CM3110
Transport Processes and Unit Operations I

Part 2:

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

CM3110 - Momentum and **Heat Transport**
CM3120 – Heat and Mass Transport

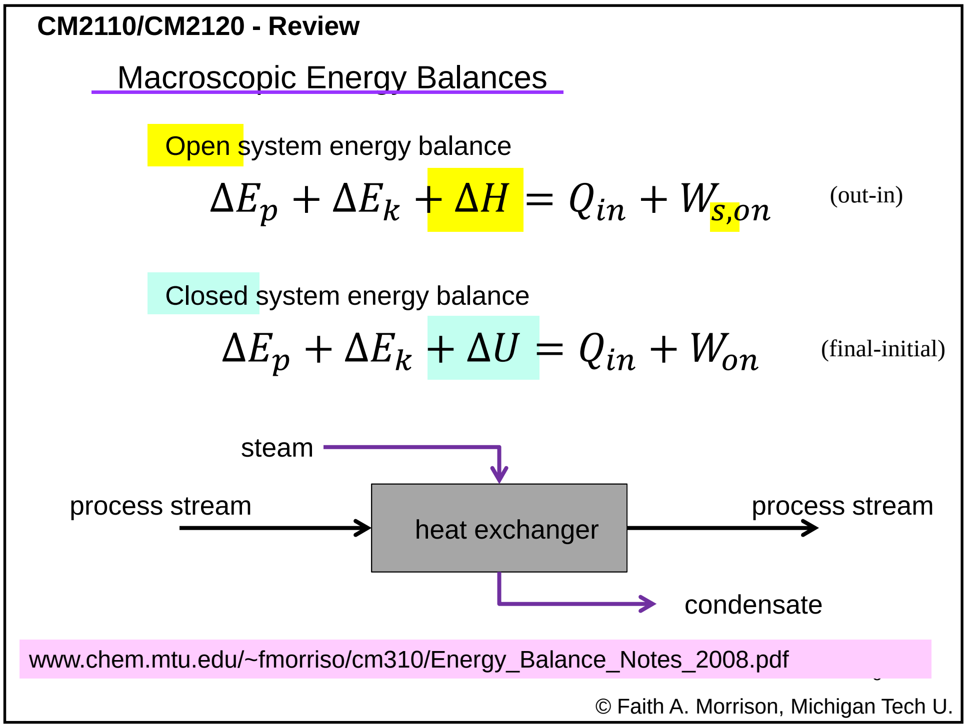
www.chem.mtu.edu/~fmorriso/cm310/cm310.html

CM3110
Transport I
Part II: Heat Transfer

Introduction to Heat Transfer

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University



CM2110/CM2120 - Review

Macroscopic Energy Balances

Open system energy balance

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on} \qquad \text{(out-in)}$$

Closed system energy balance

$$\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on} \qquad \text{(final-initial)}$$

steam

process stream

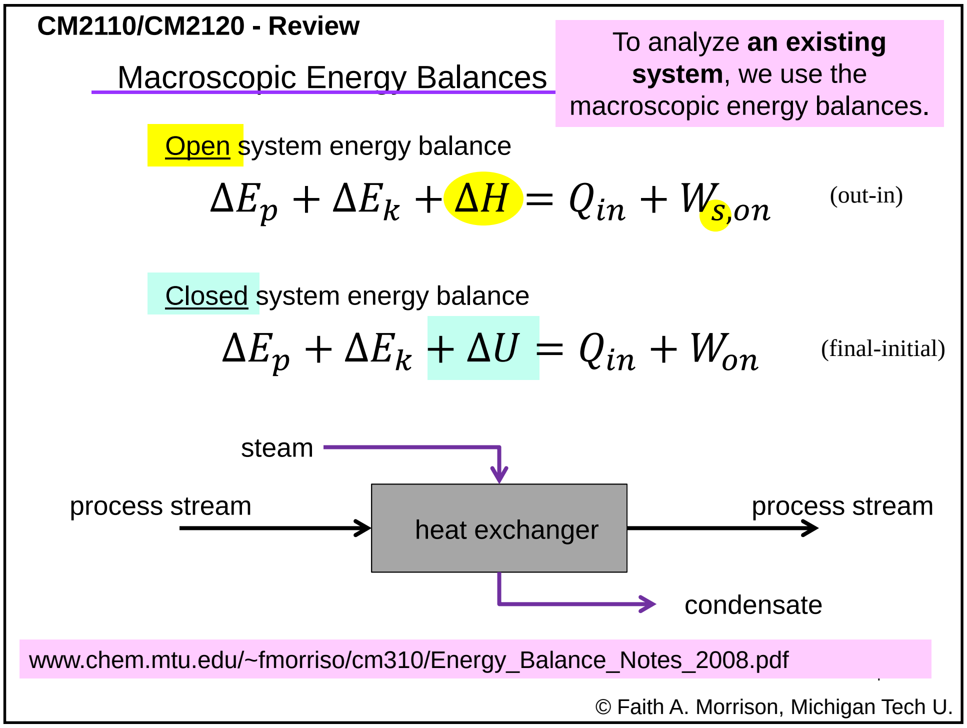
heat exchanger

process stream

condensate

www.chem.mtu.edu/~fmorriso/cm310/Energy_Balance_Notes_2008.pdf

© Faith A. Morrison, Michigan Tech U.



CM2110/CM2120 - Review

Macroscopic Energy Balances

To analyze **an existing system**, we use the macroscopic energy balances.

Open system energy balance

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on} \qquad \text{(out-in)}$$

Closed system energy balance

$$\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on} \qquad \text{(final-initial)}$$

steam

process stream

heat exchanger

process stream

condensate

www.chem.mtu.edu/~fmorriso/cm310/Energy_Balance_Notes_2008.pdf

© Faith A. Morrison, Michigan Tech U.

