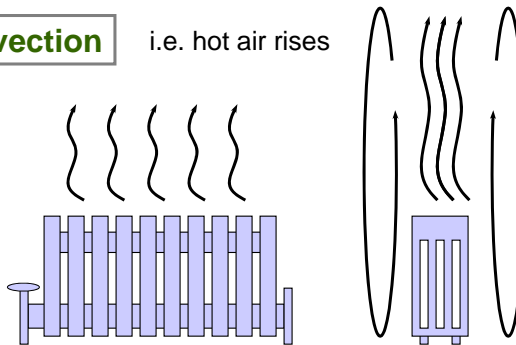
To do a situation with different physics, we must start with a different starting problem.

43

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



- heat moves from hot surface to cold air (fluid) by radiation and conduction
- increase in fluid temperature decreases fluid density
- recirculation flow begins
- **recirculation adds to the heat-transfer** from conduction and radiation

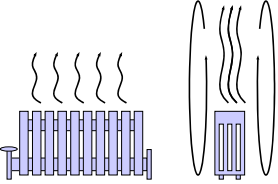
⇒ coupled heat and momentum transport

44

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



How can we solve **real** problems involving free (natural) convection?

We'll try this: Let's review how we approached solving real problems in *earlier* cases, i.e. in fluid mechanics, forced convection.

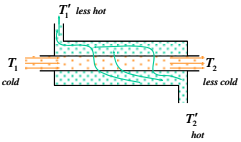
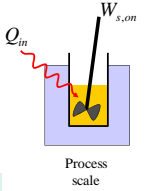


45

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Engineering Modeling

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Example: Free convection between long parallel plates or heat transfer through double-pane glass windows

$T_2 > T_1$

assumptions:

- long, wide slit
- steady state
- no source terms
- viscosity constant
- density varies with T

Calculate: T, \underline{v} profiles

47

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Example : Natural convection between vertical plates

Mass balance:

$$\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho + \rho (\nabla \cdot \underline{v}) = 0$$

Momentum balance:

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

$T_2 > T_1$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \rho g_x$$

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) + \rho g_y$$

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

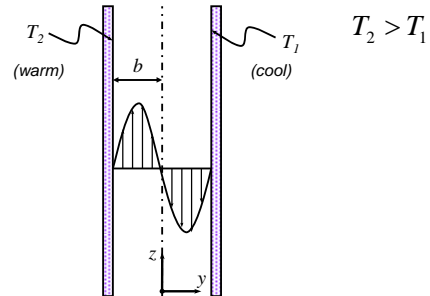
48

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Example : Natural convection between vertical plates

You try.



49

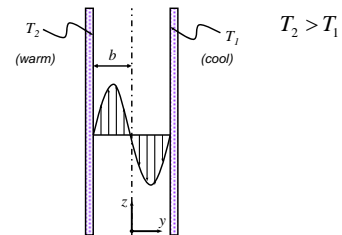
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Complex Heat Transfer – Dimensional Analysis—Free Convection

Mass balance:

$$\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho + \rho(\nabla \cdot \underline{v}) = 0$$

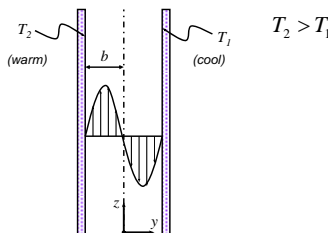
$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$



50

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Complex Heat Transfer – Dimensional Analysis—Free Convection



Mass balance:

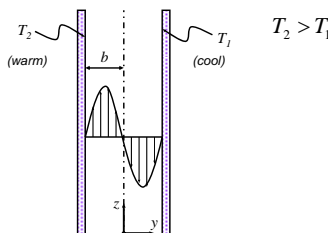
$$\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho + \rho(\nabla \cdot \underline{v}) = 0$$

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

steady $\underline{v} = v_z(y)\hat{e}_z$ tall, wide

51
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Complex Heat Transfer – Dimensional Analysis—Free Convection



