CM3110
Transport I
Part II: Heat Transfer

Complex Heat Transfer – Dimensional Analysis

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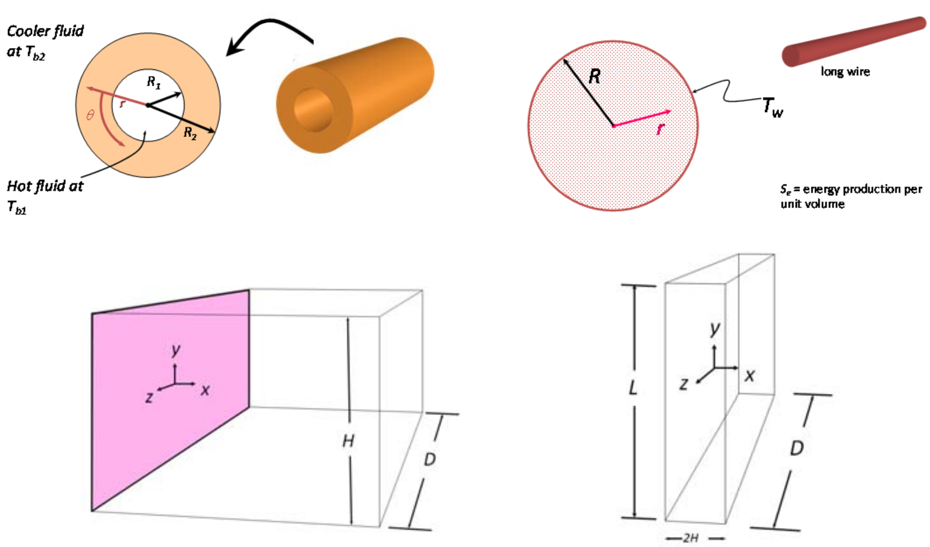
(what have we been up to?)

Examples of (simple, 1D) Heat Conduction



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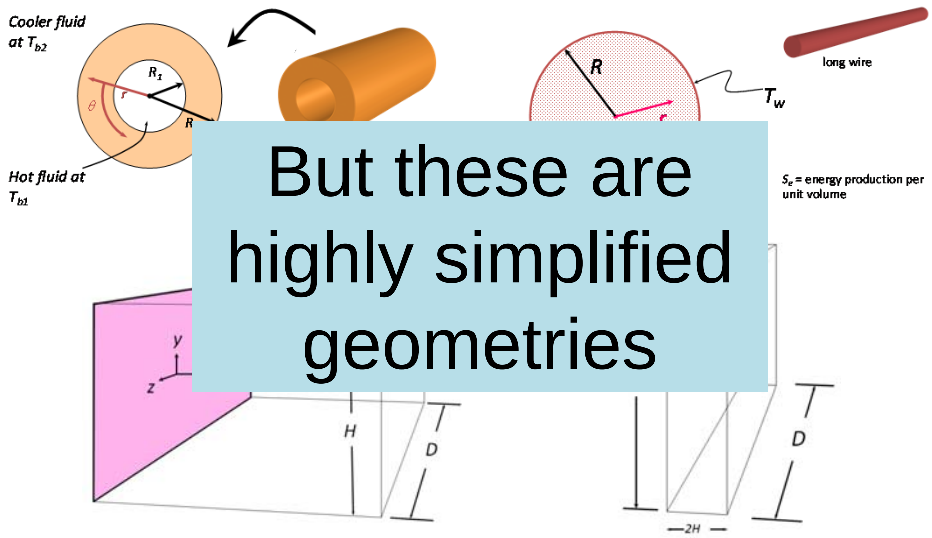
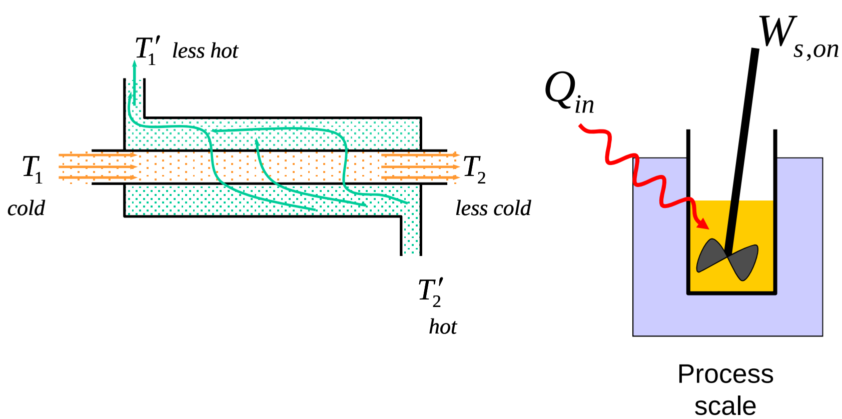
Examples of (simple, 1D) Heat Conduction



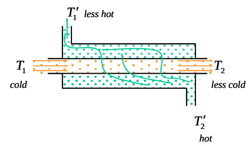
But these are highly simplified geometries

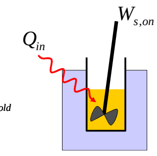
How do we handle complex geometries, complex flows, complex machinery?



Complex Heat Transfer – Dimensional Analysis

(***Answer***: *Use the same techniques we have been using in fluid mechanics)*





Engineering Modeling

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis thus far:

- Flow in pipes at all flow rates (laminar and turbulent)
  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\mathrm{Re}, L/D)$
- Rough pipes — ***Solution:*** add additional length scale; then nondimensionalize
- Non-circular conduits — ***Solution:*** Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes
  Solution: Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\mathrm{Re})$
- Boundary layers — ***Solution:*** Two components of velocity need independent lengthscales