To design **a new system**, we also use the macroscopic energy balances;

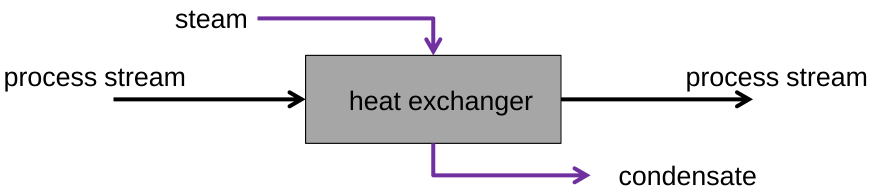
We need ***transport relationships*** to give us the heat transferred, however.

Open system energy balance

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on}$$

Closed system energy balance

$$\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on}$$



steam

process stream

heat exchanger

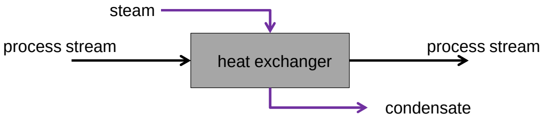
process stream

condensate

Concerned, now, with ***rates*** of heat transfer

Heat Transfer

Concerned now with ***rates*** of heat transfer



steam

process stream

heat exchanger

process stream

condensate

To track down the physics of the *rate* of heat transfer, we turn to the equations that govern the physics on the *microscopic* scale:

Microscopic Energy Balance

(first law of thermo)

Energy Balance: Body versus Control Volume

First Law of Thermodynamics:
(**on a body**)

$$\boxed{\frac{dE_B}{dt} = Q_{in,B} - W_{by,B}}$$

First Law of Thermodynamics:
(**on a control volume**)

$$\frac{dE_{CV}}{dt} = Q_{in,CV} - W_{by,CV} + \underbrace{\iint_{CS} -(\hat{n}\cdot\underline{v})\rho\hat{E}\,dS}_{\text{the usual convective term: net energy convected in}}$$

Reference: Morrison, F. A., Web Appendix D1: Microscopic Energy Balance, Supplement to *An Introduction to Fluid Mechanics* (Cambridge, 2013), www.chem.mtu.edu/~fmorriso/IFM_WebAppendixD2011.pdf

---

Energy Balance on a Control Volume

First Law of Thermodynamics:
(on a control volume)

$$\frac{dE_{CV}}{dt} + \iint_{CS} (\hat{n}\cdot\underline{v})\rho\hat{E}\,dS = \boxed{Q_{in,CV}} - \boxed{W_{by,CV}}$$

Microscopic CV:

…

$$\rho\left(\frac{\partial\hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = \underbrace{-\nabla\cdot\underline{\tilde{q}} + S_e}_{\text{Heat into CV due to conduction and reaction + electrical current}} \underbrace{-\nabla\cdot(P\underline{v}) + \nabla\cdot(\underline{\underline{\tilde{\tau}}}\cdot\underline{v})}_{\text{-Work by the fluid in the CV due to pressure/volume work and viscous dissipation}}$$

Energy Balance on a Control Volume

First Law of Thermodynamics:
(on a control volume)

$$\frac{dE_{CV}}{dt} + \iint_{CS} (\hat{n}\cdot \underline{v})\rho\hat{E}\,dS = \boxed{Q_{in,CV}} - \boxed{W_{by,CV}}$$

Microscopic CV:

…

$$\rho\left(\frac{\partial \hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = \underbrace{-\nabla\cdot\underline{\tilde{q}} + S_e}_{\text{Heat into CV due to conduction and reaction + electrical current}} \underbrace{- \nabla\cdot(P\underline{v}) + \nabla\cdot(\underline{\tilde{\tau}}\cdot\underline{v})}_{\text{-Work by the fluid in the CV due to pressure/volume work and viscous dissipation}}$$

Heat into CV due to conduction and reaction + electrical current

-Work by the fluid in the **CV** due to pressure/volume work and viscous dissipation

In heat-transfer unit operations, PV work and viscous dissipation are usually negligible

Energy Balance on a Control Volume (heat-transfer unit operations)

First Law of Thermodynamics:
(on a control volume, no work)

$$\rho\left(\frac{\partial \hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = -\nabla\cdot\underline{\tilde{q}} + S_e$$

$$\begin{pmatrix}\text{rate of}\\ \text{energy}\\ \text{accumulation}\end{pmatrix} + \begin{pmatrix}\text{net energy}\\ \underline{v}\text{ flow out}\\ \text{(convection)}\end{pmatrix} = \underbrace{\begin{pmatrix}\text{net heat}\\ \text{in,}\\ \text{conduction}\end{pmatrix}}_{} + \underbrace{\begin{pmatrix}\text{net heat in,}\\ \text{energy}\\ \text{production}\end{pmatrix}}_{}$$

conduction - Fourier's law

e.g. chemical reaction, electrical current

$$\frac{q}{A} \equiv \boxed{\underline{\tilde{q}} = -k\nabla T}$$

Energy Balance on a Control Volume (heat-transfer unit operations)

First Law of Thermodynamics:
(on a control volume, no work)

$$\rho\left(\frac{\partial\hat{E}}{\partial t}+\underline{v}\cdot\nabla\hat{E}\right)=-\nabla\cdot\underline{\tilde{q}}+S_e$$

$$\begin{pmatrix}\text{rate of}\\ \text{energy}\\ \text{accumulation}\end{pmatrix}+\begin{pmatrix}\text{net energy}\\ \underline{v}\text{ flow out}\\ \text{(convection)}\end{pmatrix}=\underbrace{\begin{pmatrix}\text{net heat}\\ \text{in,}\\ \text{conduction}\end{pmatrix}}_{\substack{\textbf{conduction}\text{ -}\\ \text{Fourier's law}}}+\underbrace{\begin{pmatrix}\text{net heat in,}\\ \text{energy}\\ \text{production}\end{pmatrix}}_{\substack{\text{e.g.}\\ \text{chemical}\\ \text{reaction,}\\ \text{electrical}\\ \text{current}}}$$