Mass balance:

$$\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho + \rho(\nabla \cdot \underline{v}) = 0$$

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

steady $\underline{v} = v_z(y)\hat{e}_z$ tall, wide

Conclusion: density must not vary with z.

$$\rho = \rho(x, y)$$

$$\rho = \rho(y)$$

52
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Complex Heat Transfer – Dimensional Analysis—Free Convection

Momentum balance:

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \rho g_x$$

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) + \rho g_y$$

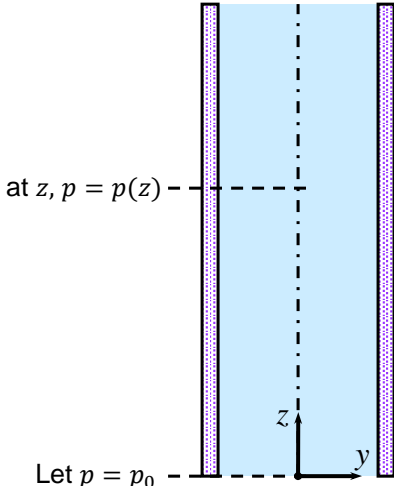
$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Is Pressure a function of z?
 YES, there should be hydrostatic pressure (due to weight of fluid)

“Pressure at the bottom of a column of fluid = pressure at top + ρgh .”



at z , $p = p(z)$

Let $p = p_0$ at $z = 0$

average density
 $p_0 = p(z) + \bar{\rho}gz$
 $p(z) = p_0 - \bar{\rho}gz$

$$\Rightarrow \frac{dP}{dz} = -\bar{\rho}g$$

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(look up the physics in the literature)

To account for the temperature variation of ρ :

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}(T - \bar{T})$$

 $\bar{\rho}$ = mean density $\bar{\beta}$ = volumetric coefficient of expansion at \bar{T}

$$\bar{T} = \frac{T_1 + T_2}{2}$$

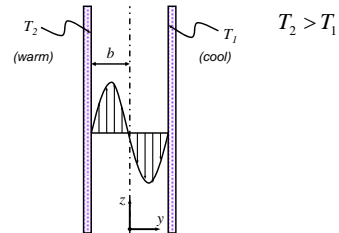
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Complex Heat Transfer – Dimensional Analysis—Free Convection

Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$



$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

56

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

(solve) ...

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

57
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Complex Heat Transfer – Dimensional Analysis—Free Convection

Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

(solve) ...

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

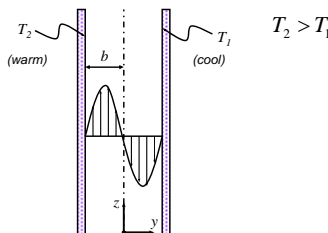
$$\rho = \bar{\rho} - \bar{\rho} \bar{\beta} (T - \bar{T})$$

$$\rho = \bar{\rho} - \bar{\rho} \bar{\beta} \left(\frac{T_1 - T_2}{2b} y \right)$$

58
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Complex Heat Transfer – Dimensional Analysis—Free Convection

Solve



Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

...

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

$$\rho = \bar{\rho} - \bar{\rho} \bar{\beta} (T - \bar{T})$$

$$\rho = \bar{\rho} - \bar{\rho} \bar{\beta} \left(\frac{T_1 - T_2}{2b} y \right)$$