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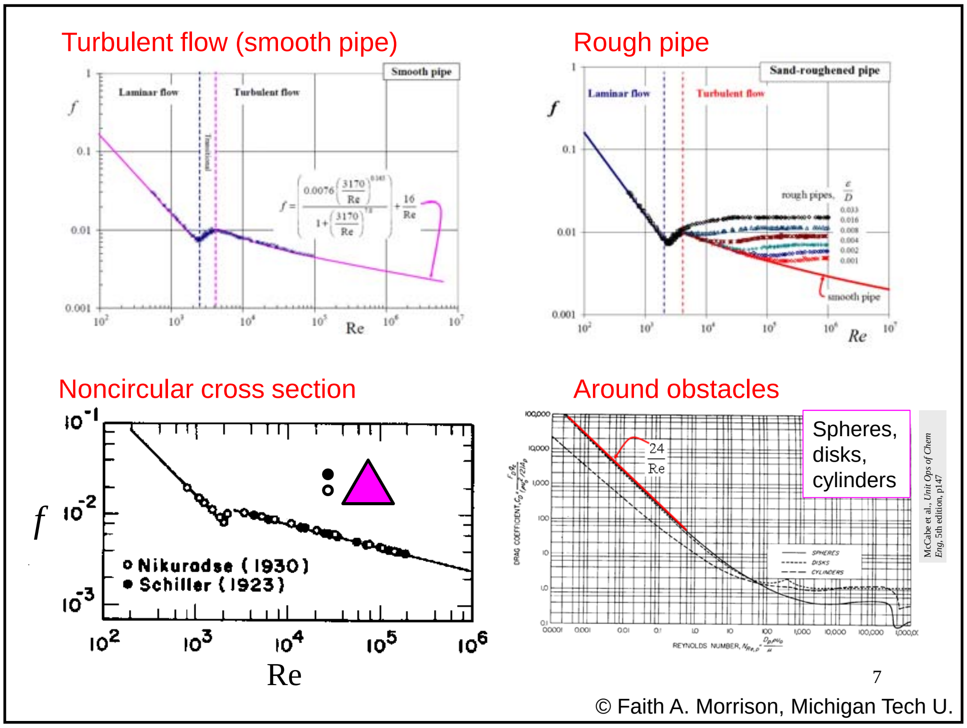
Turbulent flow (smooth pipe)
Rough pipe
Noncircular cross section
Around obstacles
Smooth pipe
Laminar flow
Turbulent flow
Sand-roughened pipe
rough pipes, ε/D
smooth pipe
Re
f
Nikuradse (1930)
Schiller (1923)
24/Re
Spheres, disks, cylinders
McCabe et al., Unit Ops of Chem Eng, 5th edition, p147
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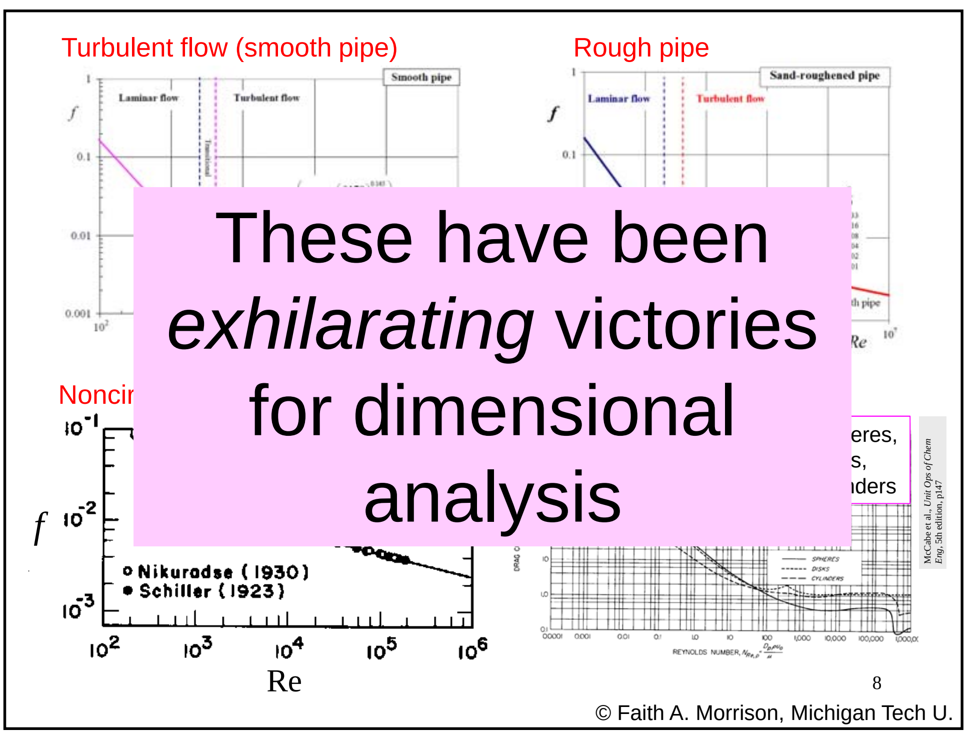
Turbulent flow (smooth pipe)
Rough pipe
These have been *exhilarating* victories for dimensional analysis
Nikuradse (1930)
Schiller (1923)
Re
f
McCabe et al., Unit Ops of Chem Eng, 5th edition, p147
8
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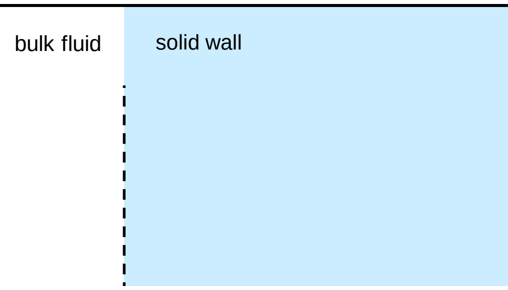
Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

- Forced convection heat transfer from fluid to wall
  Solution: ?
- Natural convection heat transfer from fluid to wall
  Solution: ?
- Radiation heat transfer from solid to fluid
  Solution: ?



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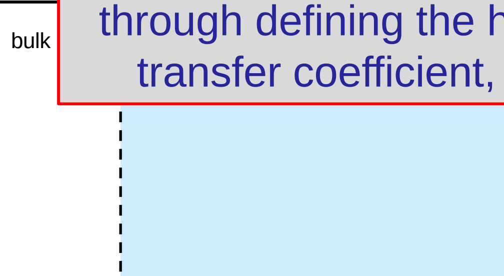
Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

- Forced convection heat transfer from fluid to wall
  Solution: ?
- Natural convection hea
  Solut
- Radiation heat transfer
  Solut

We have already started using the <u>results</u>/techniques of dimensional analysis through defining the heat transfer coefficient, h



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Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

- Forced convection heat transfer from fluid to wall

  Solution: ?

