Before turning to radiation (last topic) we will discuss a few practical applications.

How can we use Fundamental Heat Transfer to understand real devices like heat exchangers?
The Simplest Heat Exchanger:
Double-Pipe Heat exchanger - counter current

The heat transfer from the outside to the inside is just heat flux in an annular shell

Example 4: Heat flux in a cylindrical shell

Assumptions:
• long pipe
• steady state
• k = thermal conductivity of wall
• h₁, h₂ = heat transfer coefficients

Maybe we can use heat transfer coefficient to understand forced-convection heat exchangers... BUT...
**BUT:** The temperature difference between the fluid and the wall varies along the length of the heat exchanger.

The Simplest Heat Exchanger:  
Double-Pipe Heat exchanger - counter current

How can we adapt $h$ so that we can use the concept to characterize heat exchangers?

Let's look at the solution for radial conduction in an annulus

Example 4: Heat flux in a cylindrical shell

Solution:

$$q_r = \frac{c_1}{r}$$

$$T = -\frac{c_1}{k} \ln r + c_2$$

Boundary conditions?
Example 4: Heat flux in a cylindrical shell, Newton’s law of cooling boundary conditions

Results: Radial Heat Flux in an Annulus

\[ T - T_{b2} = \frac{(T_{b1} - T_{b2})}{k} \left( \frac{\ln \left( \frac{R_2}{r} \right)}{h_2 R_2} + \frac{1}{h_1 R_1} \right) \]

\[ q_r = \frac{(T_{b1} - T_{b2})}{A} \left( \frac{1}{h_2 R_2} + \frac{1}{k} \ln \left( \frac{R_2}{R_1} \right) + \frac{1}{h_1 R_1} \right) \]

Solution for Heat Flux:

Calculate Total Heat flow:

\[ Q = \frac{q_r (2\pi r L)}{A} = \frac{(T_{b1} - T_{b2})(2\pi L)}{h_2 R_2 + \frac{1}{k} \ln \left( \frac{R_2}{R_1} \right) + \frac{1}{h_1 R_1}} \]

Note that total heat flow is proportional to bulk \( \Delta T \) and (almost) area of heat transfer.
Overall Heat Transfer Coefficient, $U$

\[ Q = UA \Delta T \]
\[ = UA(T_{b1} - T_{b2}) \]

$A$ = area of heat transfer (not always unambiguous)

$\Delta T$ = driving temperature difference

Example: in a pipe

Do we use inner or outer area?

**Applied Heat Transfer**

Overall heat xfer coeffs in pipe

\[ Q = U_1 A_1 \Delta T \]
\[ = \left( \frac{1}{R_1} \right) \left( \frac{1}{h_2 R_2} + \frac{1}{k} \ln \frac{R_2}{R_1} + \frac{1}{h_1 R_1} \right) (2 \pi R_1 L) (T_{b1} - T_{b2}) \]

\[ Q = U_2 A_2 \Delta T \]
\[ = \left( \frac{1}{R_2} \right) \left( \frac{1}{h_2 R_2} + \frac{1}{k} \ln \frac{R_2}{R_1} + \frac{1}{h_1 R_1} \right) (2 \pi R_2 L) (T_{b1} - T_{b2}) \]

Area must be specified when $U$ is reported

© Faith A. Morrison, Michigan Tech U.
Heat flux in a cylindrical shell: \[ Q = UA(T_b - T_c) \]

But, in an actual heat exchanger, \( T_b \) and \( T_c \) vary along the length of the heat exchanger.

What kind of average \( \Delta T \) do we use?

