



CM3110
Transport I
Part I: Fluid Mechanics



**More Complicated Flows III:
 Boundary-Layer Flow**

(plus other applied topics)



Professor Faith Morrison

Department of Chemical Engineering
 Michigan Technological University

1

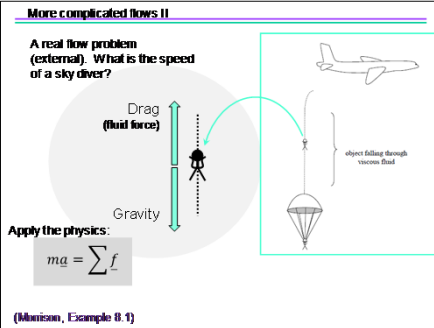
© Faith A. Morrison, Michigan Tech U.

More complicated flows II

Powerful:

More complicated flows II

A real flow problem (external). What is the speed of a sky diver?



Apply the physics:

$$m\vec{a} = \sum \vec{f}$$

(Morrison, Example 8.1)

More complicated flows II

A real flow problem (external). What is the speed of a sky diver?

$$v_{\infty} = \sqrt{\frac{4(\rho_{\text{body}} - \rho)Dg}{3\rho C_D}}$$

$v_{\infty} = 107\text{mph}$

Right!

(or close, anyway)

Solving never-before-solved problems.

With the right physics, and **dimensional analysis**

2

© Faith A. Morrison, Michigan Tech U.

 More complicated flows II
Powerful:

Solving never-before-solved problems.

Left to explore in fluid mechanics:

- What is non-creeping flow like?
(boundary layers)
- Viscosity dominates in creeping flow, what about the flow where inertia dominates?
(potential flow)
- What about mixed flows (viscous+inertial)?
(boundary layers)
- What about really complex flows (curly)?
(vorticity, irrotational+circulation)

3

© Faith A. Morrison, Michigan Tech U.

 More complicated flows II
Powerful:

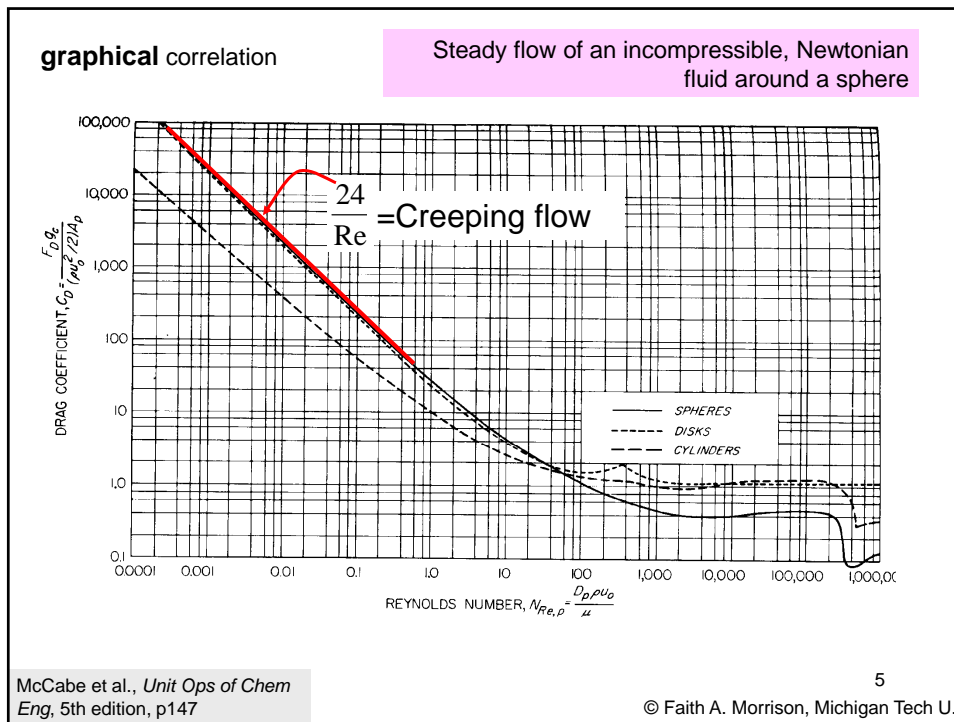
Solving never-before-solved problems.

Left to explore in fluid mechanics:

- **What is non-creeping flow like?**
(boundary layers)
- Viscosity dominates in creeping flow, what about the flow where inertia dominates?
(potential flow)
- What about mixed flows (viscous+inertial)?
(boundary layers)
- What about really complex flows (curly)?
(vorticity, irrotational+circulation)

4

© Faith A. Morrison, Michigan Tech U.