- Natural convection hea

  Solut

- Radiation heat transfer

  Solut

bulk

We have already started using the results/techniques of dimensional analysis through defining the heat transfer coefficient, h

(recall that we did this in fluids too: we used the $f(\mathrm{Re})$ correlation (Moody chart) long before we knew where that all came from)

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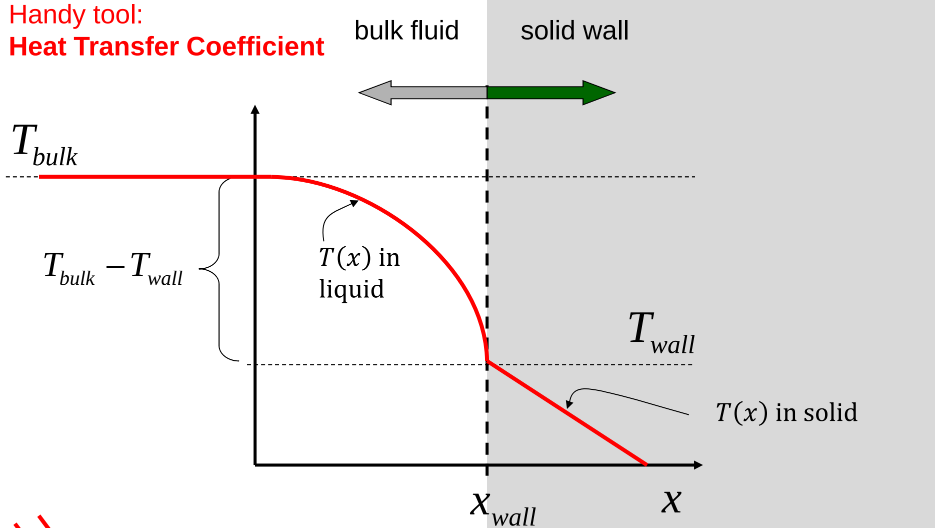
Handy tool:

Heat Transfer Coefficient



The temperature variation near-wall region is caused by complex phenomena that are lumped together into the heat transfer coefficient, *h*

Complex Heat Transfer – Dimensional Analysis

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| \equiv h \left|T_{bulk} - T_{wall}\right|$$

h **depends on:**

- geometry
- fluid velocity
- fluid properties
- temperature difference

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Complex Heat Transfer – Dimensional Analysis

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| \equiv h \left|T_{bulk} - T_{wall}\right|$$

To get values of h for various situations, we need to measure data and create data correlations (**dimensional analysis**)

h **depends on:**

- geometry
- fluid velocity
- fluid properties
- temperature difference

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Complex Heat Transfer – Dimensional Analysis

Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall

Solution: ?

•Natural convection heat transfer from fluid to wall

Solution: ?

•Radiation heat transfer from solid to fluid

Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

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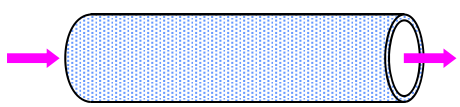
Complex Heat Transfer – Dimensional Analysis

Chosen problem: Forced Convection Heat Transfer
Solution: Dimensional Analysis



Following procedure familiar from pipe flow,

- **What are governing equations?**
- **Scale factors (dimensionless numbers)?**
- **Quantity of interest?**

Answer: Heat flux at the wall

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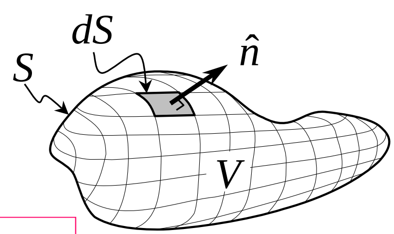
Complex Heat Transfer – Dimensional Analysis

General Energy Transport Equation

(microscopic energy balance)

As for the derivation of the microscopic momentum balance, the microscopic energy balance is derived on an arbitrary volume, V, enclosed by a surface, S.



Gibbs notation:

$$\boxed{\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S}$$

see handout for component notation

Complex Heat Transfer – Dimensional Analysis

General Energy Transport Equation

(microscopic energy balance; **in the fluid**)

$$\rho \hat{C}_p \left(\underbrace{\frac{\partial T}{\partial t}}_{\text{rate of change}} + \overbrace{\underline{v} \cdot \nabla T}^{\text{convection}} \right) = \underbrace{k \nabla^2 T}_{\text{conduction (all directions)}} + \overbrace{S}^{\text{source}}$$

(energy generated per unit volume per time)

fluid velocity must satisfy equation of motion, equation of continuity

see handout for component notation

Equation of energy

Equation of energy for Newtonian fluids of constant density, ρ, and thermal conductivity, k, with source term (source could be viscous dissipation, electrical energy, chemical energy, etc., with units of energy/(volume time)).

Source: R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Processes*, Wiley, NY, 1960, page 319.

Note: this handout is on the web:
www.chem.mtu.edu/~fmorriso/cm310/energy2013.pdf

Gibbs notation (vector notation)

$$\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=\frac{k}{\rho\hat{C}_p}\nabla^2 T+\frac{S}{\rho\hat{C}_p}$$

Cartesian (xyz) coordinates:

$$\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}=\frac{k}{\rho\hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{S}{\rho\hat{C}_p}$$

Cylindrical (rθz) coordinates:

$$\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}=\frac{k}{\rho\hat{C}_p}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{S}{\rho\hat{C}_p}$$

Spherical (rθϕ) coordinates:

$$\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}=\frac{k}{\rho\hat{C}_p}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2}\right.$$

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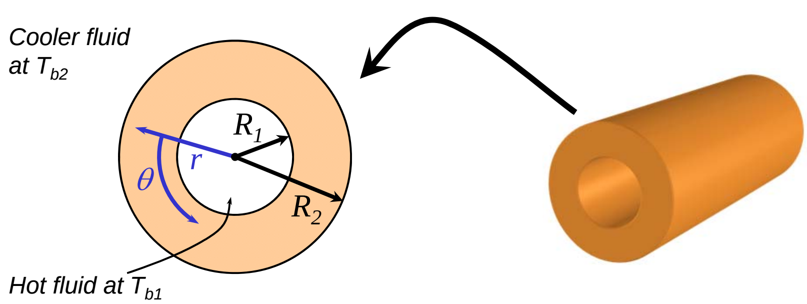
** REVIEW ** REVIEW **

Forced-convection heat transfer

Example: Heat flux in a cylindrical shell

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **fluid on the inside** is at T_{b1} and the **fluid on the outside** is at T_{b2}? ($T_{b1}>T_{b2}$)*



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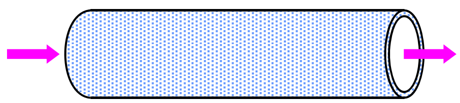
Now: How do develop correlations for *h*?