The Simplest Heat Exchanger:
Double-Pipe Heat exchanger - counter current

We will do an open-system energy balance on a differential section to determine the correct average temperature difference to use.
The Simplest Heat Exchanger:  
Double-Pipe Heat exchanger - counter current

Another way of looking at it:

\[ Q_{\text{inside}} = Q = -Q_{\text{outside}} \]

Can do three balances:

1. Balance on the inside system

\[ Q_{\text{in}} = Q = -Q_{\text{in}} \]
The Simplest Heat Exchanger:
Double-Pipe Heat exchanger - counter current

Another way of looking at it:

Can do three balances:
1. Balance on the inside system
2. Balance on the outside system
3. Overall balance

\[ Q_{\text{inside}} = Q = -Q_{\text{outside}} \]
The Simplest Heat Exchanger:
Double-Pipe Heat exchanger - counter current

Another way of looking at it:

We can do:
• a macroscopic balances over the entire heat exchanger, or
• a pseudo microscopic balance over a slice of the heat exchanger

\[ Q_{inside} = Q = -Q_{outside} \]

All the details of the algebra are here:
www.chem.mtu.edu/~fmorriso/cm310/double_pipe.pdf
Pseudo Microscopic Energy Balance on a slice of the heat exchanger

Open system energy balance on a differential volume:

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on}$$

$$\Delta H = Q_{in}$$

INSIDE BALANCE

recall: $\Delta$ is out-in

© Faith A. Morrison, Michigan Tech U.

Pseudo Microscopic Energy Balance on a slice of the heat exchanger

$$\Delta H = Q_{in}$$

INSIDE BALANCE

recall: $\Delta$ is out-in

© Faith A. Morrison, Michigan Tech U.
Pseudo Microscopic Energy Balance on a slice of the heat exchanger

\[ \Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s, on} \]

\[ \Delta H = 0 \]

This expression characterizes the rate of change of heat transferred with respect to distance down the heat exchanger.
The Simplest Heat Exchanger:
Double-Pipe Heat exchanger - counter current

Result of inside balance:
\[ \frac{dQ_{\text{inner}}}{dx} = m\hat{c}_p \frac{dT}{dx} \]

Result of outside balance:
\[ -\frac{dQ_{\text{outer}}}{dx} = m'\hat{c}'_p \frac{dT'}{dx} \]

Result of overall balance:
\[ -\frac{dQ_{\text{outer}}}{dx} = \frac{dQ_{\text{inner}}}{dx} = \frac{dQ_{\text{in}}}{dx} \]

Solve for temperature derivatives, and subtract:
\[ \frac{dQ_{\text{in}}}{dx} \left( 1 - \frac{1}{m\hat{c}_p} \right) = \frac{d(T' - T)}{dx} \]

This depends on \( T' - T \)

All the details of the algebra are here:
www.chem.mtu.edu/~fmorris/cm310/double_pipe.pdf

Question:  How can we write \( \frac{dQ_{\text{in}}}{dx} \) in terms of \( T' - T \)?

Answer: Define an “overall” heat transfer coefficient, \( U \)
Want to integrate to solve for $T' - T$, but this is a function of $T' - T$.

For the differential slice of the heat exchanger that we are considering (modeling our ideas on Newton's law of cooling),

$$\frac{dQ_{in}}{dA} = (?)(T' - T)$$

This is the missing piece that we needed.
Analysis of double-pipe heat exchanger

\[
\frac{d(T' - T)}{dx} = \frac{dQ_{in}}{dx} \left( \frac{1}{m'c'p} - \frac{1}{mc_p} \right)
\]

\[
\frac{dQ_{in}}{dx} = 2\pi RU(T' - T)
\]

\[
\frac{d(T' - T)}{T' - T} = 2\pi RU (T' - T) \left( \frac{1}{c'_pm'} - \frac{1}{c_pm} \right)
\]

\[
\frac{d(T' - T)}{(T' - T)} = \left[ 2\pi RU \left( \frac{1}{c'_pm'} - \frac{1}{c_pm} \right) \right] dx
\]

Analysis of double-pipe heat exchanger

\[
\phi \equiv T' - T
\]

\[
\alpha_0 \equiv 2\pi RU \left( \frac{1}{c'_pm'} - \frac{1}{c_pm} \right)
\]

\[
\frac{d\phi}{\Phi} = \alpha_0 dx
\]

\[
\int \frac{d\phi}{\Phi} = \alpha_0 \int dx
\]

\[
\ln \phi = \alpha_0 x + \text{constant}
\]

\[
\phi = \Phi_0 e^{\alpha_0 x}
\]