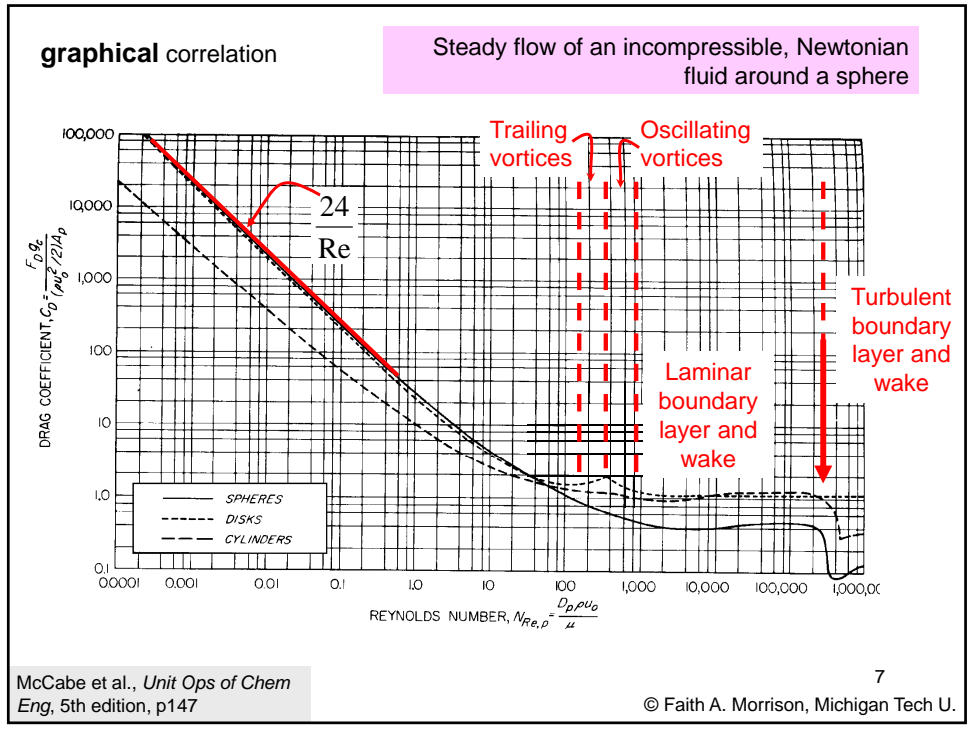
More complicated flows III

What does **non-creeping** flow look like? (Let's look in a wind tunnel)

Can we predict these flows?

6

Text, Figure 8.22, p649, from Sakamoto and Haniu, 1990 © Faith A. Morrison, Michigan Tech U.



More complicated flows II

Powerful:

Solving never-before-solved problems.

Left to explore in fluid mechanics:

- What is non-creeping flow like? (boundary layers) Can we predict these flows?
- **Viscosity dominates in creeping flow, what about the flow where inertia dominates?** (potential flow) Let's apply our methods
- What about mixed flows (viscous+inertial)? (boundary layers)
- What about really complex flows (curly)? (vorticity, irrotational+circulation)

8

© Faith A. Morrison, Michigan Tech U.

Flow where **Viscosity** Dominates:

Nondimensional Navier-Stokes Equation:

$$\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v} \cdot \nabla \underline{v})^* = -\nabla^* P + \frac{\mu}{\rho V D} (\nabla^2 \underline{v})^* + \frac{g D}{V^2} \underline{g}^*$$

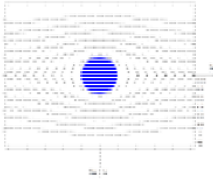
With the appropriate terms in spherical coordinates

No free surfaces

We considered the creeping flow limit:

$$\text{Re} \left(\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v} \cdot \nabla \underline{v})^* \right) = -\text{Re} \nabla^* P + (\nabla^2 \underline{v})^*$$

small Re



Solve for a sphere,

$$C_d = \frac{24}{\text{Re}}$$

9

© Faith A. Morrison, Michigan Tech U.

Flow where **Inertia** Dominates:

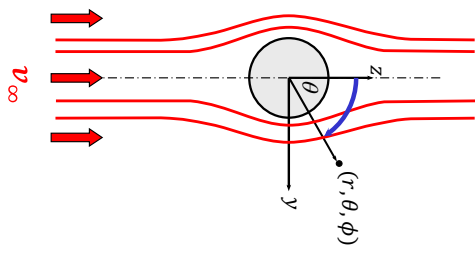
Let's predict these flows!

Consider the high Re limit:

$$\left(\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v} \cdot \nabla \underline{v})^* \right) = -\nabla^* P + \frac{1}{\text{Re}} (\nabla^2 \underline{v})^*$$

Re → ∞

Now solve for flow around a sphere



10

© Faith A. Morrison, Michigan Tech U.

Potential flow around a Sphere (high Re, no viscosity)

(equation 8.203)

$$\nabla^* \cdot \underline{v}^* = 0$$

$$\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v}^* \cdot \nabla \underline{v}^*)^* = -\frac{\partial P^*}{\partial z^*}$$

$$C_D = \frac{2}{\pi} \int_0^{2\pi} \int_0^\pi [-P^* \cos \theta]_{r^*=\frac{1}{2}} \sin \theta d\theta d\phi$$

Predictions:

(the math requires specialized expertise)

Solutions:

$$\underline{v} = \begin{pmatrix} v_\infty \left(1 - \left(\frac{R}{r}\right)^3\right) \cos \theta \\ -v_\infty \left(1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right) \sin \theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

$$P(r, \theta) = P_\infty + \frac{1}{2} \rho v_\infty^2 \left(2 \left(\frac{R}{r}\right)^3 \left(1 - \frac{3}{2} \sin^2 \theta\right) - \left(\frac{R}{r}\right)^6 \left(1 - \frac{3}{4} \sin^2 \theta\right) \right)$$

(equation 8.238-9)

11

© Faith A. Morrison, Michigan Tech U.

Potential flow around a Sphere (high Re, no viscosity)

(equation 8.203)

$$\nabla^* \cdot \underline{v}^* = 0$$

$$\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v}^* \cdot \nabla \underline{v}^*)^* = -\frac{\partial P^*}{\partial z^*}$$

$$C_D = \frac{2}{\pi} \int_0^{2\pi} \int_0^\pi [-P^* \cos \theta]_{r^*=\frac{1}{2}} \sin \theta d\theta d\phi$$

How do these results compare to what we see at high Re?

Solution:

$$\underline{v} = \begin{pmatrix} v_\infty \left(1 - \left(\frac{R}{r}\right)^3\right) \cos \theta \\ -v_\infty \left(1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right) \sin \theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

$$P(r, \theta) = P_\infty + \frac{1}{2} \rho v_\infty^2 \left(2 \left(\frac{R}{r}\right)^3 \left(1 - \frac{3}{2} \sin^2 \theta\right) - \left(\frac{R}{r}\right)^6 \left(1 - \frac{3}{4} \sin^2 \theta\right) \right)$$

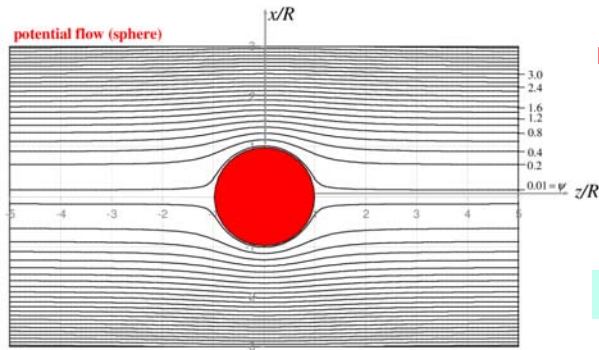
(equation 8.238-9)

12

© Faith A. Morrison, Michigan Tech U.

