Michigan Tech

CM3110

Transport Processes and Unit Operations I

Fluid Mechanics
Non-Newtonian fluids –
An Introduction

Photo by Carlos Arango Sabogal, U. Wisconsin, Madison

The *Weissenberg effect* is when a viscoelastic, non-Newtonian fluid will climb a rotating shaft.

https://www.youtube.com/watch?v=npZzlgKjs0I

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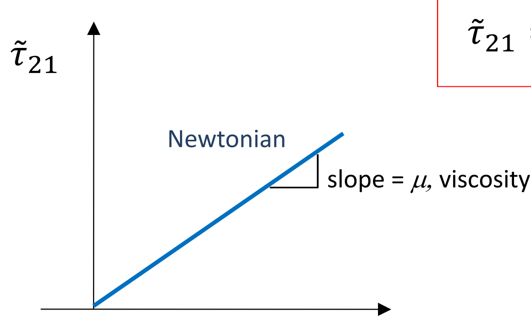
Newtonian Fluids

Newton's Law of Viscosity

(unidirectional flow)

$$\tilde{\tau}_{21} = \mu \left(\frac{\partial v_1}{\partial x_2} \right)$$



$$\dot{\gamma} = \left| \frac{\partial v_1}{\partial x_2} \right|$$

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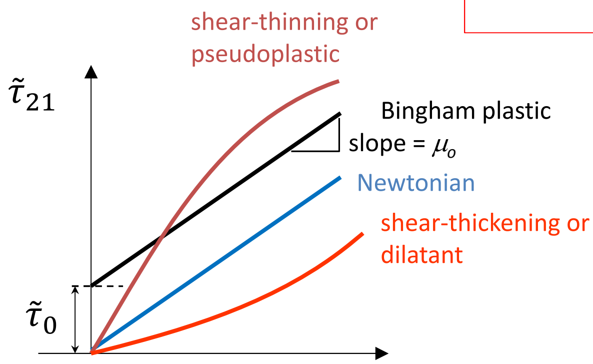
Non-Newtonian Fluids

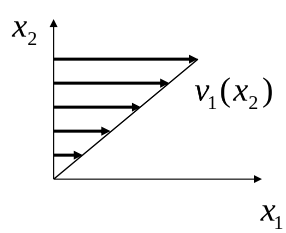
Non-Newtonian behavior (unidirectional flow)

$$\tilde{\tau}_{21} = \text{function of}\left(\frac{\partial v_1}{\partial x_2}\right)$$



$$\dot{\gamma} = \left|\frac{\partial v_1}{\partial x_2}\right|$$

What is the definition of viscosity for Non-Newtonian Fluids?

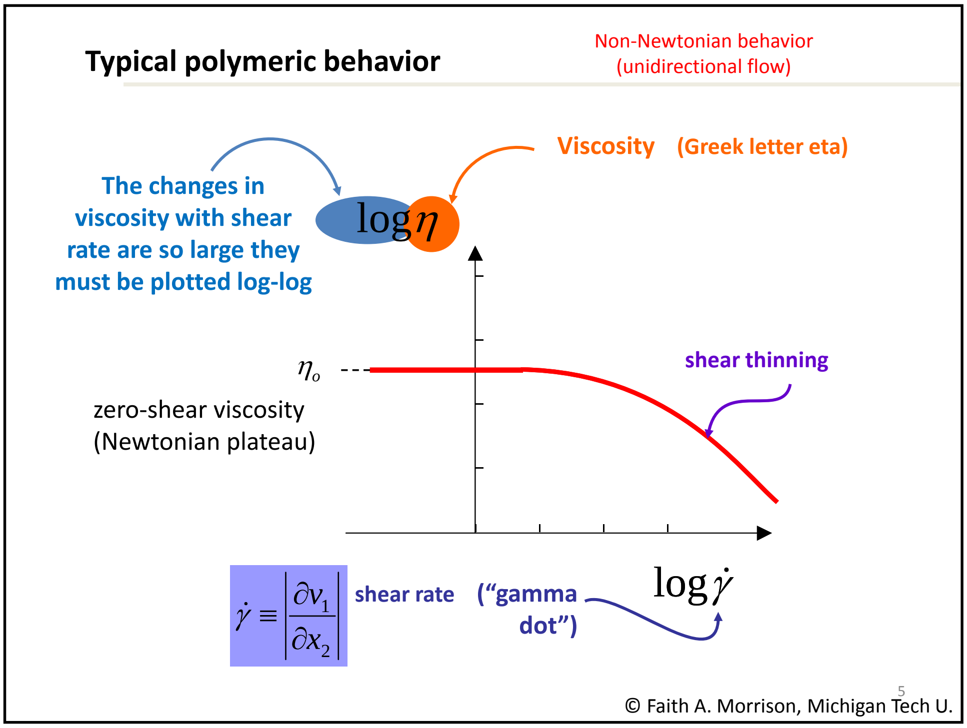


$$\dot{\gamma} = \left|\frac{\partial v_1}{\partial x_2}\right|$$

$$\eta = \frac{\tilde{\tau}_{21}}{\dot{\gamma}}$$

Non-Newtonian viscosity (unidirectional flow)