B.C: \(x = 0, T - T' = T_1 - T'_1\)
Temperature profile in a double-pipe heat exchanger:

\[ \frac{T' - T}{T'_1 - T_1} = e^{\alpha_0 x} \quad \alpha_0 = 2\pi RU \left( \frac{1}{C'_p m'} - \frac{1}{C_p m} \right) \]

Analysis of double-pipe heat exchanger

\[ \frac{T' - T}{T'_1 - T_1} = e^{\alpha_0 L \left( \frac{x}{L} \right)} \quad \alpha_0 L = 2\pi RL \left( \frac{1}{C'_p m'} - \frac{1}{C_p m} \right) \]

Note that the temperature curves are not linear.

© Faith A. Morrison, Michigan Tech U.
Temperature profile in a double-pipe heat exchanger:

\[ \frac{T' - T}{T_1' - T_1} = e^{\alpha_0 x} \]
\[ \alpha_0 = 2\pi RU \left( \frac{1}{\bar{C}_p' m'} - \frac{1}{\bar{C}_p m} \right) \]

Useful result, but what we **REALLY** want is an easy way to relate \( Q_{in, overall} \) to inlet and outlet temperatures.

At the exit: \( x = L, \ (T - T') = (T_2 - T_2') \)

\[ \ln \left( \frac{T_2' - T_2}{T_1' - T_1} \right) = U(2\pi RL) \left( \frac{1}{\bar{C}_p' m'} - \frac{1}{\bar{C}_p m} \right) \]

The \( m\hat{C}_p \) terms appear in the overall macroscopic energy balances. We can therefore rearrange this equation by replacing the \( m\hat{C}_p \) terms with \( Q_{in} \):

\[ Q_{in} = m\hat{C}_p (T_2 - T_1) \]
\[ \Rightarrow \frac{1}{m\hat{C}_p} = \frac{T_2 - T_1}{Q_{in}} \]
\[ -Q_{in} = m\hat{C}_p (T_1' - T_2') \]
\[ \Rightarrow \frac{1}{m\hat{C}_p} = \frac{(T_1' - T_2')}{Q_{in}} \]
Analysis of double-pipe heat exchanger

FINAL RESULT:

\[
Q = U\frac{(2\pi RL)(T'_1 - T_1) - (T'_2 - T_2)}{A} \ln \left( \frac{T'_1 - T_1}{T'_2 - T_2} \right)
\]

\[\equiv \Delta T_{lm}\]

\[Q = UA\Delta T_{lm}\]

\[\Delta T_{lm}\] is the correct average temperature to use for the overall heat-transfer coefficients in a double-pipe heat exchanger.

© Faith A. Morrison, Michigan Tech U.
Example: Heat Transfer in a Double-Pipe Heat Exchanger: Geankoplis 4th ed. 4.5-4

Water flowing at a rate of 13.85 kg/s is to be heated from 54.5 to 87.8°C in a double-pipe heat exchanger by 54,430 kg/h of hot gas flowing counterflow and entering at 427°C ($\rho_m = 1.005 \text{ kg/m}^3$). The overall heat-transfer coefficient based on the outer surface is $U_o = 69.1 \text{ W/m}^2 \text{ K}$. Calculate the exit-gas temperature and the heat transfer area needed.
**Summary:**

**Double-Pipe Heat Exchanger** – the driving force for heat transfer changes along the length of the heat exchanger.

For example,

\[ T_1' = 300°C \]
\[ \Delta T = (T' - T)_{x=x_0} = 340°C \]
\[ T_1 = 50°C \]
\[ T_2 = 90°C \]
\[ \Delta T = (T' - T)_{x=x_1} = 250°C \]
\[ T_2' = 430°C \]

The correct *average* driving force for the whole exchanger is the log-mean temperature difference

\[ = \Delta T_{lm} \]

Optimizing heat exchangers

**Optimizing Heat Exchangers**

\[ Q = UAD_{lm} \]

To increase \( Q \) appreciably, we must increase \( A \), i.e. \( R_i \).