Potential flow around a Sphere (high Re, no viscosity)

Solution:



How do these results compare to what we see at high Re?

(does it match?)

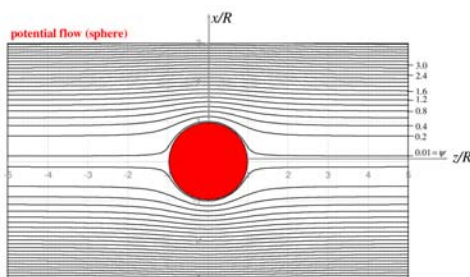
13

© Faith A. Morrison, Michigan Tech U.

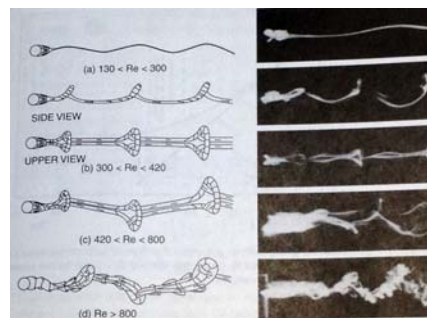
Potential flow around a Sphere (high Re, no viscosity)

(does it match?)

Solution (all high Re):



How does this compare to what we see at high Re?



14

© Faith A. Morrison, Michigan Tech U.

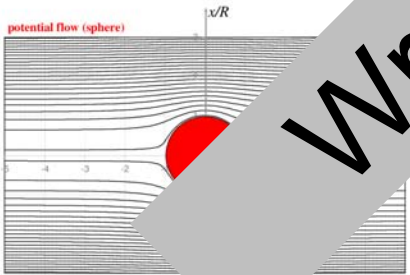
Potential flow around a Sphere (high Re, low viscosity)

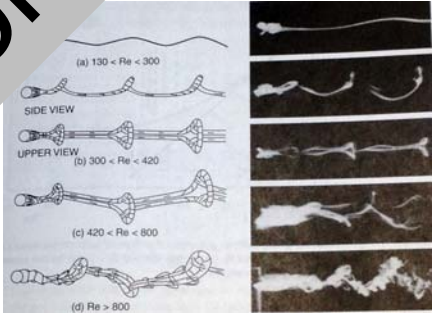
(does it match?)

No.

Does this compare to what we see at high Re?

Solution (all high Re):





(a) 130 < Re < 300
SIDE VIEW
UPPER VIEW
(b) 300 < Re < 420
(c) 420 < Re < 800
(d) Re > 800

Wrong!

15
© Faith A. Morrison, Michigan Tech U.

Potential flow around a Sphere (high Re, low viscosity)

(equation 8.20)

$$\nabla^* \cdot \underline{v}^* = 0$$

$$\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v} \cdot \nabla \underline{v})^* = -\frac{\partial P^*}{\partial x^*}$$

$$C_D = \frac{2}{\pi} \int_0^{2\pi} \int_0^\pi [-P^* \cos \theta] \sin \theta \, d\theta \, d\phi$$

Wrong!

Solution:

$$\underline{v} = \begin{pmatrix} v_\infty \left(1 - \left(\frac{R}{r}\right)^3\right) \cos \theta \\ -v_\infty \left(1 + \frac{1}{2} \left(\frac{R}{r}\right)^3\right) \sin \theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

$$F_D = \frac{1}{2} \rho v_\infty^2 \left(2 \left(\frac{R}{r}\right)^3 \left(1 - \frac{3}{2} \sin^2 \theta\right) - \left(\frac{R}{r}\right)^6 \left(1 - \frac{3}{4} \sin^2 \theta\right) \right)$$

(equation 8.238-9)

16
© Faith A. Morrison, Michigan Tech U.

Potential flow around a Sphere (high Re, no viscosity)

Wrong!

$$\frac{\partial \underline{v}^*}{\partial t} + (\underline{v} \cdot \nabla \underline{v})^* = - \frac{\partial P^*}{\partial t}$$

Predicts:

- No drag (d'Alembert's paradox)
- Slip at the wall
- Approximately right pressure profile (near the wall)
- Right velocity field away from the wall

$$P(r, \theta) = P_\infty + \frac{1}{2} \rho v_\infty^2 \left(2 \left(\frac{R}{r} \right)^3 \left(1 - \frac{3}{2} \sin^2 \theta \right) - \left(\frac{R}{r} \right)^6 \left(1 - \frac{3}{4} \sin^2 \theta \right) \right)$$

(equation 8.238-9)

17

© Faith A. Morrison, Michigan Tech U.

Potential flow around a Sphere (high Re, no viscosity)

Wrong!

$$\frac{\partial \underline{v}^*}{\partial t} + (\underline{v} \cdot \nabla \underline{v})^* = - \frac{\partial P^*}{\partial t}$$

Predicts:

- No drag (d'Alembert's paradox)
 - Slip at the wall
 - *Approximately right pressure profile (near the wall)*
 - *Right velocity field away from the wall*
- partially right.**

$$P(r, \theta) = P_\infty + \frac{1}{2} \rho v_\infty^2 \left(2 \left(\frac{R}{r} \right)^3 \left(1 - \frac{3}{2} \sin^2 \theta \right) - \left(\frac{R}{r} \right)^6 \left(1 - \frac{3}{4} \sin^2 \theta \right) \right)$$

(equation 8.238-9)

18

© Faith A. Morrison, Michigan Tech U.