59
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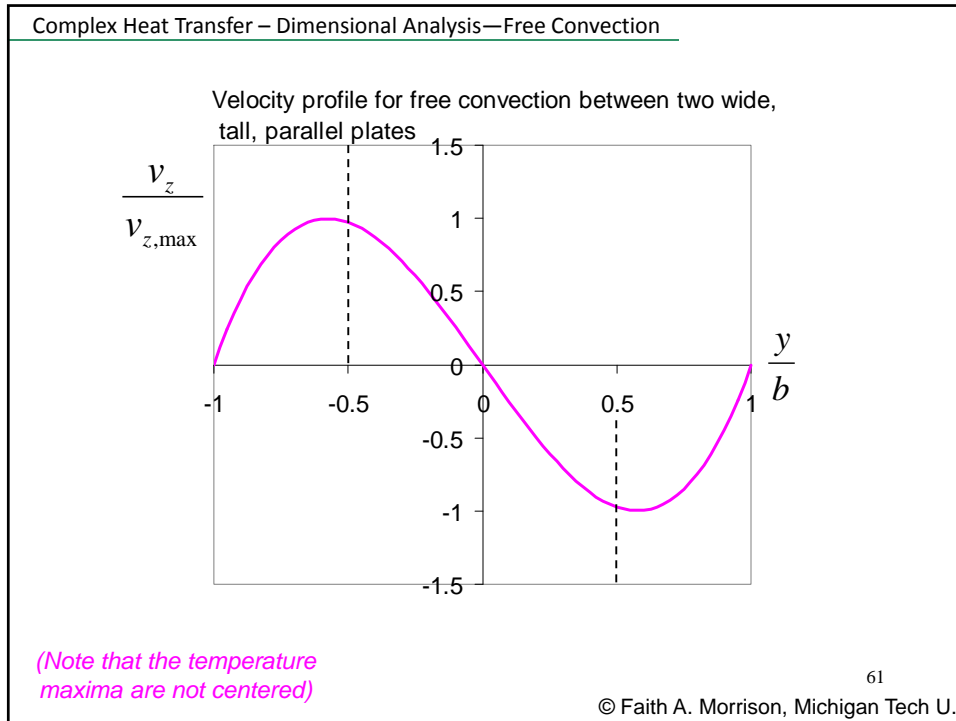
Complex Heat Transfer – Dimensional Analysis—Free Convection

Final Result: (free convection between two slabs)

$$v_z(y) = \frac{\bar{\rho} \bar{\beta} g (T_2 - T_1) b^2}{12 \mu} \left[\left(\frac{y}{b} \right)^3 - \left(\frac{y}{b} \right) \right]$$

(see next slide for plot)

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Free Convection i.e. hot air rises

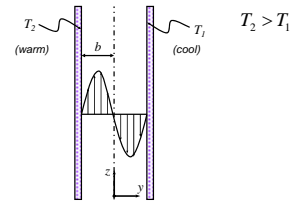
Engineering Modeling

- ✓ Choose an idealized problem and solve it
 - From insight obtained from **ideal** problem, identify governing equations of **real** problem
 - Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
 - Design experiments to test modeling thus far
 - Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
 - Design additional experiments
 - Iterate until useful correlations result

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Free Convection

i.e. hot air rises



Mass balance:

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Momentum balance:

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

Energy balance:

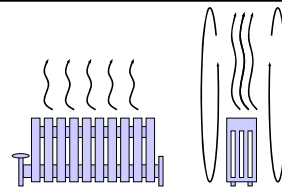
$$\rho \hat{c}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

63

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Free Convection

i.e. hot air rises

**Engineering Modeling**

- ✓ Choose an idealized problem and solve it
- ✓ From insight obtained from **ideal** problem, identify governing equations of **real** problem
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Return to Dimensional Analysis...

Nondimensionalize the governing equations; deduce dimensionless scale factors

To nondimensionalize the Navier-Stokes for **free convection** problems, we follow the simple problem we just completed: $\rho = \rho(T), \langle v_z \rangle = 0$.

density not constant

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla P + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

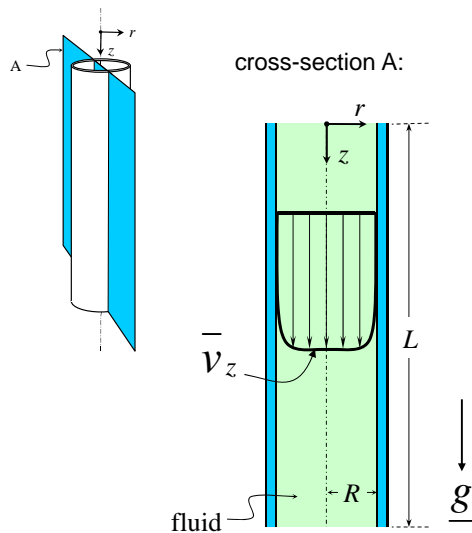
driving the flow

there was a trick for this

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How did we nondimensionalize the Navier-Stokes before?