**But:**
- only small increases possible
- increasing \( R_i \) decreases \( h \)
Optimizing heat exchangers

1-1 Shell and Tube Heat Exchanger
(Same as double pipe H.E.)

1-2 Shell and Tube Heat Exchanger

1 shell
1 tube

Geankoplis 4th ed., p292

Cross Baffles in Shell-and-Tube Heat Exchangers

© Faith A. Morrison, Michigan Tech U.
And other more complex arrangements:

2 shell
4 tube

Optimizing heat exchangers

For double-pipe heat exchanger:

\[ Q = UA\Delta T_{lm} \]

For shell-and-tube heat exchangers:

\[ Q = UA\Delta T_{lm}(F_T) \equiv \Delta T_m \]

- calculated correction factor (obtain from experimentally determined charts)
- correct mean temperature difference for shell-and-tube heat exchangers
Lectures 7+8 CM3110

Optimizing heat exchangers

Shell-and-Tube Heat Exchangers

(1-1 exchanger, \( F_T = 1 \))

\[ T_{h_i} = \text{hot, in} \]
\[ T_{h_o} = \text{hot, out} \]
\[ T_{c_i} = \text{cold, in} \]
\[ T_{c_o} = \text{cold, out} \]

Efficiency is low when \( F_T \) is below \( F_{T,\text{min}} \)

Heat Exchanger Design

To calculate \( Q \), we need both inlet and outlet temperatures:

\[ Q = UA \Delta T_m = UA \left( F_T \Delta T_{lm} \right) \]

What if the outlet temperatures are unknown?

i.e. the design/spec problem.
Water flowing at a rate of 0.723 \textit{kg/s} enters the inside of a countercurrent, double-pipe heat exchanger at 300.\textit{K} and is heated by an oil stream that enters at 385 \textit{K} at a rate of 3.2 \textit{kg/s}. The heat capacity of the oil is 1.89 \textit{kJ/kgK}, and the average heat capacity of the water of the temperature range of interest is 4.192 \textit{kJ/kgK}. The overall heat-transfer coefficient of the exchanger is 300. \textit{W/m}^2\textit{K}, and the area for heat transfer is 15.4 \textit{m}^2. What is the total amount of heat transferred?

You try.
Example Problem:
How will this heat exchanger perform?

To calculate unknown outlet temperatures:

Procedure:
1. Guess Q
2. Calculate outlet temperatures
3. Calculate $\Delta T_{lm}$
4. Calculate Q
5. Compare, adjust, repeat

Example Problem:

How will this heat exchanger perform?