Potential flow around a Sphere (high Re, no viscosity)

Wrong!

$$\frac{\partial v^*}{\partial t} + (v \cdot \nabla) v^* = -\frac{\partial P^*}{\partial x}$$

Predicts:

- No drag
- Slip at the wall partially right.
- *Approximately right pressure profile (near the wall)*
- *Right velocity field away from the wall*

$$P(r, \theta) = P_\infty + \frac{1}{2} \rho v_\infty^2 \left(2 \left(\frac{R}{r} \right)^3 \left(1 - \frac{3}{2} \sin^2 \theta \right) - \left(\frac{R}{r} \right)^6 \left(1 - \frac{3}{4} \sin^2 \theta \right) \right)$$

(equation 8.238-9)

?

What now?

19

© Faith A. Morrison, Michigan Tech U.

More complicated flows III

partially right.

Predicts:

- No drag (d'Alembert's paradox)
- Slip at the wall
- *Approximately right pressure profile (near the wall)* (near the wall)
- *Right velocity field away from the wall* away from the wall

outer solution

inner solution

solutions match at boundary

Break into two parts?

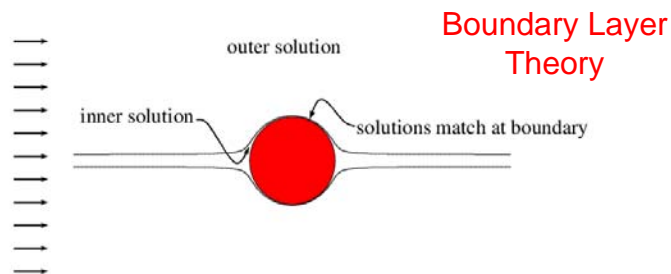
20

© Faith A. Morrison, Michigan Tech U.

More complicated flows III

Prandtl's Great Idea (1904):

- Keep the good parts of the potential flow solution: \underline{v} in free stream, $p(r, \theta)$ near the surface
- Throw away the bad parts: slip at the wall
- Solve a new problem near the wall with $p(r, \theta)$ from the potential-flow solution



21

© Faith A. Morrison, Michigan Tech U.

What can we do to understand Boundary Layers?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

22

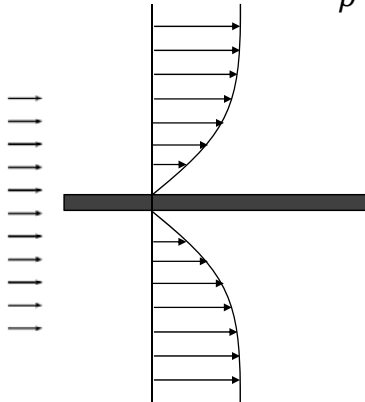
© Faith A. Morrison, Michigan Tech U.

More complicated flows III

(Section 8.2)

Boundary Layer Theory

- Choose simplest boundary layer (flat plate)
- Nondimensionalize Navier-Stokes
- Eliminate small terms
- Solve



$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

Characteristic values:

- U in principal flow direction v_1
- V in direction perpendicular to wall, v_2
- L length of plate for x_1
- δ boundary layer thickness for x_2

23

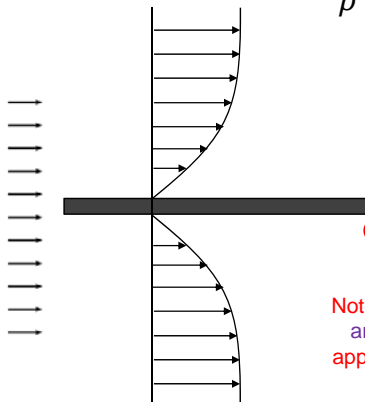
© Faith A. Morrison, Michigan Tech U.

More complicated flows III

(Section 8.2)

Boundary Layer Theory

- Choose simplest boundary layer (flat plate)
- Nondimensionalize Navier-Stokes
- Eliminate small terms
- Solve



$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

Characteristic values:

- U in principal flow direction v_1
- V in direction perpendicular to wall, v_2
- L length of plate for x_1
- δ boundary layer thickness for x_2

Note that for this flow, **two length** scales
and **two velocities** were found to be
appropriate for the dimensional analysis.

24

© Faith A. Morrison, Michigan Tech U.

More complicated flows III

It works!

Boundary Layer Theory

- Apply to uniform flow approaching a sphere

outer solution

inner solution

solutions match at bounda

$$\frac{y}{R} \sqrt{\frac{\rho v_\infty R}{\mu}}$$

(see text, p710)

Boundary layer velocity profiles as you progress from the stagnation point (0°) to the top of the sphere (90°) and beyond.

25

© Faith A. Morrison, Michigan Tech U.

separation

More complicated flows III

It works!

Boundary Layer Theory

- Explains boundary-layer separation
- Golf ball problem
- BL separation caused by adverse pressure gradient

pressure pushes flow along

pressure slows the flow and causes reversal

smooth ball

rough ball

H. Schlichting, Boundary Layer Theory (McGraw-Hill, NY 1955).

26

© Faith A. Morrison, Michigan Tech U.

separation

More complicated flows III

Boundary Layer Theory

- Explains boundary-layer separation
- Golf ball problem
- BL separation caused by adverse pressure gradient

pressure pushes flow along

pressure slows the flow and causes reversal

It works!