FORCED CONVECTION

EXAMPLE 1: Pressure-driven flow of a Newtonian fluid in a tube:

- steady state
- well developed
- long tube

There was an average velocity used as the characteristic velocity

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FORCED CONVECTION FORCED CONVECTION FORCED CONVECTION

z-component of the Navier-Stokes Equation:

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

Choose:

D = characteristic length
 V = characteristic velocity
 D/V = characteristic time
 ρV^2 = characteristic pressure

This velocity is an imposed (forced) average velocity

We do not have such an imposed velocity in natural convection

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FORCED CONVECTION FORCED CONVECTION FORCED CONVECTION

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

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FORCED CONVECTION FORCED CONVECTION FORCED CONVECTION

z-component of the nondimensional Navier-Stokes Equation:

$\frac{1}{Re}$

$\frac{1}{Fr}$

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{\mu}{\rho V D} (\nabla^2 v_z)^* + \frac{gD}{V^2} g^*$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^* \frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*} \frac{\partial v_z^*}{\partial \theta} + v_z^* \frac{\partial v_z^*}{\partial z^*} \right)$$

$$(\nabla^2 v_z)^* \equiv \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial v_z^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

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FREE CONVECTION FREE CONVECTION

We do not have such an imposed velocity in natural convection

For free convection, what is the average velocity?

for forced convection we used: $v_z^* = \frac{v_z}{V}$ $V \equiv \langle v \rangle$

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FREE CONVECTION **FREE CONVECTION** We do not have such an imposed velocity in natural convection

For free convection, what is the average velocity?

for forced convection we used: $v_z^* = \frac{v_z}{V}$ $V \equiv \langle v \rangle$

ZERO

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FREE CONVECTION **FREE CONVECTION** We do not have such an imposed velocity in natural convection

For free convection, what is the average velocity?
Answer: zero!

for forced convection we used: $v_z^* = \frac{v_z}{V}$ $V \equiv \langle v \rangle$

For free convection $\langle v \rangle = 0$; what V should we use for free convection?

Solution: use a Reynolds-number type expression so that no characteristic velocity imposes itself (we'll see now how that works):

$$v_z^* = \frac{v_z}{V} = \frac{\bar{\rho} v_z D}{\mu} \quad \Rightarrow \quad V \equiv \frac{\rho}{D \bar{\rho}}$$

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FREE CONVECTION FREE CONVECTION FREE CONVECTION

When non-dimensionalizing the Navier-Stokes, what do I use for ρ ? (answer from idealized problem)

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

here we use $\bar{\rho}$ because the issue is volumetric flow rate

as before, for pressure gradient we use $-\bar{\rho}g$

here we use $\rho(T)$ because the issue is driving the flow by density differences affected by gravity

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FREE CONVECTION FREE CONVECTION FREE CONVECTION

non-dimensional variables:

<p>time:</p> $t^* \equiv \frac{t\mu}{D^2\bar{\rho}}$	<p>position:</p> $r^* \equiv \frac{r}{D}$ $z^* \equiv \frac{z}{D}$	<p>velocity:</p> $v_z^* \equiv \frac{v_z D \bar{\rho}}{\mu}$ $v_r^* \equiv \frac{\bar{\rho} v_r D}{\mu}$ $v_\theta^* \equiv \frac{\bar{\rho} v_\theta D}{\mu}$	<p>driving force:</p> $T^* = \frac{T - \bar{T}}{T_2 - \bar{T}}$
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FREE CONVECTION FREE CONVECTION FREE CONVECTION