(NOTE on coordinate system: Viscosity definition is written for shear flow in x_1 direction and gradient in x_2 direction)


Typical polymeric behavior
Non-Newtonian behavior
(unidirectional flow)
Viscosity (Greek letter eta)
The changes in viscosity with shear rate are so large they must be plotted log-log
$\log \eta$
η_o
zero-shear viscosity
(Newtonian plateau)
shear thinning
$\dot{\gamma} \equiv \left| \frac{\partial v_1}{\partial x_2} \right|$
shear rate ("gamma dot")
$\log \dot{\gamma}$
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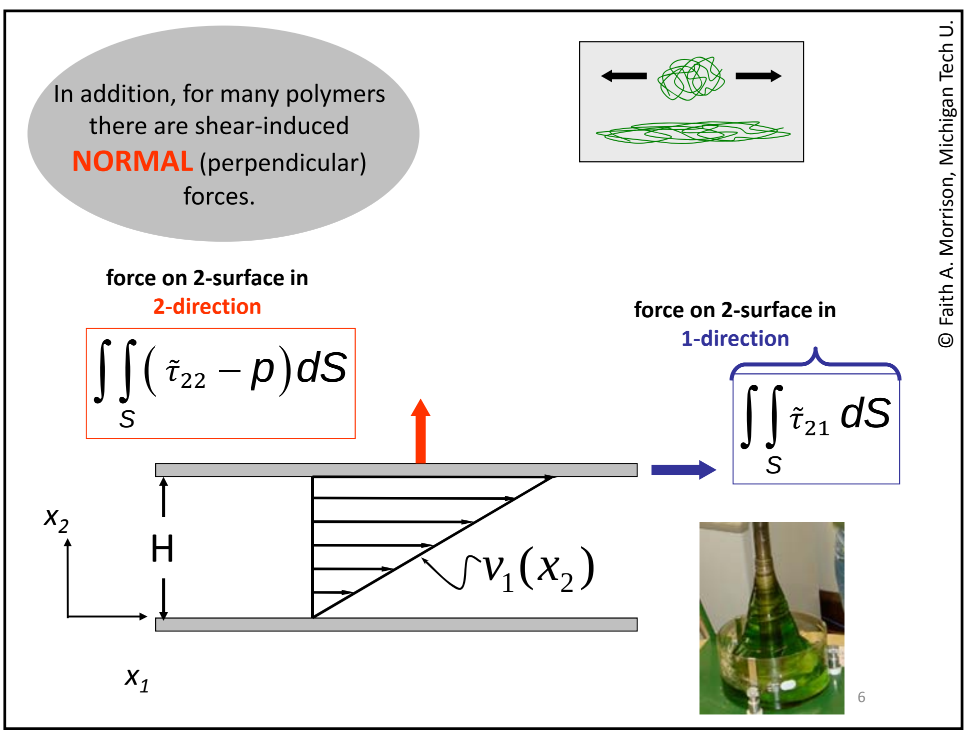
In addition, for many polymers there are shear-induced NORMAL (perpendicular) forces.
force on 2-surface in 2-direction
$\iint_S (\tilde{\tau}_{22} - p)\, dS$
force on 2-surface in 1-direction
$\iint_S \tilde{\tau}_{21}\, dS$
x_2
x_1
H
$v_1(x_2)$
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Non-Newtonian behavior
(unidirectional flow)

$\tilde{\tau}_{21} = \text{function of} \left(\frac{\partial v_1}{\partial x_2}\right)$

Power-Law Model

(does **not** model normal stresses)

$$\tilde{\tau}_{21} = m\left|\frac{dv_1}{dx_2}\right|^{n-1}\frac{dv_1}{dx_2}$$

m or *K* = consistency index (m = μ for Newtonian)
n = power-law index (*n* = 1 for Newtonian)

$$\dot{\gamma} \equiv \left|\frac{\partial v_1}{\partial x_2}\right| = shear\ rate$$

What does the power-law model predict for viscosity?