Note the two different q's (watch units)

$$\frac{q}{A}\equiv\boxed{\underline{\tilde{q}}=-k\nabla T}$$

Part I: Momentum Transfer

Momentum transfer:

$$\tau_{21}=\underbrace{(-\tilde{\tau}_{21})}_{\text{momentum flux}}=-\overset{\text{viscosity}}{\mu}\underbrace{\left(\frac{dv_1}{dx_2}\right)}_{\text{velocity gradient}}$$

Part II: Heat Transfer

Heat transfer:

$$\underbrace{\frac{q_x}{A}}_{\text{heat flux}}=-\overset{\text{thermal conductivity}}{k}\underbrace{\frac{dT}{dx}}_{\substack{\text{temperature}\\ \text{gradient}}}$$

Part I: Momentum Transfer

Momentum transfer:

viscosity

$$\underbrace{\tau_{21} = (-\tilde{\tau}_{21})}_{\text{momentum flux}} = -\mu \underbrace{\left(\frac{dv_1}{dx_2}\right)}_{\text{velocity gradient}}$$

Newton's law of viscosity

Part II: Heat Transfer

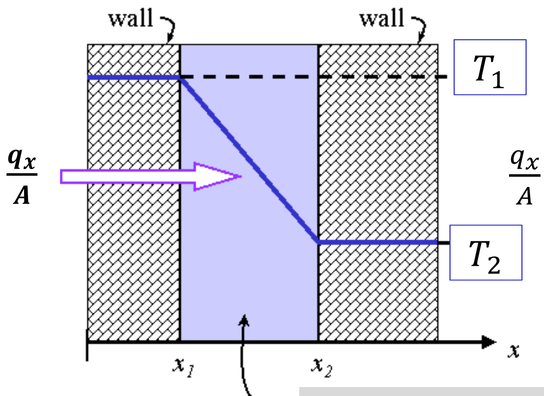
Heat transfer:

thermal conductivity

$$\underbrace{\frac{q_x}{A}}_{\text{heat flux}} = -k \underbrace{\frac{dT}{dx}}_{\text{temperature gradient}}$$

Fourier's law of conduction

Fourier's Experiments: Simple One-dimensional Heat Conduction



$$\frac{q_x}{A} = k\,\frac{T_1 - T_2}{x_2 - x_1} = -k\,\frac{dT}{dx}$$

Fourier's law of conduction

Homogeneous material of thermal conductivity, k

Fourier's law of Heat Conduction:

makes reference to a coordinate system

Allows you to solve for temperature profiles

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

Gibbs notation: $\frac{\underline{q}}{A} = -k\nabla T$

Fourier's law

$$\tilde{\underline{q}} = \frac{\underline{q}}{A} = \begin{pmatrix} -k\frac{\partial T}{\partial x} \\ -k\frac{\partial T}{\partial y} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

- Heat flows **down** a temperature gradient
- Flux is proportional to magnitude of temperature gradient

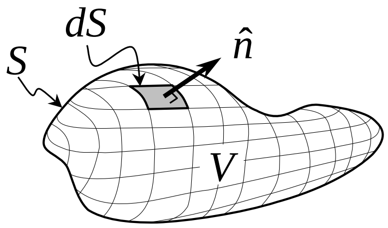
As was true in momentum transfer (fluid mechanics) solving problems with shell balances on individual control volumes is tedious, and it is easy to make errors.

Instead, we use the general equation, derived for all circumstances:

General Energy Transport Equation
(microscopic energy balance)

Recall Microscopic Momentum Balance:

Equation of Motion



Microscopic **momentum** balance written on an arbitrarily shaped control volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\boxed{\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \nabla\cdot\underline{\underline{\tilde{\tau}}} + \rho\underline{g}}$$ **general fluid**

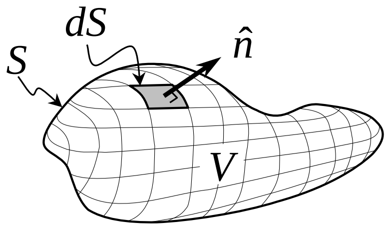
Gibbs notation: $$\boxed{\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}}$$ **Newtonian fluid**

Navier-Stokes Equation

Microscopic momentum balance is a vector equation.

Microscopic Energy Balance:

Equation of Thermal Energy



Microscopic **energy** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\boxed{\rho\left(\frac{\partial \hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = -\nabla\cdot\underline{\tilde{q}} + S_e}$$ **general conduction**

Gibbs notation: $$\boxed{\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e}$$ **Only Fourier conduction**

(incompressible fluid, constant pressure, neglect $\hat{E}_k, \hat{E}_p$, viscous dissipation)

Equation of Energy

(microscopic energy balance)

$$\rho\hat{C}_p\left(\underbrace{\frac{\partial T}{\partial t}}_{\text{rate of change}}+\overbrace{\underline{v}\cdot\nabla T}^{\text{convection}}\right)=\underbrace{k\nabla^2 T}_{\substack{\text{conduction}\\\text{(all directions)}}}+\overbrace{S_e}^{\text{source}}$$

(energy generated per unit volume per time)

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

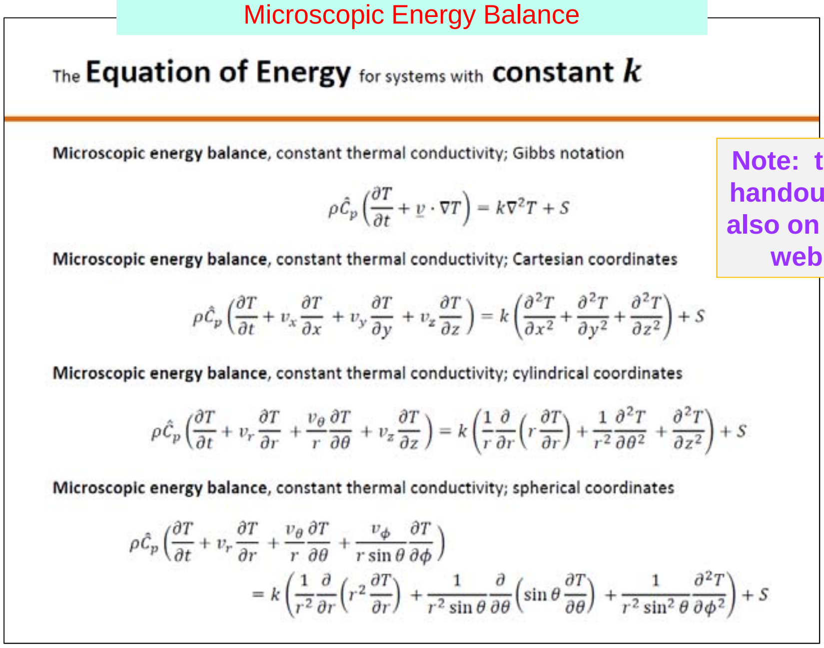
Problem-Solving Procedure - heat-transfer problems