smooth ball

rough ball

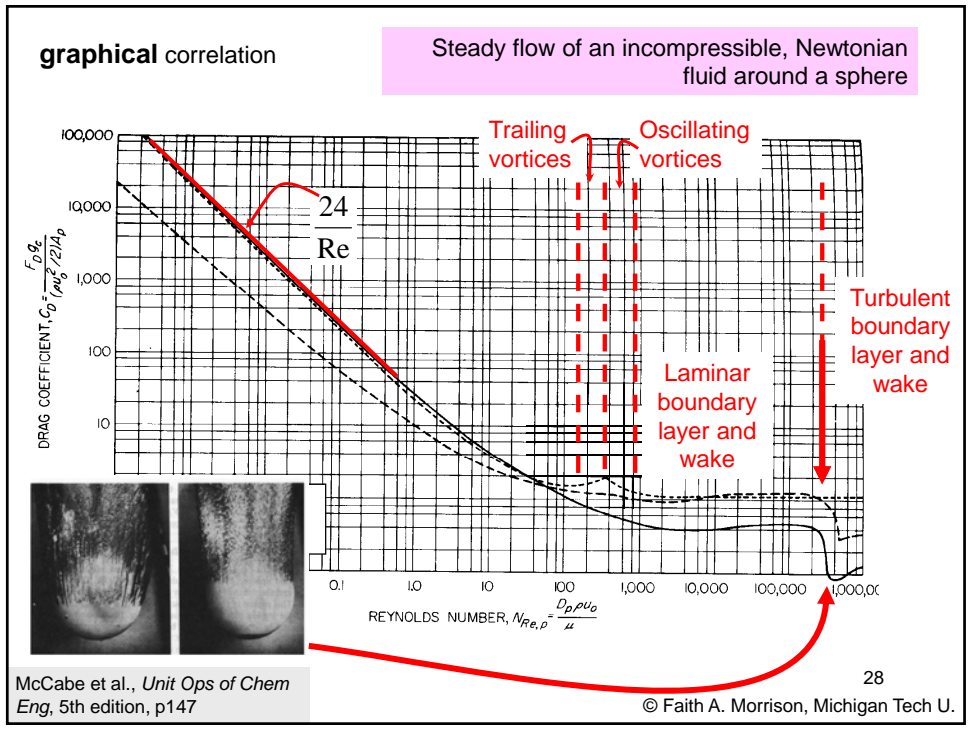
H. Schlichting, Boundary Layer Theory (McGraw-Hill, NY 1955).

The pressure distribution is like a storage mechanism for momentum in the flow; as other momentum sources die out, the pressure drives the flow.

separation

27

© Faith A. Morrison, Michigan Tech U.



What do we do to understand complex flows?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles
- **Boundary Layers**

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

**Solve Real Problems.
Powerful.**

29

© Faith A. Morrison, Michigan Tech U.

More complicated flows II

Powerful:

Solving never-before-solved problems.

Left to explore in fluid mechanics:

- What is non-creeping flow like?
(boundary layers)
- Viscosity dominates in creeping flow, what about the flow where inertia dominates?
(potential flow)

- See text
- What about mixed flows (viscous+inertial)?
(boundary layers)
 - What about really complex flows (curly)?
(vorticity, irrotational+circulation)

30

© Faith A. Morrison, Michigan Tech U.

What do we do to understand complex flows?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles
- Boundary Layers
- Curvy flows,

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

**Solve Real Problems.
Powerful.**

31

© Faith A. Morrison, Michigan Tech U.

**CM3110
Transport I
Part I: Fluid Mechanics**

MichiganTech

***Applied Topics:
Fluidized Beds***



Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

32

© Faith A. Morrison, Michigan Tech U.

ChemE Application of Ergun Equation

Fluidized beds

- ion exchange columns
- packed bed reactors
- packed distillation columns
- filtration
- flow through soil (environmental issues, enhanced oil recovery)
- fluidized bed reactors

gas velocity →

33
© Faith A. Morrison, Michigan Tech U.

Image from: fluidizedbed2008.webs.com

ChemE Application of Ergun Equation

Calculate the minimum superficial velocity at which a bed becomes fluidized.

In a fluidized bed reactor, the flow rate of the gas is adjusted to overcome the force of gravity and fluidize a bed of particles; in this state heat and mass transfer is good due to the chaotic motion.

The Δp vs Q relationship can come from the **Ergun eqn** at small Re_p

$$\frac{150}{Re_p} + 1.75 = f_p$$

neglect

dominates

© Faith A. Morrison, Michigan Tech U. 34

note: Re_p vs Re_{DH}

More Complex Applications II: Fluidized beds

Now we perform a force balance on the bed:

$$m\bar{a} = \sum \bar{f}$$

bed volume = $(1 - \epsilon)AL$

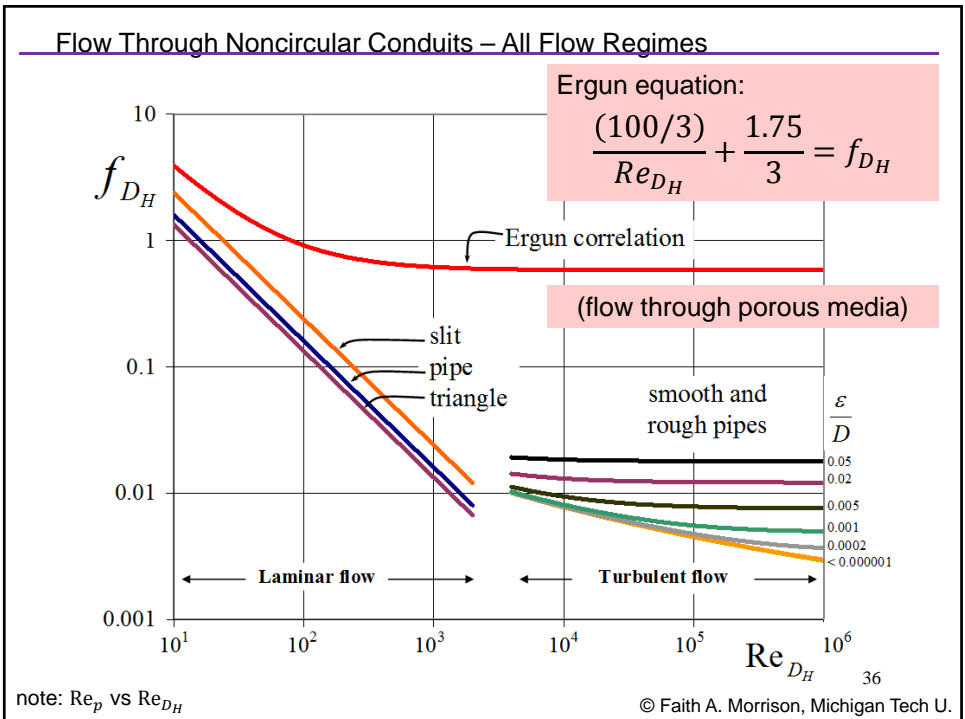
When the forces balance, *incipient fluidization*

net effect of gravity and buoyancy is:

$$\underbrace{(\rho_p - \rho)}_{\substack{\text{mass} \\ \text{volume}}} \underbrace{(1 - \epsilon)AL}_{\text{volume}} g$$