SOLUTION: z-component of the **nondimensional** Navier-Stokes Equation (free convection):

$$\frac{Dv_z^*}{Dt} = (\nabla^2 v_z)^* + \left[\frac{gD^3 \bar{\rho}^2 \bar{\beta} (T_2 - \bar{T})}{\mu^2} \right] T^*$$

Or any appropriate characteristic ΔT

≡ Grashof number

$$(\nabla^2 v_z)^* \equiv \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial v_z^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^* \frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*} \frac{\partial v_z^*}{\partial \theta} + v_z^* \frac{\partial v_z^*}{\partial z^*} \right)$$

75
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FREE CONVECTION FREE CONVECTION

$$Gr \equiv \frac{gD^3 \bar{\rho}^2 \bar{\beta} \Delta T}{\mu^2}$$

Dimensionless Equation of Motion (free convection)

$$\frac{Dv_z^*}{Dt^*} = (\nabla^2 v_z)^* + Gr T^*$$

Dimensionless Energy Equation (free convection; Re = 1)

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^* \right) = \frac{1}{Pr} \nabla^{*2} T^*$$

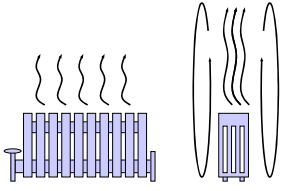
$$Nu = Nu \left(T^*, \frac{L}{D} \right) \Rightarrow Nu = Nu \left(Pr, Gr, \frac{L}{D} \right)$$

No Pe
No Re

76
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Free Convection

i.e. hot air rises



Engineering Modeling

- ✓ Choose an idealized problem and solve it
- ✓ From insight obtained from **ideal** problem, identify governing equations of **real** problem
- ✓ Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)

• Design experiments to test modeling thus far

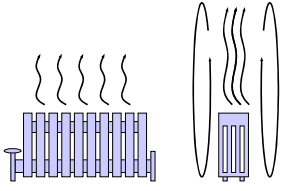
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

77

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Free Convection

i.e. hot air rises



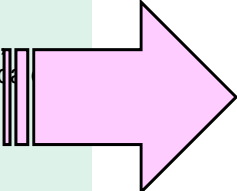
Engineering Modeling

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- Design additional experiments
- Iterate until useful correlations result

Done (see literature)



78

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Literature Results:

$$\text{Gr} \equiv \frac{gD^3 \bar{\rho}^2 \bar{\beta} \Delta T}{\mu^2}$$

Example: Natural convection from vertical planes and cylinders

$$\text{Nu} = \frac{hL}{k} = a\text{Gr}^m \text{Pr}^m$$

- a, m are given in Table 4.7-1, page 255 Geankoplis for several cases
- L is the height of the plate
- all physical properties evaluated at the **film temperature**, T_f

Free convection correlations use the **film temperature** for calculating the physical properties

$$T_f = \frac{T_w + T_b}{2}$$

Free convection correlations use the **film temperature** for calculating the physical properties

79

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Complex Heat Transfer – Correlations for Nu

Natural convection
Vertical planes and cylinders

$$\text{Nu} = \frac{hL}{k} = a\text{Gr}^m \text{Pr}^m$$

- all physical properties evaluated at the **film temperature**, T_f

compare with:

Forced convection
Heat Transfer in Laminar flow in pipes

$$\text{Nu}_a = \frac{h_a L}{k} = 1.86 \left(\text{RePr} \frac{D}{L} \right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

- all physical properties (except μ_w) evaluated at the **bulk mean temperature**
- (true also for turbulent flow correlation)

Physical Properties evaluated at:

$$T_f = \frac{T_w + T_b}{2}$$

Physical Properties evaluated at:

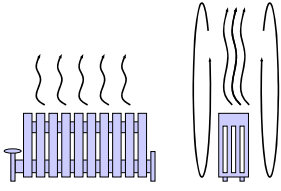
$$\frac{T_{b,in} + T_{b,out}}{2}$$

80

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Free Convection

i.e. hot air rises



Engineering Modeling

- ✓ Choose an idealized problem and solve it
- ✓ From insight obtained from **ideal** problem, identify governing equations of **real** problem
- ✓ Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- ✓ Design experiments to test modeling thus far
 - ~~Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness)~~
 - ~~Design additional experiments~~
- ✓ ~~Iterate until~~ Useful correlations result

Success!
(Dimensional Analysis wins again)

81

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Practice Heat-Transfer Problems:

Forced Convection

Free Convection

82

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Practice 1: A wide, deep rectangular oven (1.0 ft tall) is used for baking loaves of bread. During the baking process the temperature of the air in the oven reaches a stable value of $100^{\circ}F$. The oven side-wall temperature is measured at this time to be a stable $450^{\circ}F$. Please estimate the heat flux from the wall per unit width.

Reference: Geankoplis Ex. 4.7-1 page 279

83

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Practice 2: A hydrocarbon oil enters a pipe (0.0303 ft inner diameter; 15.0 ft long) at a flow rate of $80 \text{ lb}_m/h$. Steam condenses on the outside of the pipe, keeping the inside pipe surface at a constant $350^{\circ}F$. If the temperature of the entering oil is $150^{\circ}F$, what is temperature of the oil at the outlet of the pipe?

Hydrocarbon oil properties:

$$\text{Mean heat capacity} = 0.50 \frac{\text{BTU}}{\text{lb}_m^{\circ}F}$$

$$\text{Thermal conductivity} = 0.083 \frac{\text{BTU}}{\text{h ft }^{\circ}F}$$

Viscosity =

$$6.50 \text{ cp, } 150^{\circ}F$$

$$5.05 \text{ cp } 200^{\circ}F$$

$$3.80 \text{ cp } 250^{\circ}F$$

$$2.82 \text{ cp } 300^{\circ}F$$

$$1.95 \text{ cp } 350^{\circ}F$$

Reference: Geankoplis Ex. 4.5-5 page 269

84

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Practice 3: Air flows through a tube (25.4 mm inside diameter, long tube) at 7.62 m/s. Steam condenses on the outside of the tube such that the inside surface temperature of the tube is 488.7 K. If the air pressure is 206.8 kPa and the mean bulk temperature of the air is $(T_{\text{out}} + T_{\text{in}})/2 = 477.6 \text{ K}$, what is the steady-state heat flux to the air?

Reference: Geankoplis Ex. 4.5-1 page 262

85

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Practice 4: Hard rubber tubing (inside radius = 5.0mm; outside radius = 20.0mm) is used as a cooling coil in a reaction bath. Cold water is flowing rapidly inside the tubing; the inside wall temperature is 274.9 K and the outside wall temperature is 297.1 K. To keep the reaction in the bath under control, the required cooling rate is 14.65 W. What is the minimum length of tubing needed to accomplish this cooling rate? What length would be needed if the coil were copper?

Hard rubber properties:

$$\text{Density} = 1198 \frac{\text{kg}}{\text{m}^3}$$

$$\text{Thermal conductivity (0}^\circ\text{C)} = 0.151 \frac{\text{W}}{\text{mK}}$$

Reference: Geankoplis Ex. 4.2-1 page 243, but don't do it his way—follow class methods.

86

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Practice 5: A cold-storage room is constructed of an inner layer of pine (thickness = 12.7 mm), a middle layer of cork board (thickness = 101.6 mm), and an outer layer of concrete (thickness = 76.2 mm). The inside wall surface temperature is 255.4 K and the outside wall surface temperature is 297.1 K. What is the heat loss per square meter through the walls and what is the temperature at the interface between the wood and the cork board?