Consider: Heat-transfer to from flowing fluid inside of a tube – forced-convection heat transfer



T_1= core bulk temperature
T_o= wall temperature
$T(r,\theta,z)$ = temp distribution in the fluid

In principle, with the right math/computer tools, we could calculate the complete temperature and velocity profiles in the moving fluid.

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Complex Heat Transfer – Dimensional Analysis

What are governing equations?

Microscopic energy balance plus Navier-Stokes, continuity

Scale factors?

Re, Fr, L/D plus whatever comes from the rest of the analysis

Quantity of interest (like wall force, drag)?

Heat transfer coefficient

The quantity of interest in forced-convection heat transfer is *h*

How is the heat transfer coefficient related to the full solution for $T(r,\theta,z)$ in the fluid?

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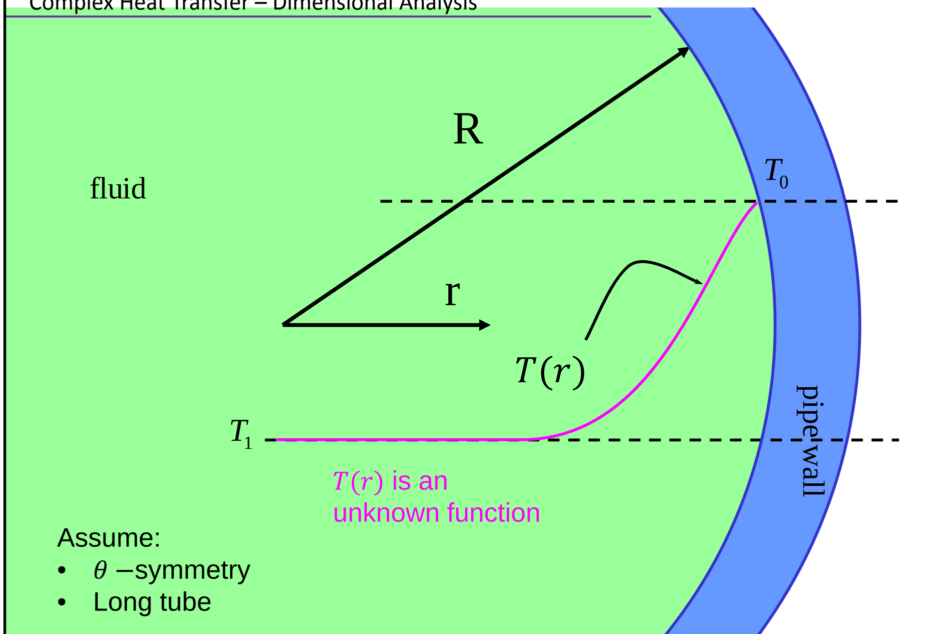
Complex Heat Transfer – Dimensional Analysis



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Complex Heat Transfer – Dimensional Analysis

At the boundary, (Newton's Law of Cooling is the **boundary condition**)

Total heat flow through (at) the wall in terms of *h*

$$\left|\frac{q_r}{A}\right| = h|T_1 - T_0|$$

$$Q = (2\pi RL)(h)(T_1 - T_0)$$

We can calculate the total heat transferred from $T(r)$ in the fluid:

Total heat conducted to the wall from the fluid

$$Q = \iint_S \left[\hat{n} \cdot \underline{\tilde{q}}\right]_{surface} dS$$

$$\tilde{q} = \frac{q_r}{A} = -k\frac{\partial T}{\partial r}$$

We need $T(r)$ ***in the fluid***

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Complex Heat Transfer – Dimensional Analysis

Equate these two: **Total** heat flow through the wall

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S \left[\hat{e}_r \cdot \underline{\tilde{q}}\right]_{surface} dS$$

$$\underbrace{(2\pi RL)(h)(T_1 - T_0)}_{\textbf{Total}\text{ heat flow at the wall in terms of } h} = \boxed{Q} = \underbrace{\int_0^{2\pi}\int_0^L -k\left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta}_{\textbf{Total}\text{ heat conducted to the wall from the fluid}}$$

Complex Heat Transfer – Dimensional Analysis

Equate these two: **Total** heat flow through the wall

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S \left[\hat{e}_r \cdot \underline{\tilde{q}}\right]_{surface} dS$$

$$(2\pi RL)(h)(T_1 - T_0) = \boxed{Q} = \int_0^{2\pi}\int_0^L -k\left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta$$

Now, non-dimensionalize this expression

Complex Heat Transfer – Dimensional Analysis

Non-dimensionalize

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$$

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Complex Heat Transfer – Dimensional Analysis

$$h(\pi \cancel{D} L)\cancel{(T_1 - T_o)} = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{(T_1 - T_o)}}{\cancel{D}} \frac{\cancel{D^2}}{2} dz^* d\theta$$

$$2\pi \underbrace{\left(\frac{hD}{k}\right)}\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$\boxed{Nu = Nu\left(T^*, \frac{L}{D}\right)}$$

one additional dimensionless group

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Complex Heat Transfer – Dimensional Analysis

$$h(\pi \cancel{D} L)\cancel{(T_1 - T_o)} = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{(T_1 - T_o)}}{\cancel{D}} \frac{\cancel{D^2}}{2} dz^* d\theta$$

$$2\pi \underbrace{\left(\frac{hD}{k}\right)}\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} - \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

This is a function of Re through fluid $\underline{v}$ distribution

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$\boxed{Nu = Nu\left(T^*, \frac{L}{D}\right)}$$

one additional dimensionless group

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Complex Heat Transfer – Dimensional Analysis

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^* \frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^* \frac{\partial T^*}{\partial z^*}\right) = \frac{1}{\mathrm{Pe}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\boxed{\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr} g^*}$$

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Non-dimensional Continuity Equation

$$\boxed{\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0}$$

Quantity of interest

$$\boxed{Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta}$$

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Complex Heat Transfer – Dimensional Analysis

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ three dimensionless groups:

no free surfaces

$$\boxed{Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)}$$

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Now, do the experiments.

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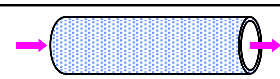
Complex Heat Transfer – Dimensional Analysis



Now, do the experiments.