$\Delta p A$

35
© Faith A. Morrison, Michigan Tech U.



More Complex Applications II: Fluidized beds

When the forces balance, *incipient fluidization*

eliminate Δp ;
solve for v_0

}

$$\Delta p A = (\rho_p - \rho)(1 - \varepsilon) A L g$$

$$\frac{150}{Re_p} = f_p$$

note: Re_p vs Re_{DH}

$$v_0 = \frac{(\rho_p - \rho) g D_p^2 \varepsilon^3}{150 \mu (1 - \varepsilon)}$$

velocity at the point of
incipient fluidization

Complete solution steps in Denn, Process Fluid Mechanics (Prentice Hall, 1980) 37

© Faith A. Morrison, Michigan Tech U.

Fluidized beds?

What do we do to understand complex flows?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles
- Boundary Layers

}

1. Find a simple problem that allows us to identify the physics (flow through packed bed)
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate (Ergun equation)
5. Solve real problems

Solve Real Problems.

Powerful.

Model the slow flow;
calculate incipient fluidization criteria; take data on the more complex cases

See Perry's Handbook for more on Fluidized Beds

38
© Faith A. Morrison, Michigan Tech U.

CM3110
Transport I
Part I: Fluid Mechanics

MichiganTech

Applied Topics:
Compressible Flow



Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

39

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

- most fluids are somewhat compressible
- in chemical-engineering processes, compressibility is unimportant at most operating pressures
- even gases may be modeled as incompressible if $\Delta p < p_{mean}$

EXCEPT:

When the fluid velocity approaches the speed of sound

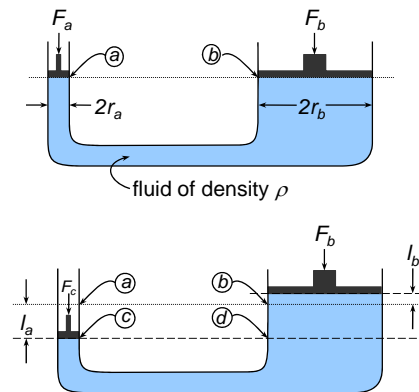
© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

How is pressure information transmitted in liquids and gases?

The Hydraulic Lift operates on Pascal's principle

Pressure exerted on an enclosed liquid is transmitted equally to every part of the liquid and to the walls of the container.



© Faith A. Morrison, Michigan Tech U.

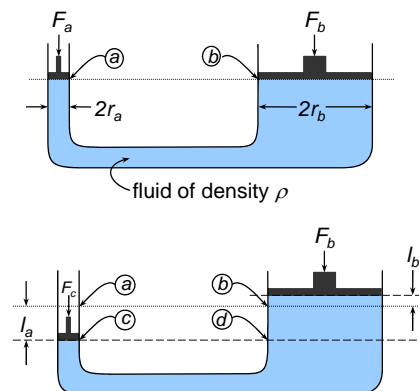
Compressible Fluids

For static incompressible liquids,

The Hydraulic Lift operates on Pascal's principle

*Pressure exerted on an enclosed liquid **is transmitted equally** to every part of the liquid and to the walls of the container.*

and essentially, instantaneously



© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

For static compressible fluids (gases), pressure causes volume change.

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

For moving **incompressible** liquids and gases,

The presence of the obstacle is felt by the upstream fluid (pressure) and that information is transmitted very rapidly throughout the fluid.

The streamlines adjust according to momentum conservation.

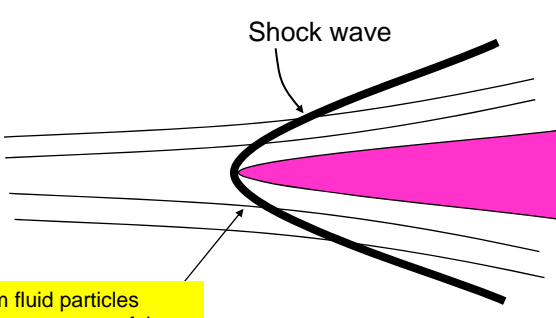
These fluid particles are not blocked by the sphere, but they feel its presence due to the pressure field.

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

For **compressible fluids** moving near sonic speeds, information (pressure) and the gas itself are moving at comparable speeds.

Pressure piles up at the shock wave



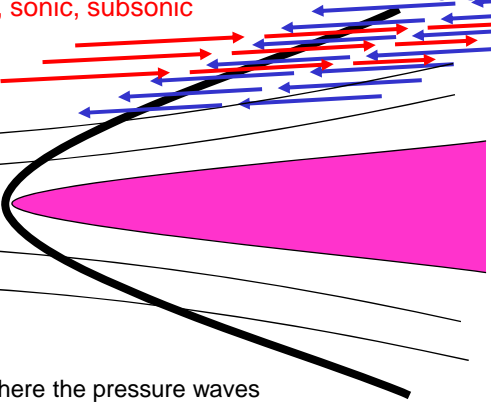
The upstream fluid particles cannot feel the presence of the object because it's **outrunning** the pressure field.