1. sketch system
2. choose coordinate system
3. choose a control volume - small dimension in the direction of flux
4. perform an energy balance (will contain energy flux)
5. substitute in *Fourier's law of conduction*, e.g. $\boxed{\frac{q_x}{A}=-k\left(\frac{dT}{dx}\right)}$
6. solve the differential equation for temperature profile
7. apply boundary conditions

IF we do the CV manually; usually we use the micro E bal

Does this seem familiar?

Microscopic Energy Balance



The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)+S$$

Note: this handout is also on the web

© Faith A. Morrison, Michigan Tech U.

---

heat transfer?

What do we do to understand complex ~~flows?~~

Same strategy as:

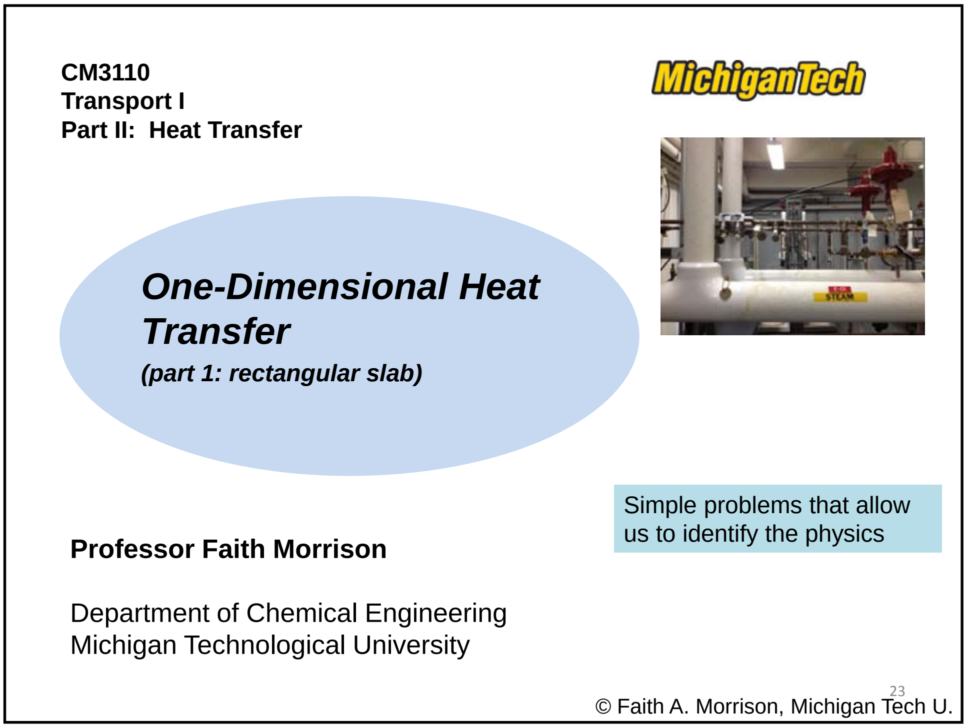
- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles
- **Boundary Layers**

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

Solve Real Problems.

Powerful.

© Faith A. Morrison, Michigan Tech U.



CM3110
Transport I
Part II: Heat Transfer

MichiganTech

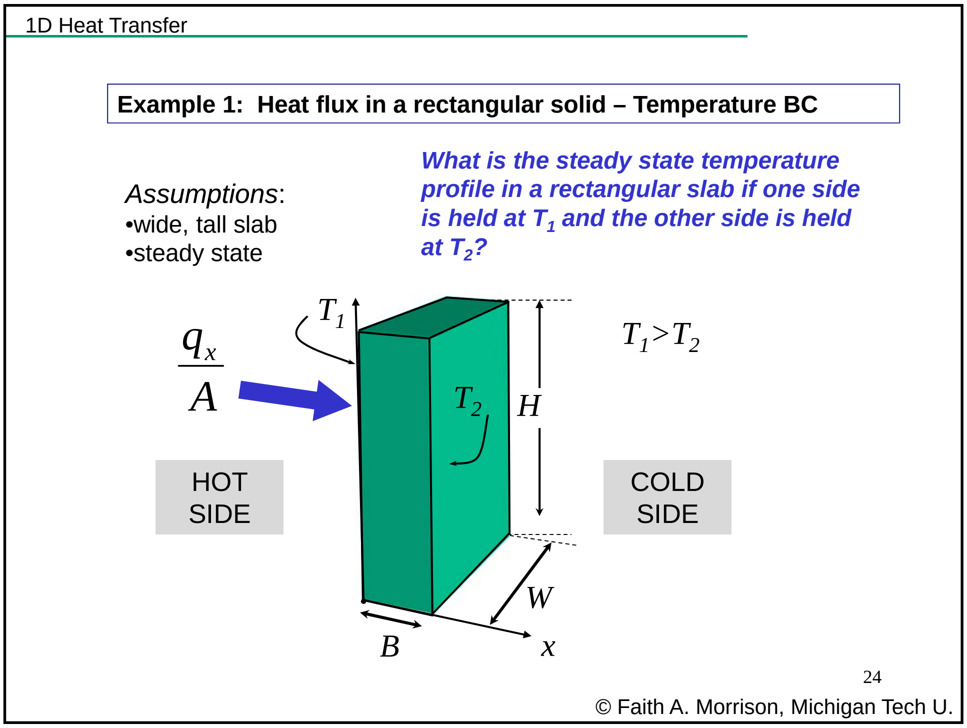
One-Dimensional Heat Transfer

(part 1: rectangular slab)