<table>
<thead>
<tr>
<th>U</th>
<th>0.3 [kW/m²K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>30 K</td>
</tr>
<tr>
<td>T_prime_1</td>
<td>368 K</td>
</tr>
<tr>
<td>Delta_left</td>
<td>52 K</td>
</tr>
<tr>
<td>Delta_Right</td>
<td>52 K</td>
</tr>
<tr>
<td>Delta_Tm</td>
<td>33 K</td>
</tr>
<tr>
<td>Q_new</td>
<td>276.5 [kW]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guess Q</th>
<th>100 [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_2</td>
<td>333 K</td>
</tr>
<tr>
<td>T_prime_1</td>
<td>352 K</td>
</tr>
<tr>
<td>Delta_left</td>
<td>52 K</td>
</tr>
<tr>
<td>Delta_Right</td>
<td>52 K</td>
</tr>
<tr>
<td>Delta_Tm</td>
<td>33 K</td>
</tr>
<tr>
<td>Q_new</td>
<td>276.5 [kW]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guess Q</th>
<th>200 [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_2</td>
<td>366 K</td>
</tr>
<tr>
<td>T_prime_1</td>
<td>352 K</td>
</tr>
<tr>
<td>Delta_left</td>
<td>52 K</td>
</tr>
<tr>
<td>Delta_Right</td>
<td>52 K</td>
</tr>
<tr>
<td>Delta_Tm</td>
<td>33 K</td>
</tr>
<tr>
<td>Q_new</td>
<td>151.4 [kW]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guess Q</th>
<th>150 [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_2</td>
<td>349 K</td>
</tr>
<tr>
<td>T_prime_1</td>
<td>352 K</td>
</tr>
<tr>
<td>Delta_left</td>
<td>52 K</td>
</tr>
<tr>
<td>Delta_Right</td>
<td>52 K</td>
</tr>
<tr>
<td>Delta_Tm</td>
<td>33 K</td>
</tr>
<tr>
<td>Q_new</td>
<td>151.4 [kW]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guess Q</th>
<th>170 [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_2</td>
<td>359 K</td>
</tr>
<tr>
<td>T_prime_1</td>
<td>359 K</td>
</tr>
<tr>
<td>Delta_left</td>
<td>55 K</td>
</tr>
<tr>
<td>Delta_Right</td>
<td>55 K</td>
</tr>
<tr>
<td>Delta_Tm</td>
<td>39 K</td>
</tr>
<tr>
<td>Q_new</td>
<td>176.1 [kW]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guess Q</th>
<th>180 [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_2</td>
<td>359 K</td>
</tr>
<tr>
<td>T_prime_1</td>
<td>359 K</td>
</tr>
<tr>
<td>Delta_left</td>
<td>55 K</td>
</tr>
<tr>
<td>Delta_Right</td>
<td>55 K</td>
</tr>
<tr>
<td>Delta_Tm</td>
<td>39 K</td>
</tr>
<tr>
<td>Q_new</td>
<td>176.1 [kW]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guess Q</th>
<th>178.1 [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_2</td>
<td>359 K</td>
</tr>
<tr>
<td>T_prime_1</td>
<td>359 K</td>
</tr>
<tr>
<td>Delta_left</td>
<td>55 K</td>
</tr>
<tr>
<td>Delta_Right</td>
<td>55 K</td>
</tr>
<tr>
<td>Delta_Tm</td>
<td>39 K</td>
</tr>
<tr>
<td>Q_new</td>
<td>179.3 [kW]</td>
</tr>
</tbody>
</table>

2013HeatExchEffecExample.xlsx

© Faith A. Morrison, Michigan Tech U.
Example Problem: How will this heat exchanger perform?

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>$0.3 \text{ kW/m}^2\text{K}$</td>
</tr>
<tr>
<td>$A$</td>
<td>$15.4 \text{ m}^2$</td>
</tr>
<tr>
<td>$T_{i}$</td>
<td>$300 \text{ K}$</td>
</tr>
<tr>
<td>$T_{prime}$</td>
<td>$385 \text{ K}$</td>
</tr>
<tr>
<td>$m_{\text{water}}$</td>
<td>$0.723 \text{ kg/s}$</td>
</tr>
<tr>
<td>$m_{\text{oil}}$</td>
<td>$3.2 \text{ kg/s}$</td>
</tr>
<tr>
<td>$c_p_{\text{water}}$</td>
<td>$4.192 \text{ kJ/kgK}$</td>
</tr>
<tr>
<td>$c_p_{\text{oil}}$</td>
<td>$1.89 \text{ kJ/kgK}$</td>
</tr>
<tr>
<td>$Q_{\text{new}}$</td>
<td>$276.5 \text{ kW}$</td>
</tr>
<tr>
<td>$Q_{\text{new}}$</td>
<td>$151.4 \text{ kW}$</td>
</tr>
<tr>
<td>$Q_{\text{new}}$</td>
<td>$216.1 \text{ kW}$</td>
</tr>
</tbody>
</table>

This procedure can be sped up considerably by the use of the concept of Heat-Exchanger Effectiveness, $\varepsilon$.