$$\tilde{\tau}_{21} = m\left|\frac{dv_1}{dx_2}\right|^{n-1}\frac{dv_1}{dx_2}$$

$$\eta \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}} = \frac{\tilde{\tau}_{21}}{\left|\frac{dv_1}{dx_2}\right|} = m\left|\frac{dv_1}{dx_2}\right|^{n-1}$$

On a log-log plot, this would give a straight line:

$$\underbrace{\log\eta}_{Y} = \underbrace{\log m}_{B} + \underbrace{(n-1)}_{M}\underbrace{\log\left|\frac{dv_1}{dx_2}\right|}_{X}$$

$$Y = B + M\ X$$

Power-Law Fluid

Non-Newtonian viscosity

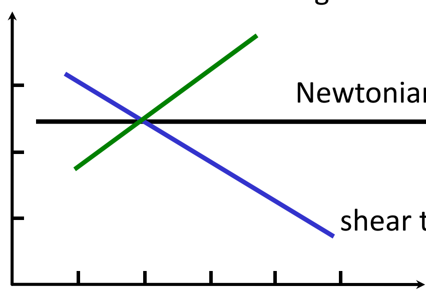
$$\eta \equiv \frac{\tilde{\tau}_{21}}{\dot{\gamma}}$$

$$\dot{\gamma} = \left|\frac{\partial v_1}{\partial x_2}\right|$$

$\log \eta$

$\eta = m\dot{\gamma}^{n-1}, \quad n > 1$

shear thickening



Newtonian $\eta = m\dot{\gamma}^{n-1}, \quad n = 1$

shear thinning $\eta = m\dot{\gamma}^{n-1}, \quad n < 1$

$\log \dot{\gamma}$

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Where do we use the power-law expression?

The Equation of Continuity and the Equation of Motion in Cartesian, cylindrical, and spherical coordinates

Continuity Equation, Cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z}\right) + \rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = 0$$

Continuity Equation, cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

$$\frac{\partial \rho}{\partial t} + \underline{v}\cdot\nabla\rho = -\rho(\nabla\cdot\underline{v})$$

Continuity Equation, spherical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\phi)}{\partial \phi} = 0$$

Equation of Motion for an incompressible fluid, 3 components in Cartesian coordinates

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \left(\frac{\partial \tilde{\tau}_{xx}}{\partial x} + \frac{\partial \tilde{\tau}_{yx}}{\partial y} + \frac{\partial \tilde{\tau}_{zx}}{\partial z}\right) + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y} + \left(\frac{\partial \tilde{\tau}_{xy}}{\partial x} + \frac{\partial \tilde{\tau}_{yy}}{\partial y} + \frac{\partial \tilde{\tau}_{zy}}{\partial z}\right) + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \left(\frac{\partial \tilde{\tau}_{xz}}{\partial x} + \frac{\partial \tilde{\tau}_{yz}}{\partial y} + \frac{\partial \tilde{\tau}_{zz}}{\partial z}\right) + \rho g_z$$

The one with "$\tilde{\tau}$" is for non-Newtonian fluids

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www.chem.mtu.edu/~fmorriso/cm310/Navier.pdf

Where do we use the power-law expression?

e.g., Poiseuille flow in a tube:

Newtonian

$$\tilde{\tau}_{rz} = \mu\left(\frac{dv_z}{dr}\right)$$

$$\tilde{\tau}_{rz} = \left(\frac{L\rho g + (P_o - P_L)}{2L}\right)r$$

non-Newtonian, power-law

$$\tilde{\tau}_{rz} = m\left|\frac{dv_z}{dr}\right|^{n-1}\frac{dv_z}{dr}$$

$\Longrightarrow$ **solve for $v_z(r)$**

1-direction = *r*
2-direction = *z*

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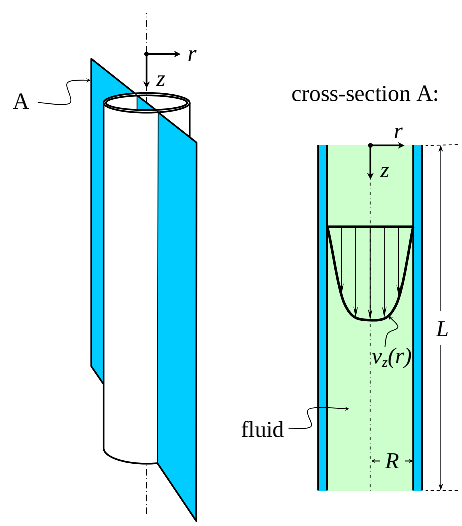
EXAMPLE III: Pressure-driven flow of a Power-law fluid in a tube