Simple problems that allow us to identify the physics

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

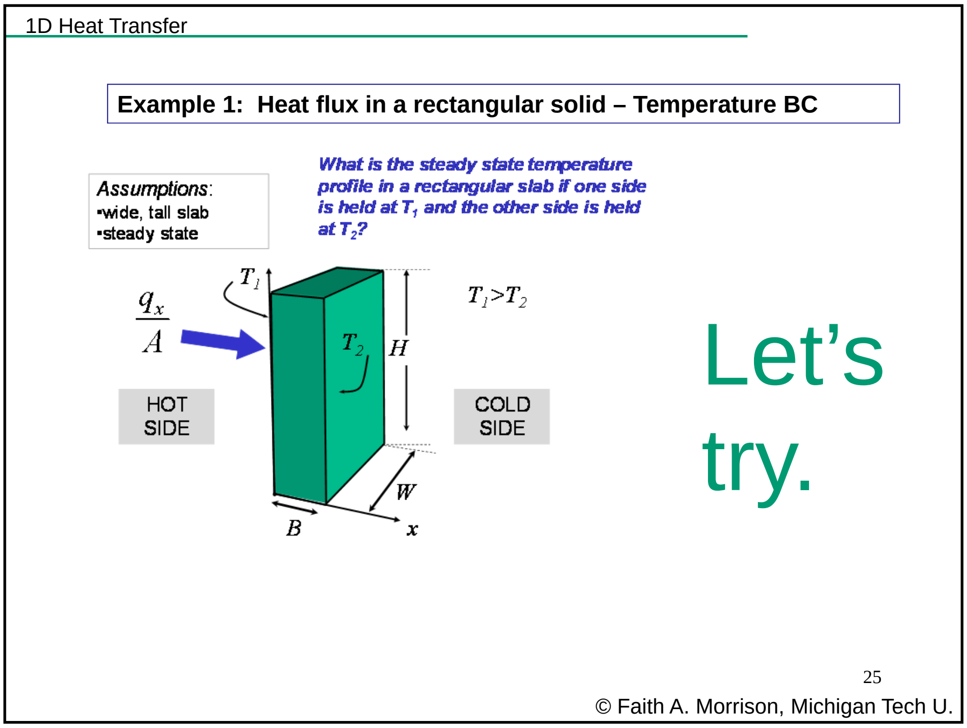


Example 1: Heat flux in a rectangular solid – Temperature BC

Assumptions:
- wide, tall slab
- steady state

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

$T_1 > T_2$

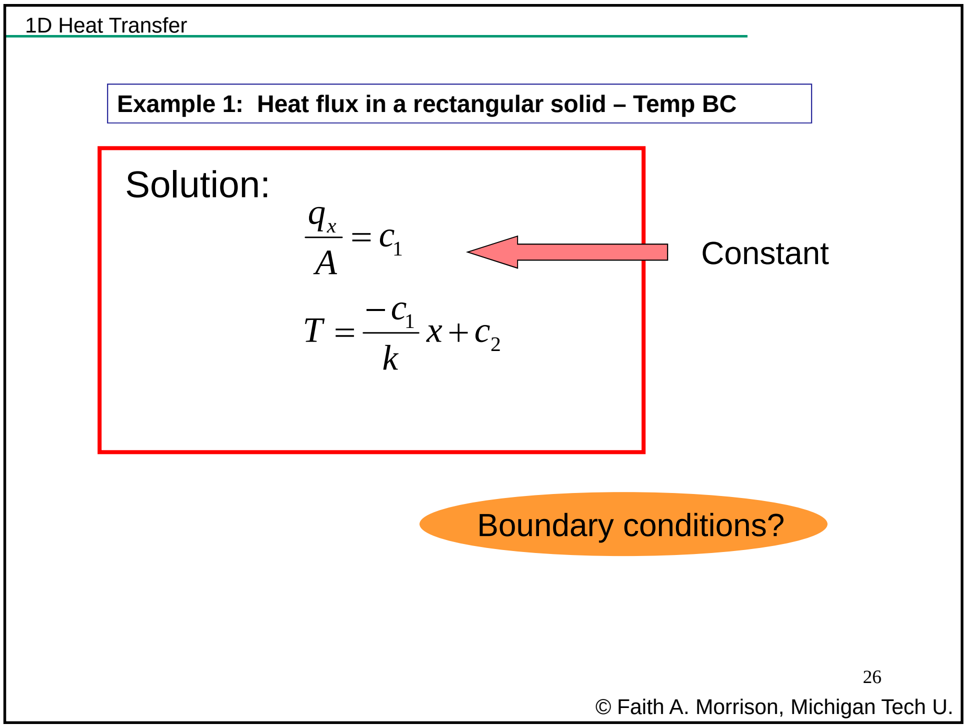


Example 1: Heat flux in a rectangular solid – Temperature BC

Assumptions:
- wide, tall slab
- steady state

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Let's try.



Example 1: Heat flux in a rectangular solid – Temp BC

Solution:

$$\frac{q_x}{A} = c_1$$

Constant

$$T = \frac{-c_1}{k}x + c_2$$

Boundary conditions?