Heat Exchanger Effectiveness

Consider a counter-current double-pipe heat exchanger:

$$
\begin{align*}
T_{hi} & \rightarrow T_{co} \\
T_{ho} & \rightarrow T_{ci}
\end{align*}
$$

distance along the exchanger
Heat Exchanger Effectiveness

Energy balance cold side:

\[ Q_{in,cold} = Q = (mC_p)_{cold} (T_{co} - T_{ci}) \]

Energy balance hot side:

\[ Q_{in,hot} = -Q = (mC_p)_{hot} (T_{ho} - T_{hi}) \]

Equate:

\[ (mC_p)_{cold} (T_{co} - T_{ci}) = -(mC_p)_{hot} (T_{ho} - T_{hi}) \]

\[ \frac{(mC_p)_{hot}}{(mC_p)_{cold}} = \frac{(T_{co} - T_{ci})}{-(T_{ho} - T_{hi})} = \frac{\Delta T_c}{\Delta T_h} \]

We want to compare the amount of heat transferred in this case to the amount of heat transferred in a **PERFECT** heat exchanger.

Case 1:

\[ \begin{cases} 
(mC_p)_{hot} > (mC_p)_{cold} \\
\Delta T_c > \Delta T_h 
\end{cases} \]

cold fluid = minimum fluid
If the heat exchanger were perfect, \( T_{hi} = T_{co} \) (no heat left un-transferred)

**cold side:**

\[
Q_{A=\infty} = (mC_p)_{cold}(T_{co} - T_{ci})
\]

This temperature difference only depends on inlet temperatures

**Heat Exchanger Effectiveness**

Heat Exchanger Effectiveness, \( \varepsilon \)

\[
\varepsilon \equiv \frac{Q}{Q_{A=\infty}}
\]

\[
\Rightarrow Q = \varepsilon (mC_p)_{cold}(T_{hi} - T_{ci})
\]

cold fluid = minimum fluid

If \( \varepsilon \) is known, we can calculate \( Q \) without iterations
Heat Exchanger Effectiveness

Case 2:
\[
\begin{align*}
&\frac{(m C_p)_{\text{hot}}}{(m C_p)_{\text{cold}}} < \frac{\Delta T_{\text{c}}}{\Delta T_{\text{h}}} \\
&\Delta T_{\text{c}} < \Delta T_{\text{h}}
\end{align*}
\]

hot fluid = minimum fluid

![Diagram of heat exchanger](image)

distance along the exchanger

© Faith A. Morrison, Michigan Tech U.

---

Heat Exchanger Effectiveness

If the heat exchanger were perfect, \( T_{\text{ho}} = T_{ci} \) (no heat left un-transferred)

hot side:
\[
Q_{A=\infty} = -(m C_p)_{\text{hot}} (T_{ho} - T_{hi})
\]

The same temperature difference as before (inlets)

\[
Q_{A=\infty} = (m C_p)_{\text{hot}} (T_{hi} - T_{ci})
\]

![Diagram of heat exchanger](image)

distance along the exchanger

© Faith A. Morrison, Michigan Tech U.
Heat Exchanger Effectiveness

\[ \varepsilon \equiv \frac{Q}{Q_{A=\infty}} \]

\[ \Rightarrow Q = \varepsilon \left( mC_p \right)_{\text{hot}} \left( T_{hi} - T_{ci} \right) \]

hot fluid = minimum fluid

in general,

\[ Q = \varepsilon \left( mC_p \right)_{\text{min}} \left( T_{hi} - T_{ci} \right) \]

if \( \varepsilon \) is known, we can calculate \( Q \) without iterations

But where do we get \( \varepsilon \)?