Forced Convection Heat Transfer

- Build apparatus (*several* actually, with different D, L)
- Run fluid through the inside (at different $\underline{v}$; for different fluids $\rho, \mu, \hat{C}_p, k$)
- Measure T_{bulk} on inside; T_{wall} on inside
- Measure rate of heat transfer, Q
- Calculate h: $|Q| = hA|T_{bulk} - T_{wall}|$
- Report h values in terms of dimensionless correlation:

$$\mathrm{Nu} = \frac{hD}{k} = f\left(\mathrm{Re}, \mathrm{Pr}, \frac{L}{D}\right)$$

It should only be a function of these dimensionless numbers (***if*** our Dimensional Analysis is correct…..)

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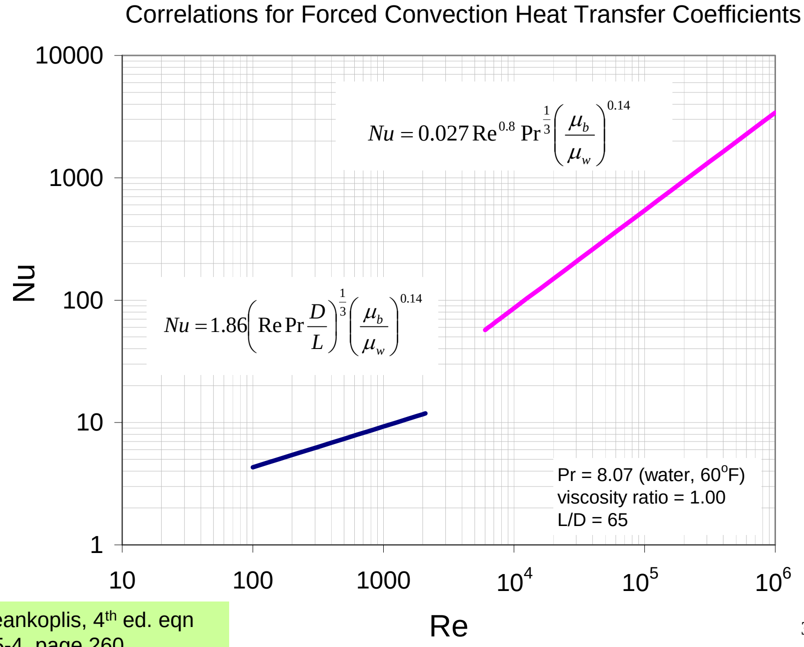
Complex Heat Transfer – Dimensional Analysis



Geankoplis, 4th ed. eqn 4.5-4, page 260

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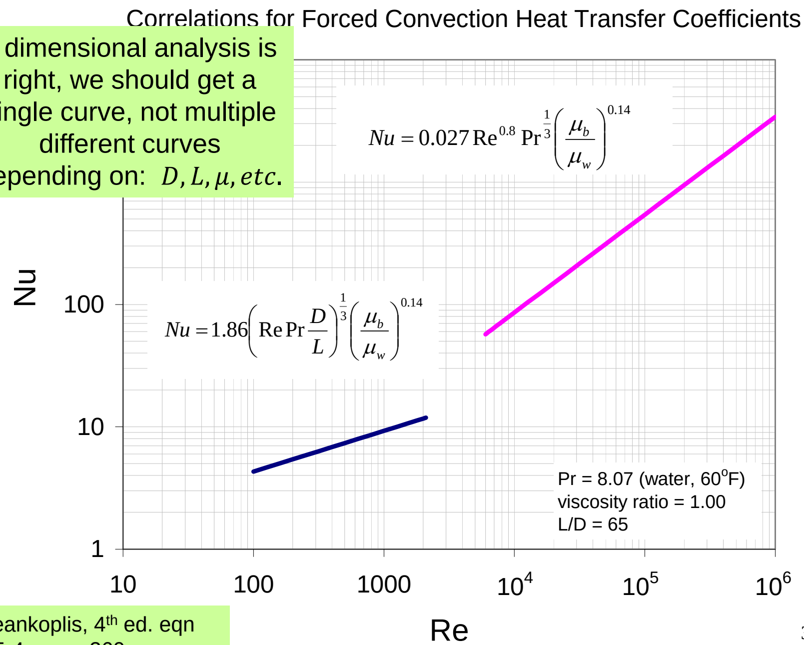
Complex Heat Transfer – Dimensional Analysis

If dimensional analysis is right, we should get a single curve, not multiple different curves depending on: $D, L, \mu, etc.$



Geankoplis, 4th ed. eqn 4.5-4, page 260

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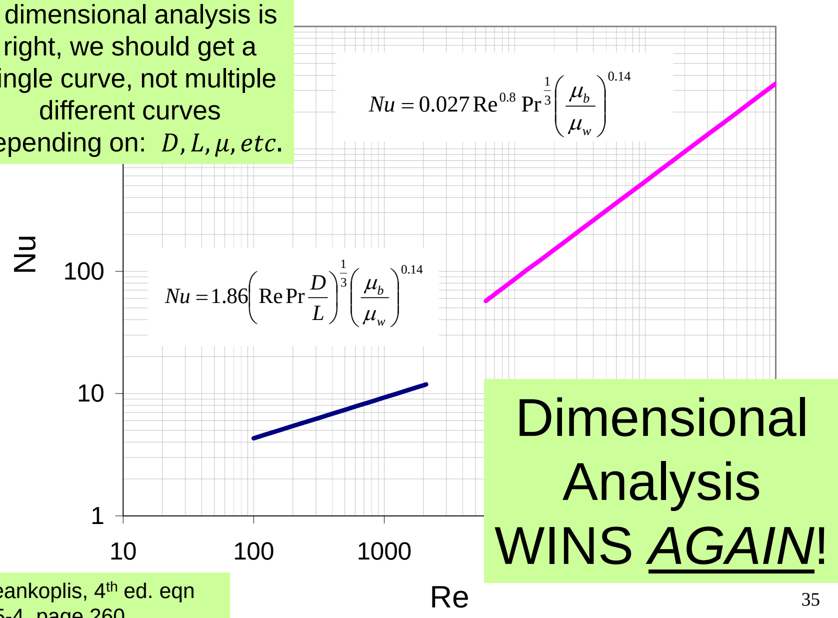
Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients

If dimensional analysis is right, we should get a single curve, not multiple different curves depending on: $D, L, \mu, etc.$



Dimensional Analysis WINS ***AGAIN***!