Example 1: Heat flux in a rectangular solid – Temp BC

Solution:

$$\frac{q_x}{A} = -k\left(\frac{T_2 - T_1}{B}\right)$$

$$T = \left(\frac{T_2 - T_1}{B}\right)x + T_1$$

Flux is constant, and depends on *k*

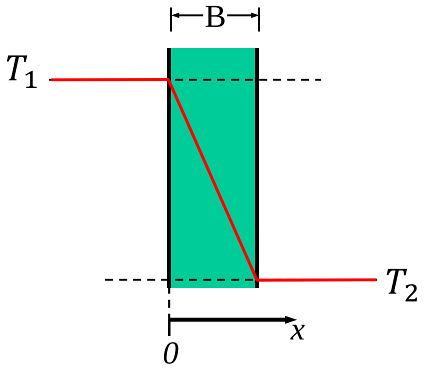
Temp. profile varies linearly, and **does not** depend on *k*

Example 1: Heat flux in a rectangular solid – Temp BC

SOLUTION:

$$T = \frac{(T_2 - T_1)}{B}x + T_1$$

$$\frac{q_x}{A} = -k\frac{(T_2 - T_1)}{B}$$

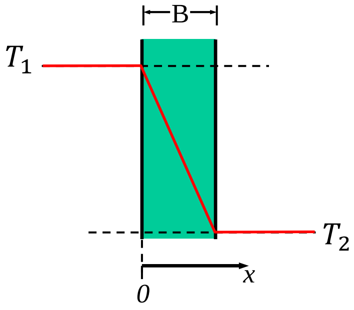


1D Heat Transfer

Example 1: Heat flux in a slab

Using the solution (conceptual):

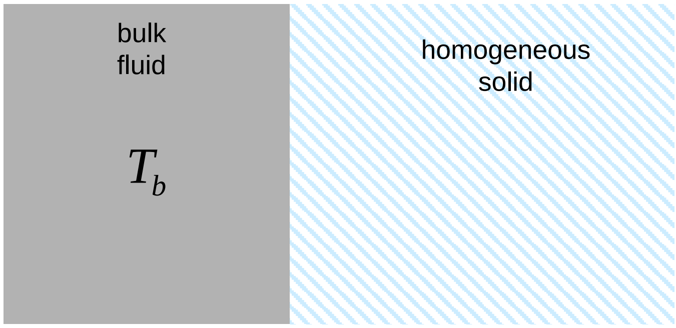
For heat conduction in a slab with temperature boundary conditions, we sketched the solution as shown. **If the thermal conductivity k of the slab became larger**, how would the sketch change? What are the predictions for $T(x)$ and the flux for this case?



What about this case?

Example 2: Heat flux in a rectangular solid – *Fluid* BC

What is the steady state temperature profile in a wide rectangular slab if one side is exposed to fluid at T_b?



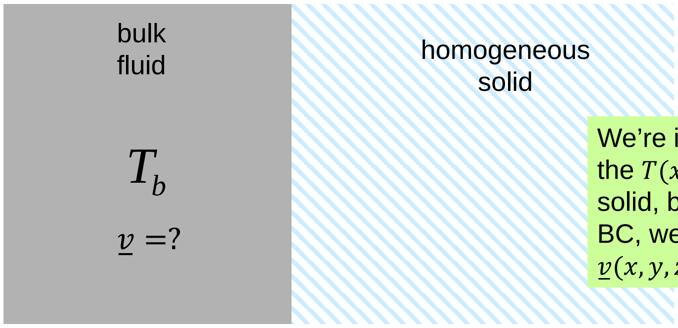
$$T_b \neq T_{wall}$$

What is the flux at the wall?

What about this case?

Example 2: Heat flux in a rectangular solid – *Fluid* BC

What is the steady state temperature profile in a wide rectangular slab if one side is exposed to fluid at T_b?



We're interested in the $T(x)$ profile in the solid, but to know the BC, we need to know $\underline{v}(x,y,z)$ in the fluid.

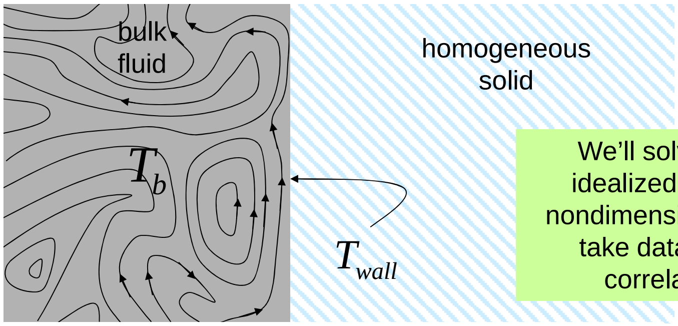
$$T_b \neq T_{wall}$$

What is the flux at the wall?

An Important Boundary Condition in Heat Transfer: Newton's Law of Cooling

We want an easier way to handle this common situation.

The fluid is **in motion**

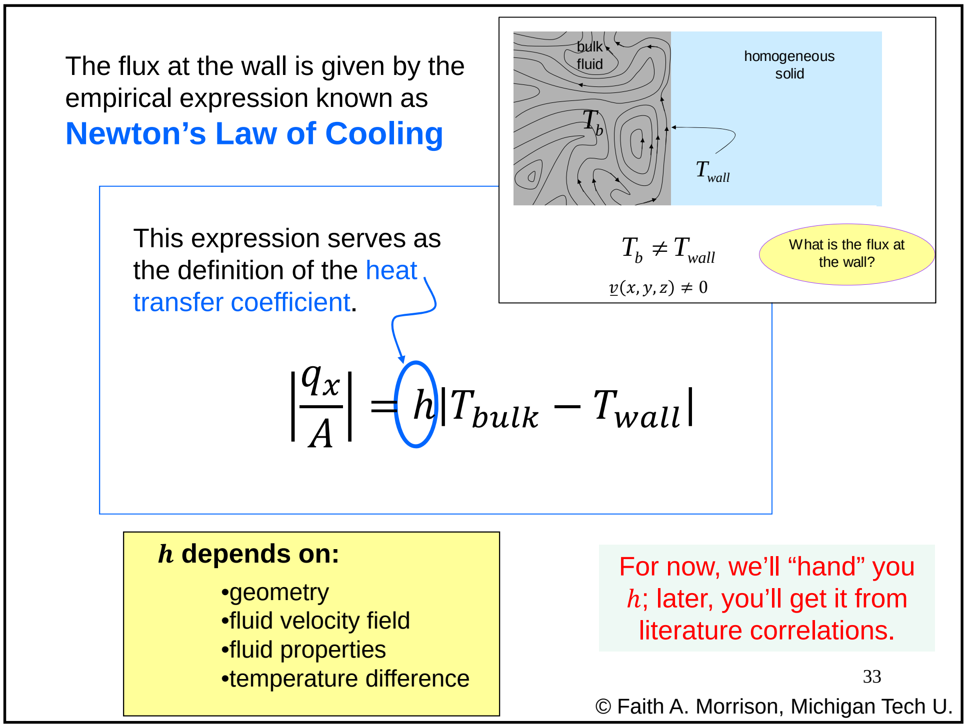


We'll solve an idealized case, nondimensionalize, take data and correlate!

$$T_b \neq T_{wall}$$

$$\underline{v}(x,y,z) \neq 0$$

What is the flux at the wall?