The same equations we use in the trial-and-error solution can be combined algebraically to give \( \varepsilon \) as a function of \( (mC_p)_{\text{min}}, (mC_p)_{\text{max}} \).

counter-current flow:

\[ \varepsilon = \begin{cases} 
1 - e^{-UA \left( \frac{mC_p}{mC_p}_{\text{min}} \right) \left( 1 - \frac{mC_p}{mC_p}_{\text{min}} \right)}} & \text{if } \left( \frac{mC_p}{mC_p}_{\text{min}} \right) \text{ is known} \\
1 - \left( \frac{mC_p}{mC_p}_{\text{min}} \right) \left( 1 - e^{\frac{-UA}{mC_p}_{\text{max}}} \right) & \text{otherwise}
\end{cases} \]

This relation is plotted in Geankoplis, as is the relation for co-current flow.
Water flowing at a rate of 0.723 kg/s enters the inside of a countercurrent, double-pipe heat exchanger at 300 K and is heated by an oil stream that enters at 365 K at a rate of 3.2 kg/s. The heat capacity of the oil is 1.89 kJ/kgK, and the average heat capacity of the water of the temperature range of interest is 4.192 kJ/kgK. The overall heat-transfer coefficient of the exchanger is 380 W/m²K, and the area for heat transfer is 15.4 m². What is the total amount of heat transferred?

You try.
Heat Exchanger Fouling

- material deposits on hot surfaces
- rust, impurities
- strong effect when boiling occurs

Clean vs. Fouled Scale

Scale adds an additional resistance to heat transfer

Heat Exchanger Effectiveness

Heat transfer resistances

\[
U_{i \text{or } o} = \frac{1}{R_{i \text{or } o}} = \frac{1}{\frac{1}{h_i R_i} + \frac{1}{h_{di} R_{di}} + \frac{1}{k \ln \left( \frac{R_o}{R_i} \right)} + \frac{1}{h_{do} R_{do}} + \frac{1}{h_o R_o}}
\]

Resistance due to interface
Resistance due to limited thermal conductivity

Add effect of fouling

\[
U_{i \text{or } o} = \frac{1}{R_{i \text{or } o}} = \frac{1}{\frac{1}{h_i R_i} + \frac{1}{h_{di} R_{di}} + \frac{1}{k \ln \left( \frac{R_o}{R_i} \right)} + \frac{1}{h_{do} R_{do}} + \frac{1}{h_o R_o}}
\]

See Perry’s Handbook, or Geankoplis 4th ed. Table 4.9-1, page 300 for values of \( h_d \)

© Faith A. Morrison, Michigan Tech U.
Heat Exchanger Fouling

Next:

- Heat transfer with phase change
- Evaporators
- Radiation
- DONE

TABLE 4.9.1. Typical Fouling Coefficients ($f$, $N$)

<table>
<thead>
<tr>
<th>$f$ (ft$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distilled and seawater</td>
</tr>
<tr>
<td>City water</td>
</tr>
<tr>
<td>Muddy water</td>
</tr>
<tr>
<td>Geese</td>
</tr>
<tr>
<td>Vapourizing liquids</td>
</tr>
<tr>
<td>Vegetable and gas oils</td>
</tr>
</tbody>
</table>

TABLE 4.9.2. Typical Values of Overall Heat-Transfer Coefficients in Shell-and-Tube Exchangers (ft$^{-2}$·°F·Btu$^{-1}$)

<table>
<thead>
<tr>
<th>$U$ (ft$^{-2}$·°F·Btu$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water to water</td>
</tr>
<tr>
<td>Water to brine</td>
</tr>
<tr>
<td>Water to organic liquids</td>
</tr>
<tr>
<td>Water to condensing steam</td>
</tr>
<tr>
<td>Water to gasoline</td>
</tr>
<tr>
<td>Water to gas oil</td>
</tr>
<tr>
<td>Water to vegetable oil</td>
</tr>
<tr>
<td>Gas oil to gas oil</td>
</tr>
<tr>
<td>Steam to boiling water</td>
</tr>
<tr>
<td>Water to air (finned tube)</td>
</tr>
<tr>
<td>Light organics to light organics</td>
</tr>
<tr>
<td>Heavy organics to heavy organics</td>
</tr>
</tbody>
</table>