Material properties:

$$\text{Thermal conductivity pine} = 0.151 \frac{W}{mK}$$

$$\text{Thermal conductivity cork board} = 0.0433 \frac{W}{mK}$$

$$\text{Thermal conductivity concrete} = 0.762 \frac{W}{mK}$$

Reference: Geankoplis Ex. 4.3-1 page 245, but don't do it his way—follow class methods.

87

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Practice 6: A thick-walled tube (stainless steel; 0.0254 m inner diameter; 0.0508 m outer diameter; length 0.305 m) is covered with a 0.0254 m thickness of insulation. The inside-wall temperature of the pipe is 811.0 K and the outside surface temperature of the insulation is 310.8 K. What is the heat loss and the temperature at the interface between the steel and the insulation?

Material properties of stainless steel:

$$\text{Thermal conductivity} = 21.63 \frac{W}{mK}$$

$$\text{Density} = 7861 \frac{kg}{m^3}$$

$$\text{Heat Capacity} = 490 \frac{J}{kg K}$$

Material properties of insulation:

$$\text{Thermal conductivity} = 0.2423 \frac{W}{mK}$$

Reference: Geankoplis Ex. 4.3-2 page 247, but don't do it his way—follow class methods.

88

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Experience with Dimensional Analysis thus far:

- ✓ •Flow in pipes at all flow rates (laminar and turbulent)
Solution: Navier-Stokes, Re , Fr , L/D ,
dimensionless wall force = f ; $f=f(Re, L/D)$
- ✓ •Flow around obstacles (spheres, other complex shapes)
Solution: Navier-Stokes, Re ,
dimensionless drag = C_D ; $C_D = C_D(Re)$
- ✓ •Forced convection heat transfer from fluid to wall
Solution: Microscopic energy, Navier-Stokes, Re , Pr , L/D ,
heat transfer coefficient= h ; $h = h(Re, Pr, L/D)$
- ✓ •Natural convection heat transfer from fluid to wall
Solution: Microscopic energy, Navier-Stokes, Gr , Pr , L/D ,
heat transfer coefficient= h ; $h = h(Gr, Pr, L/D)$

89

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Experience with Dimensional Analysis thus far:

- ✓ •Flow in pipes at all flow rates (laminar and turbulent)
Solution: Navier-Stokes, Re , Fr , L/D ,
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- ✓ •Flow around obstacles (spheres, other complex shapes)
Solution: Navier-Stokes, Re ,
dimensionless drag = C_D ; $C_D = C_D(Re)$
- ✓ •Forced convection heat transfer from fluid to wall
Solution: Microscopic energy, Navier-Stokes, Re , Pr , L/D ,
heat transfer coefficient= h ; $h = h(Re, Pr, L/D)$
- ✓ •Natural convection heat transfer from fluid to wall
Solution: Microscopic energy, Navier-Stokes, Gr , Pr , L/D ,
heat transfer coefficient= h ; $h = h(Gr, Pr, L/D)$

Now, move to last heat-transfer mechanism:

- Radiation heat transfer from solid to fluid?

Solution: ?

90

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Experience with Dimensional Analysis thus far:

- ✓ • Flow in pipes at all flow rates (laminar and turbulent)
Solution: Navier-Stokes, Re , Fr , L/D , dimensionless wall force = f ; $f = f(Re, L/D)$
- ✓ • **Actually, we'll hold off on radiation and spend some time on heat exchangers and other practical concerns** L/D,
- ✓ • Natural convection heat transfer from fluid to wall
Solution: Microscopic energy, Navier-Stokes, Gr , Pr , L/D , heat transfer coefficient= h ; $h = h(Gr, Pr, L/D)$

Now, move to last heat-transfer mechanism:

- Radiation heat transfer from solid to fluid?
Solution: ?


91
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Next:

CM3110
Transport I
Part II: Heat Transfer

MichiganTech

*Applied Heat Transfer:
Heat Exchanger Modeling,
Sizing, and Design*



➔

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92
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