The flux at the wall is given by the empirical expression known as

Newton's Law of Cooling

$T_b \neq T_{wall}$

$\underline{v}(x,y,z) \neq 0$

What is the flux at the wall?

This expression serves as the definition of the heat transfer coefficient.

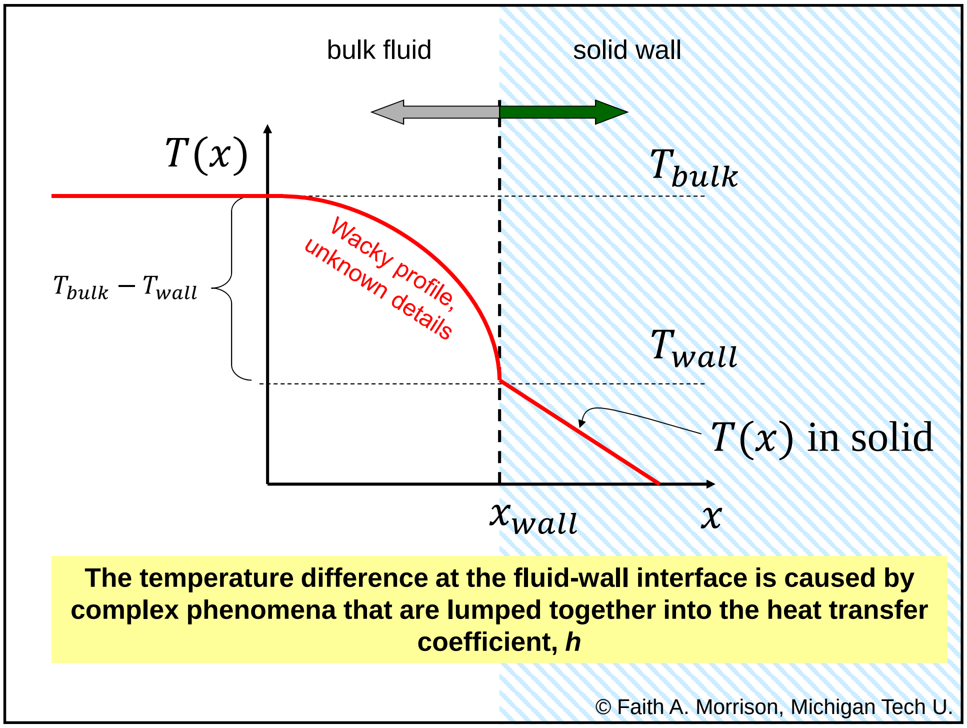
$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

***h* depends on:**

- geometry
- fluid velocity field
- fluid properties
- temperature difference

For now, we'll "hand" you h; later, you'll get it from literature correlations.

33



The temperature difference at the fluid-wall interface is caused by complex phenomena that are lumped together into the heat transfer coefficient, *h*

How do we handle the absolute value signs?

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

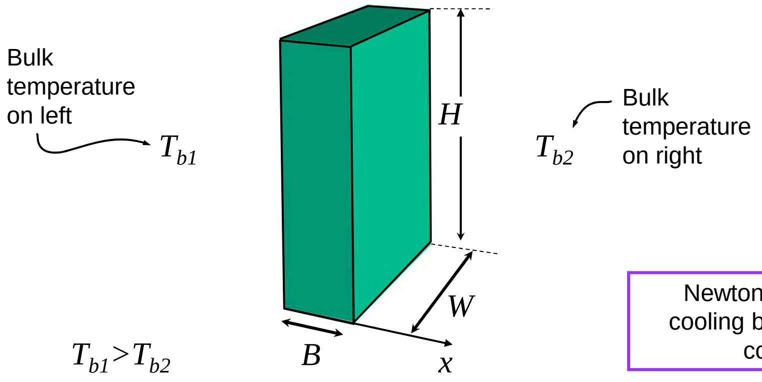
- Heat flows from hot to cold
- The coordinate system determines if the flux is positive or negative

---

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Assumptions:
- wide, tall slab
- steady state
- *h_1 and h_2 are the heat transfer coefficients of the left and right walls*

What is the steady state temperature profile in a rectangular slab if the __fluid on one side__ is held at T_{b1} and the __fluid on the other side__ is held at T_{b2}?



Newton's law of cooling boundary conditions

Problem-Solving Procedure – microscopic heat-transfer problems

1. sketch system
2. choose coordinate system
3. Apply the microscopic energy balance
4. solve the differential equation for temperature profile
5. apply boundary conditions
6. Calculate the flux from Fourier's law

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Solution:

$$\frac{q_x}{A} = c_1$$

Constant

$$T = \frac{-c_1}{k}x + c_2$$

Boundary conditions?

1D Heat Transfer

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

This is the same as Example 1, EXCEPT there are different boundary conditions.

With Newton's law of cooling boundary condition, we know the flux at the boundary in terms of the heat transfer coefficient, *h*:

$$\left.\frac{q_x}{A}\right|_{x=0} = h_1\left(T_{b1} - T_{w1}\right) > 0$$

$$\left.\frac{q_x}{A}\right|_{x=B} = h_2\left(T_{w2} - T_{b2}\right) > 0$$

The flux is **positive** (heat flows in the +*x*-direction)

but, we do not know these temps

1D Heat Transfer

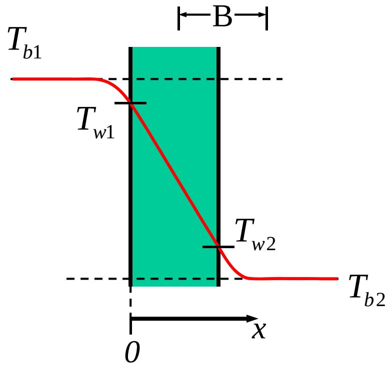
How do we apply these boundary conditions?