- •steady state
- •incompressible
- •well developed
- •long tube

Calculate velocity and stress profiles

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Non-Newtonian behavior
(unidirectional flow)

$$\tilde{\tau}_{21} = \text{PL function of}\left(\frac{\partial v_1}{\partial x_2}\right)$$

Calculate the velocity field for Pressure-driven flow of a power-law (PL) fluid:

$$\tilde{\tau}_{rz} = m\left|\frac{dv_z}{dr}\right|^{n-1}\frac{dv_z}{dr} = \left(\frac{L\rho g + (P_o - P_L)}{2L}\right)r$$

$$\left|\frac{dv_z}{dr}\right| = -\frac{\partial v_z}{\partial r} \quad \forall r \qquad \left(\frac{L\rho g + (P_o - P_L)}{2L}\right) \equiv \alpha$$

$$m\left(-\frac{dv_z}{dr}\right)^{n-1}\frac{dv_z}{dr} = -m\left(-\frac{dv_z}{dr}\right)^{n} = \alpha r$$

Solve for $v_z(r)$

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Boundary Conditions: ?

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Non-Newtonian behavior
(unidirectional flow)

$\tilde{\tau}_{21} = \text{PL function of} \left(\frac{\partial v_1}{\partial x_2}\right)$

Velocity field
Poiseuille flow of a power-law fluid:

$$v_z(r) = \left(\frac{R(L\rho g + P_o - P_L)}{2Lm}\right)^{\frac{1}{n}} \left(\frac{R}{\frac{1}{n}+1}\right)\left(1 - \left(\frac{r}{R}\right)^{\frac{1}{n}+1}\right)$$

$$\langle v \rangle = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R v_z(r) r dr d\theta = R\left(\frac{n}{1+3n}\right)\left[\frac{R(P_o - P_L)}{2mL}\right]^{\frac{1}{n}}$$

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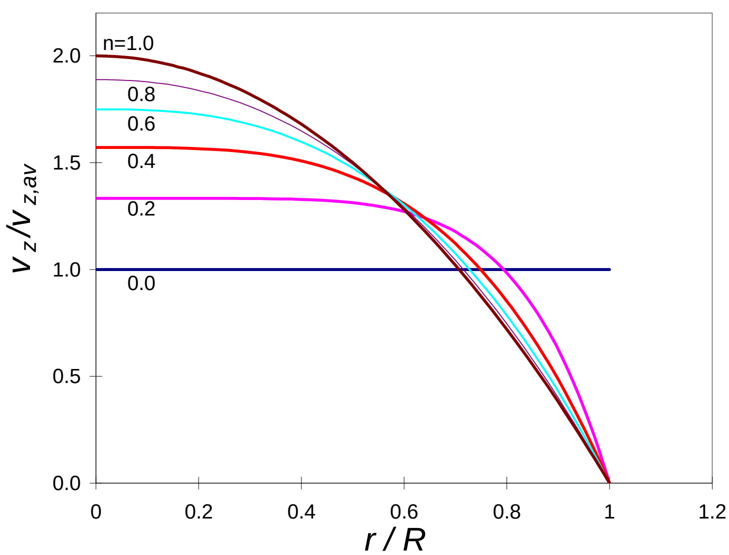
Non-Newtonian behavior
(unidirectional flow)

$\tilde{\tau}_{21} = \text{PL function of} \left(\frac{\partial v_1}{\partial x_2}\right)$

Solution to Poiseuille flow in a tube
incompressible, power-law fluid



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Non-Newtonian behavior (all flows)

$\tilde{\tau}_{21}$ = nonlinear function of $\nabla \underline{v}$

Rheology (Non-Newtonian Fluid Mechanics)

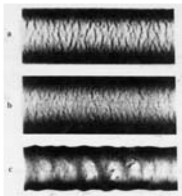
Rheology affects:

•End use (food texture, product pour, motor-oil function)

•Processing (design, costs, production rates)

www.corrugatorman.com/pic/akron%20extruder.JPG

www.math.utwente.nl/mpcm/aamp/examples.html



Pomar et al. JNNFM 54 143 1994

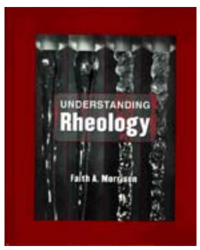
•Product quality (surface distortions, anisotropy, strength, structure development)

Rheology (Non-Newtonian Fluid Mechanics)

At Michigan Tech:

CM4650 Polymer Rheology (Even years spring, MW 4-5:20pm)
www.chem.mtu.edu/~fmorriso/cm4650/cm4650.html

CM4655 Polymer Rheology Lab (Fall, when possible)
www.chem.mtu.edu/~fmorriso/cm4655/cm4655.html



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