Geankoplis, 4th ed. eqn 4.5-4, page 260

Complex Heat Transfer – Dimensional Analysis

Heat Transfer in Laminar flow in pipes:

data correlation for *forced convection heat transfer coefficients*

$$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

the subscript "*a*" refers to *the type of average temperature* used in calculating the heat flow, *q*

$$q = h_a A \Delta T_a$$

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

Geankoplis, 4th ed. eqn 4.5-4, page 260

$Re < 2100$, $(RePr\frac{D}{L}) > 100$, horizontal pipes; all physical properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the (constant) wall temperature.

Complex Heat Transfer – Dimensional Analysis

Forced convection
Heat Transfer in Laminar flow in pipes

$$Nu_a = \frac{h_a D}{k} = 1.86 \left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Forced convection
Heat Transfer in Turbulent flow in pipes

$$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Physical Properties evaluated at:

$$\frac{T_{b,in} + T_{b,out}}{2}$$

May have to be estimated

$$\frac{T_{b,in} + T_{b,out}}{2}$$

bulk mean temperature

Fine print matters!

•all physical properties (except μ_w) evaluated at the **bulk mean temperature**
•Laminar or turbulent flow

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Complex Heat Transfer – Dimensional Analysis

Forced convection Heat Transfer in Laminar flow in pipes

? In our dimensional analysis, we assumed constant ρ, k, μ, etc. Therefore we did not predict a viscosity-temperature dependence. If viscosity is not assumed constant, the dimensionless group shown below is predicted to appear in correlations.

$$Nu_a = \frac{h_a D}{k} = 1.86 \left(\mathrm{Re\,Pr}\frac{D}{L}\right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

(reminiscent of pipe wall roughness; needed to modify dimensional analysis to correlate on roughness)

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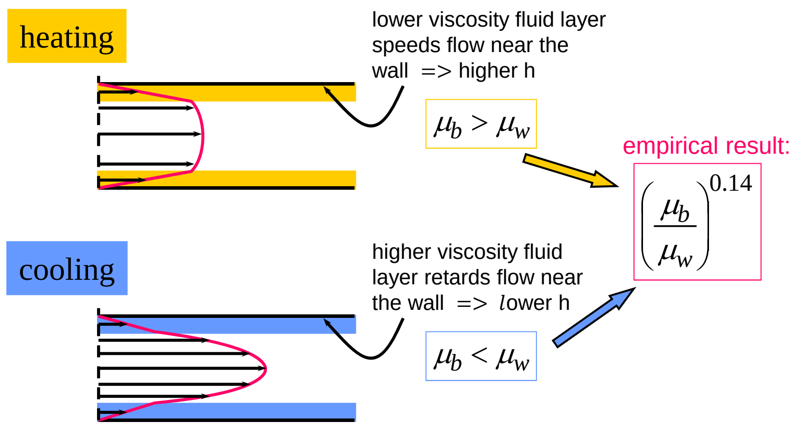
Complex Heat Transfer – Dimensional Analysis

Viscous fluids with large ΔT



ref: McCabe, Smith, Harriott, 5th ed, p339

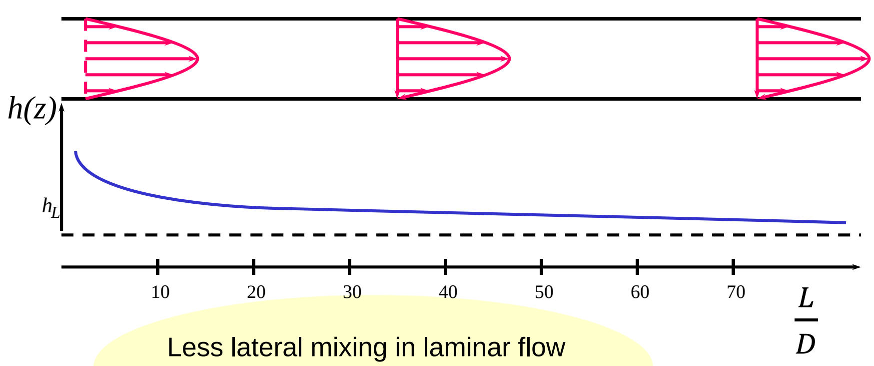
Complex Heat Transfer – Dimensional Analysis

Forced convection
Heat Transfer in Laminar flow in pipes

$$\mathrm{Nu}_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Why does $\frac{L}{D}$ appear in laminar flow correlations and not in the turbulent flow correlations?

LAMINAR



Less lateral mixing in laminar flow means more variation in $h(x)$.

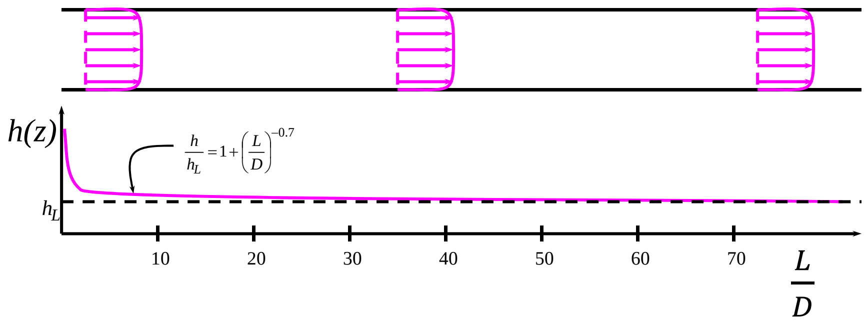
Complex Heat Transfer – Dimensional Analysis

Forced convection

Heat Transfer in Turbulent flow in pipes

$$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027 \mathrm{Re}^{0.8} \mathrm{Pr}^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

TURBULENT



In turbulent flow, good lateral mixing reduces the variation in h along the pipe length.

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Example of *partial* solution to Homework (bring to tests)

laminar flow in pipes	$Nu_a = \frac{h_a D}{k} = 1.86 \left(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L}\right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re<2100, (RePrD/L)>100, horizontal pipes, eqn 4.5-4, page 238; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.	(T_{bulk} mean)
turbulent flow in smooth tubes	$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re>6000, 0.7 <Pr <16,000, L/D>60 , eqn 4.5-8, page 239; all properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the wall temperature. The mean is the average of the inlet and outlet bulk temperatures; not valid for liquid metals.	
air at 1atm in turbulent flow in pipes	$h_{lm} = \frac{3.52 V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = \frac{0.5 V(ft/s)^{0.8}}{D(ft)^{0.2}}$	equation 4.5-9, page 239	
water in turbulent flow in pipes	$h_{lm} = 1429\left(1 + 0.0146 T\left({}^{\circ}C\right)\right)\frac{V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = 150\left(1 + 0.011 T\left({}^{\circ}F\right)\right)\frac{V(ft/s)^{0.8}}{D(ft)^{0.2}}$	4 < T(°C)<105, equation 4.5-10, page 239	

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Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

We started with a forced-convection pipe problem, did dimensional analysis, and found the dimensionless numbers.