Soln from Example 1:

$$\frac{q_x}{A} = c_1$$

$$T = \frac{-c_1}{k}x + c_2$$

2 unknown constants to solve for: c_1, c_2.



We can eliminate the wall temps from the two equations for the BC by using the solution for *T(x)*.

then solve for c_1, c_2. (2 eqns, 2 unknowns)

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

After some algebra,

$$c_1 = \frac{\left(T_{b1} - T_{b2}\right)}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

$$c_2 = \frac{T_{b1}\left(\frac{1}{h_2} + \frac{B}{k}\right) + \frac{1}{h_1} T_{b2}}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

Substituting back into the solution, we obtain the final result.

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Solution: **(temp profile, flux)**

Temperature profile:

(linear)

$$\frac{T_{b1} - T}{T_{b1} - T_{b2}} = \frac{\frac{x}{k} + \frac{1}{h_1}}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

Flux:

(constant)

$$\frac{q_x}{A} = \frac{T_{b1} - T_{b2}}{\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}}$$

Rectangular slab with Newton's law of cooling BCs

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Solution: (temp profile, flux)

Temperature profile:

(linear)

$$\frac{T_{b1}-T}{T_{b1}-T_{b2}}=\frac{\frac{x}{k}+\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}$$

$$\boxed{T=T_{b1}-\left(\frac{(T_{b1}-T_{b2})\frac{1}{k}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}\right)x+\left(\frac{(T_{b1}-T_{b2})\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}\right)}$$

Rectangular slab with Newton's law of cooling BCs

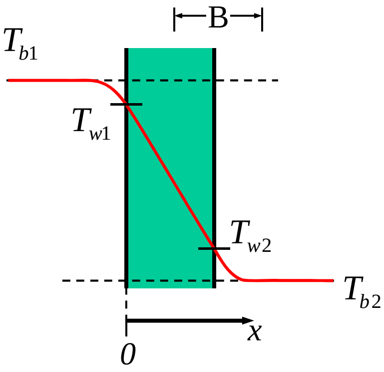
Using the solution (with numbers):

What is the temperature in the middle of a slab (thickness = B, thermal conductivity= $26\ BTU/\ h\ ft^oF$ if the left side is exposed to a fluid of temperature 120^oF and the right side is exposed to a fluid of temperature 50^oF? The heat transfer coefficients at the two faces are the same and are equal to $2.0\ BTU/h\ ft^2\ {}^oF$.

Example 4: Heat flux in a slab

Using the solution (conceptual):

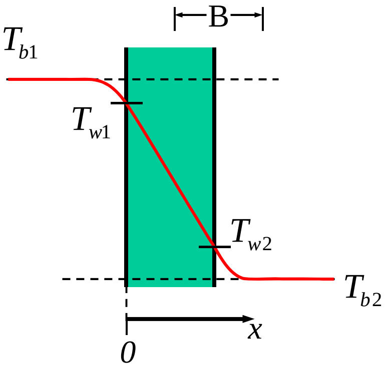
For heat conduction in a slab with Newton's law of cooling boundary conditions, we sketched the solution as shown. **If the heat transfer coefficients became infinitely large**, (no change in bulk temperatures) how would the sketch change? What are the predictions for ***T(x)*** and the **flux** for this case?



Example 4: Heat flux in a slab

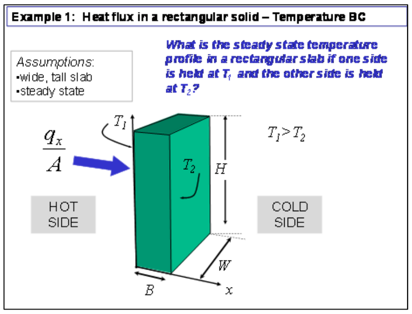
Using the solution (conceptual):

For heat conduction in a slab with Newton's law of cooling boundary conditions, we sketched the solution as shown. **If only the heat transfer coefficient on the right side became infinitely large**, (no change in bulk temperatures) how would the sketch change? What are the predictions for ***T(x)*** and the **flux** for this case?



Heat transfer to:

- ✓ Slab
-
-



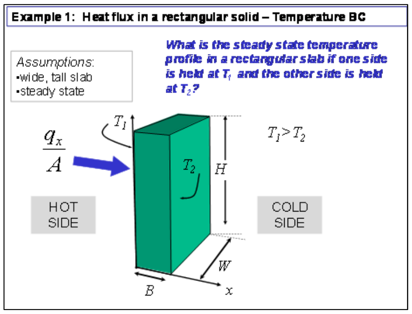
Example 1: Heat flux in a rectangular solid – Temperature BC

Assumptions:
- wide, tall slab
- steady state

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Heat transfer to:

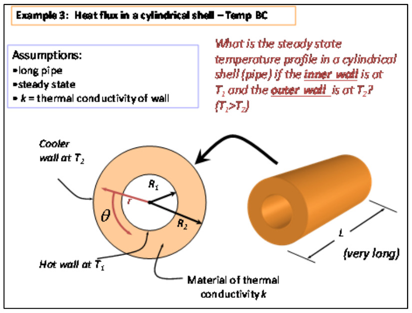
- ✓ Slab
- Cylindrical Shell
-



Example 1: Heat flux in a rectangular solid – Temperature BC

Assumptions:
- wide, tall slab
- steady state

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?



Example 3: Heat flux in a cylindrical shell – Temp BC

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall

What is the steady state temperature profile in a cylindrical shell (pipe) if the inner wall is at T_1 and the outer wall is at T_2? ($T_1 > T_2$)