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

Velocity of a fluid = variable =
supersonic, sonic, subsonic

Velocity of a pressure wave = constant = speed of sound



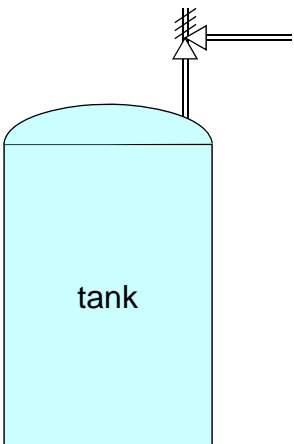
A shock forms where the pressure waves from the obstacle stack up, and the speed of the pressure wave traveling upstream equals the speed of the fluid traveling downstream.

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

Super-sonic flows in Chem Eng:
Relief Valves (Safety Valves)

The rapid flows in relief valves can become sonic.



For supersonic flow, the flow rate is constant no matter what the pressure drop is.

(pressure waves pile up)

Choked Flow

Choked flow can be understood from basic equations of compressible fluid mechanics.

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

Momentum and Energy in Compressible Fluids

Microscopic momentum balance:

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

incompressible

$$\underline{\underline{\tau}} = \mu (\nabla \underline{v} + (\nabla \underline{v})^T)$$

Mechanical energy balance:

$$\frac{\Delta p}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\delta z + F = \frac{W_{s,on}}{\dot{m}}$$

incompressible

compressible?

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

Momentum and Energy in Compressible Fluids

Microscopic momentum balance:

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

incompressible $\underline{\underline{\tau}} = \mu(\nabla \underline{v} + (\nabla \underline{v})^T)$

$$\underline{\underline{\tau}} = \mu(\nabla \underline{v} + (\nabla \underline{v})^T) - \left(\frac{2}{3} \mu - \kappa \right) \nabla \cdot \underline{v}$$

compressible

$\kappa = \text{bulk viscosity}$

Mechanical energy balance:

$$\frac{\Delta p}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\delta z + F = \frac{W_{s,on}}{\dot{m}}$$

incompressible

MEB for compressible?

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

Mechanical energy balance (compressible)

Back up one step

in the derivation and reintegrate without constant ρ assumption.

$$\frac{dp}{\rho} + VdV + gdz + dF = \frac{dW_{s,on}}{\dot{m}}$$

Assume:

- constant cross section
- constant mass flow $\rho VA = GA$
- neglect gravity
- no shaft work

$G \equiv \rho V = \text{mass velocity}$

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

Mechanical energy balance (compressible)

Ideal Gas Law $pV = NRT$

$$\frac{V}{N} = \frac{RT}{p}$$

$$\frac{V}{MN} = \frac{RT}{pM}$$

$\frac{1}{\rho} = \frac{RT}{pM}$

For isothermal flow:

$$p_1 V_1 = NRT$$

$$p_2 V_2 = NRT$$

$\frac{p_1}{p_2} = \frac{V_2}{V_1}$

Also, $\frac{\rho_{av}}{p_{av}} = \frac{M}{RT}$

$\frac{2\rho_{av}}{p_1 + p_2} = \frac{M}{RT}$

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

Mechanical energy balance (compressible)

$G \equiv \rho V = \text{mass velocity}$

$$(p_2 - p_1) + \frac{G^2}{\rho_{av}} \ln \frac{p_1}{p_2} + \frac{2fG^2}{\rho_{av}D} (L_2 - L_1) = 0$$

The compressible MEB predicts that there is a maximum velocity at (see book)

$$V_{\max} = \sqrt{\frac{p_2}{\rho_2}} = \sqrt{\frac{RT}{M}} = \text{isothermal speed of sound}$$

© Faith A. Morrison, Michigan Tech U.

Compressible Fluids

A better assumption than isothermal flow is adiabatic flow (no heat transferred). For this case,

$$V_{\max} = \sqrt{\frac{\gamma p_2}{\rho_2}} = \sqrt{\frac{\gamma RT}{M}} = \text{adiabatic speed of sound}$$

$$\gamma = \frac{C_p}{C_v}$$

(see book)


© Faith A. Morrison, Michigan Tech U.

CM3110
Transport I
Part I: Fluid Mechanics

MichiganTech

**More Complicated Flows III:
Boundary-Layer Flow**

(plus other applied topics)



Done!

Professor Faith Morrison
Department of Chemical Engineering
Michigan Technological University

*Just one
more thing*



54

© Faith A. Morrison, Michigan Tech U.

Numerical PDE Solving with Comsol 5.2

MichiganTech



www.comsol.com

Finite-element numerical differential equation solver. Applications include fluid mechanics and heat transfer.

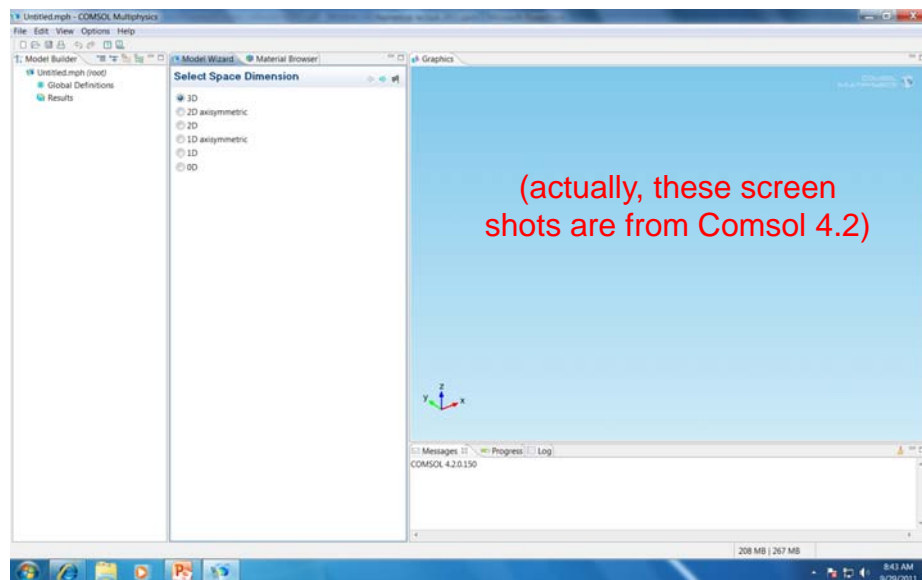
1. Choose the physics (2D, 2D axisymmetric, laminar, steady/unsteady, etc.)
2. Choose flow geometry and fluid (shape of the flow domain)
3. Define boundary conditions
4. Design and generate mesh
5. Solve the problem
6. Calculate and plot engineering quantities of interest.