To do a situation with different physics, we must start with a different starting problem.

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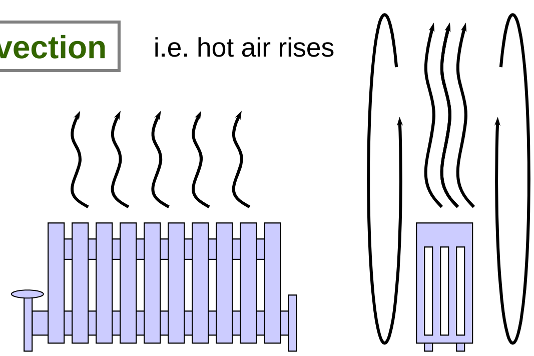
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



•heat moves from hot surface to cold air (fluid) by radiation and conduction
•increase in fluid temperature decreases fluid density
•recirculation flow begins
•**recirculation adds to the heat-transfer** from conduction and radiation

$\Rightarrow$ coupled heat and momentum transport

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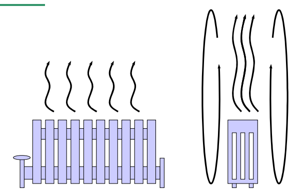
Complex Heat Transfer – Dimensional Analysis—Free Convection

Free Convection i.e. hot air rises



How can we solve **real** problems involving free (natural) convection?

We'll try this: Let's review how we approached solving real problems in *earlier* cases, i.e. in fluid mechanics, forced convection.

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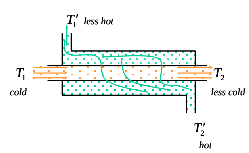
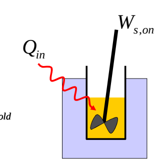
Complex Heat Transfer – Dimensional Analysis—Free Convection





Engineering Modeling

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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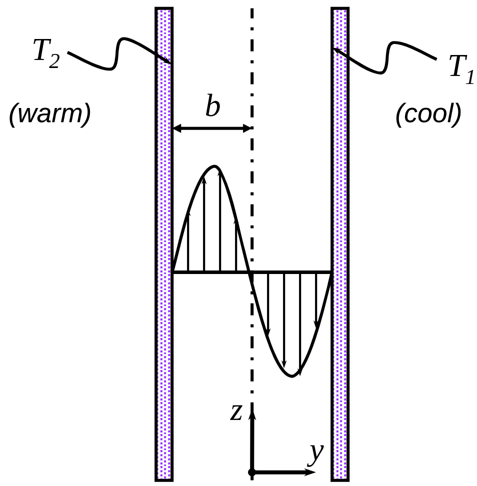
Complex Heat Transfer – Dimensional Analysis—Free Convection

Example: Free convection between long parallel plates *or* heat transfer through double-pane glass windows



$$T_2 > T_1$$

assumptions:

- long, wide slit
- steady state
- no source terms
- viscosity constant
- density varies with T

Calculate: $T, \underline{v}$ profiles

Complex Heat Transfer – Dimensional Analysis—Free Convection

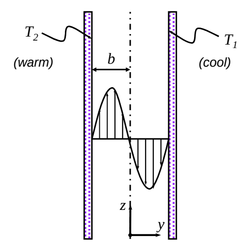
Example : Natural convection between vertical plates

Mass balance:

$$\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho + \rho(\nabla \cdot \underline{v}) = 0$$

Momentum balance:

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$



Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$

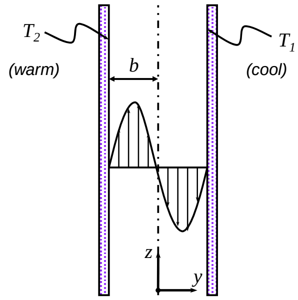
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

Complex Heat Transfer – Dimensional Analysis—Free Convection

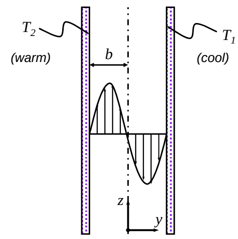
Example : Natural convection between vertical plates



$T_2 > T_1$

You try.

Complex Heat Transfer – Dimensional Analysis—Free Convection



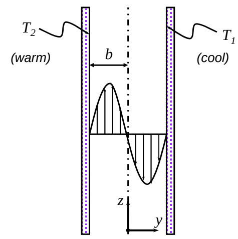
$T_2 > T_1$

Mass balance:

$$\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho + \rho(\nabla \cdot \underline{v}) = 0$$

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Complex Heat Transfer – Dimensional Analysis—Free Convection



$T_2 > T_1$

Mass balance:

$$\frac{\partial\rho}{\partial t} + \underline{v}\cdot\nabla\rho + \rho(\nabla\cdot\underline{v}) = 0$$

$$\cancel{\frac{\partial\rho}{\partial t}} + \left(\cancel{v_x}\frac{\partial\rho}{\partial x} + \cancel{v_y}\frac{\partial\rho}{\partial y} + v_z\frac{\partial\rho}{\partial z}\right) + \rho\left(\cancel{\frac{\partial v_x}{\partial x}} + \cancel{\frac{\partial v_y}{\partial y}} + \cancel{\frac{\partial v_z}{\partial z}}\right) = 0$$

steady $\underline{v} = v_z(y)\hat{e}_z$ tall, wide

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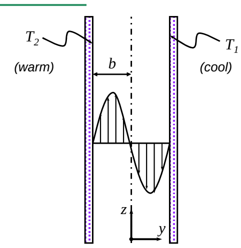
Complex Heat Transfer – Dimensional Analysis—Free Convection



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steady $\underline{v} = v_z(y)\hat{e}_z$

Conclusion: density must not vary with z.

tall, wide

$$\rho = \rho(\cancel{x}, y)$$

$$\rho = \rho(y)$$

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Complex Heat Transfer – Dimensional Analysis—Free Convection

Momentum balance:

$$\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla p+\mu\nabla^2\underline{v}+\rho\underline{g}$$

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}+\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}+\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

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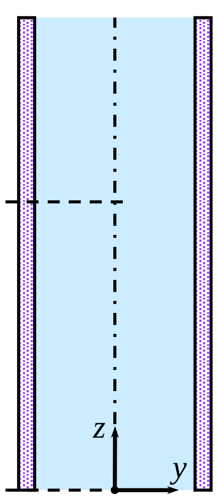
Complex Heat Transfer – Dimensional Analysis—Free Convection

Is Pressure a function of z?