55
© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics 5.2

Launch the program

0

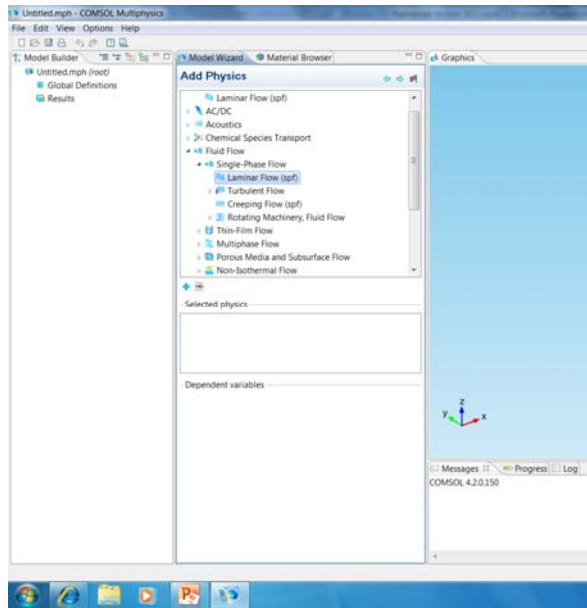


© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics 5.2

Choose the physics

1



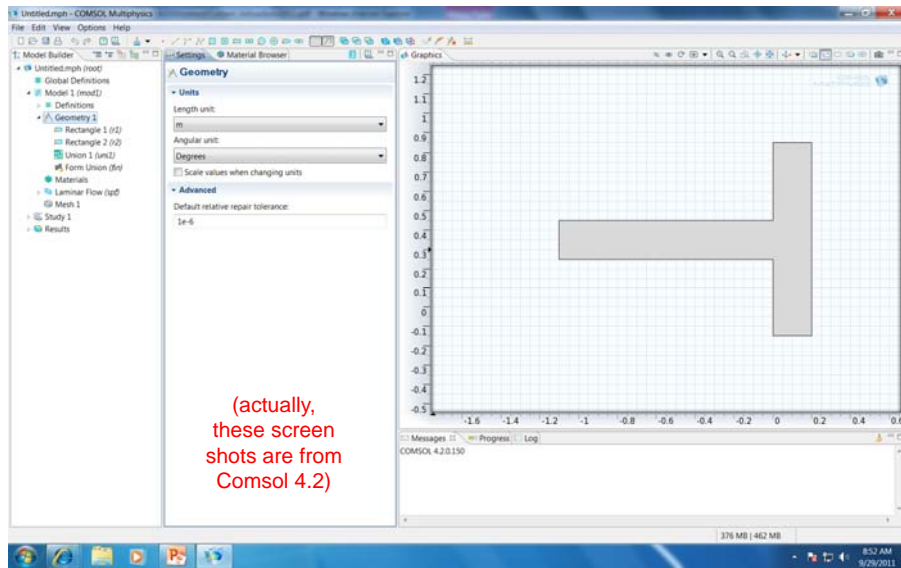
(actually, these screen shots are from Comsol 4.2)

© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics 5.2

Choose flow geometry and fluid

2



(actually, these screen shots are from Comsol 4.2)

© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics 5.2

Define boundary conditions

3

(actually, these screen shots are from Comsol 4.2)

© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics 5.2

Design and generate mesh

4

(actually, these screen shots are from Comsol 4.2)

© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics 5.2

Solve the problem

5

(actually, these screen shots are from Comsol 4.2)

© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics 5.2

View the solution

5

(actually, these screen shots are from Comsol 4.2)

© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics 5.2

Calculate engineering problems of interest **6**

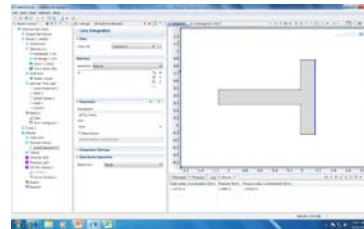
The screenshot shows the Comsol Multiphysics 5.2 interface. On the left is the Model Builder tree. The central pane shows the 'Line Integration' settings for 'Solution 1'. The 'Expression' field is set to 'spfA_stress', and the 'Unit' is 'N/m'. The 'Integration Settings' are set to 'None'. On the right, a 'Convergence Plot' window displays a 2D plot of a T-junction geometry. A red circle highlights the data table at the bottom of the plot, which contains the following values:

Total stress, x component (N/m)	Pressure (N/m)	Viscous stress, x component (N/m)
-2.3071e-6	2.288e-6	-1.9116e-8

Red text overlaid on the plot reads: "(actually, these screen shots are from Comsol 4.2)".

© Faith A. Morrison, Michigan Tech U.

Comsol Multiphysics




Comsol project:

- Due last day of classes
 - Individual work
 - 2 points for part 1 (instructions given)
 - 3 points for part 2 (no instructions)
 - Coming soon
- } Adds on top of your course grade

Part II: Heat Transfer

CM3110
Transport I
Part II: Heat Transfer

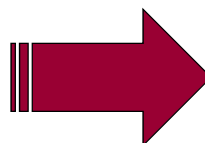


Michigan Tech

Introduction to Heat Transfer



Professor Faith Morrison
Department of Chemical Engineering
Michigan Technological University



65

© Faith A. Morrison, Michigan Tech U.