YES, there should be hydrostatic pressure (due to weight of fluid)

"Pressure at the bottom of a column of fluid = pressure at top $+\rho g h$."

at z, $p = p(z)$



Let $p = p_0$ at $z = 0$

average density

$$p_0 = p(z) + \bar{\rho} g z$$

$$p(z) = p_0 - \bar{\rho} g z$$

$$\boxed{\Rightarrow \frac{dP}{dz} = -\bar{\rho} g}$$

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Complex Heat Transfer – Dimensional Analysis—Free Convection

To account for the temperature variation of ρ:

(look up the physics in the literature)

$$\boxed{\rho = \bar{\rho} - \bar{\rho}\bar{\beta}(T - \bar{T})}$$

$\bar{\rho} =$ mean density

$\bar{\beta} =$ volumetric coefficient of expansion at $\bar{T}$

$$\bar{T} = \frac{T_1 + T_2}{2}$$

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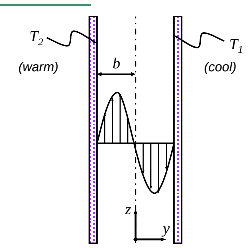
Complex Heat Transfer – Dimensional Analysis—Free Convection



$T_2 > T_1$

Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

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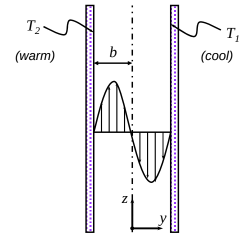
Complex Heat Transfer – Dimensional Analysis—Free Convection



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(solve) …

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

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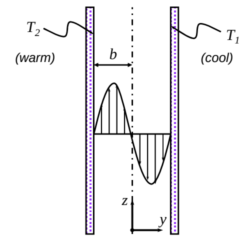
Complex Heat Transfer – Dimensional Analysis—Free Convection



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$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

(solve) …

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}(T - \bar{T})$$

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}\left(\frac{T_1 - T_2}{2b} y\right)$$

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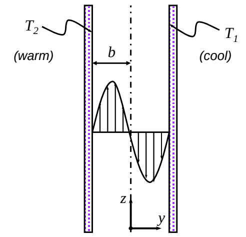
Complex Heat Transfer – Dimensional Analysis—Free Convection

Solve



$T_2 > T_1$

Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

...

$$T(y) = \frac{T_1 - T_2}{2b} y + \frac{T_1 + T_2}{2}$$

$$T(y) = \frac{T_1 - T_2}{2b} y + \bar{T}$$

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}(T - \bar{T})$$

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}\left(\frac{T_1 - T_2}{2b} y \right)$$

Complex Heat Transfer – Dimensional Analysis—Free Convection

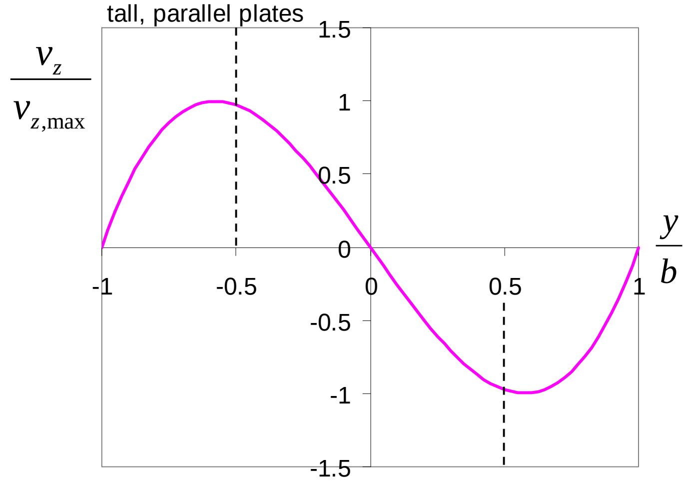
Final Result: (free convection between two slabs)

$$v_z(y) = \frac{\bar{\rho}\bar{\beta} g (T_2 - T_1) b^2}{12\mu} \left[\left(\frac{y}{b} \right)^3 - \left(\frac{y}{b} \right) \right]$$

(see next slide for plot)

Complex Heat Transfer – Dimensional Analysis—Free Convection

Velocity profile for free convection between two wide, tall, parallel plates



(Note that the temperature maxima are not centered)

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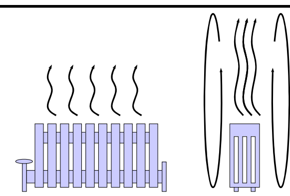
Free Convection i.e. hot air rises



Engineering Modeling

- ✓ •Choose an idealized problem and solve it
- •**From insight obtained from ideal problem, identify governing equations of real problem**
- •Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- •Design experiments to test modeling thus far
- •Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- •Design additional experiments
- •Iterate until useful correlations result

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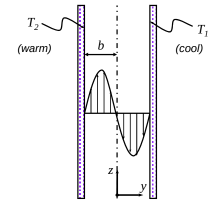
Free Convection

i.e. hot air rises



$T_2 > T_1$

Mass balance:

$$\frac{\partial \rho}{\partial t} + \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

Momentum balance:

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} + \right) = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

Energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

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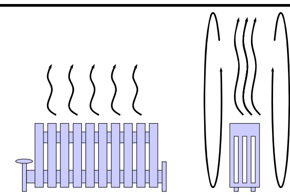
Free Convection

i.e. hot air rises



Engineering Modeling

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- **Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)**
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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Return to Dimensional Analysis…

Nondimensionalize the governing equations; deduce dimensionless scale factors

To nondimensionalized the Navier-Stokes for free convection problems, we follow the simple problem we just completed: $\rho = \rho(T), \langle v_z \rangle = 0$.

density not constant

driving the flow

$$\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla P+\mu\nabla^2\underline{v}+\rho\underline{g}$$

there was a trick for this

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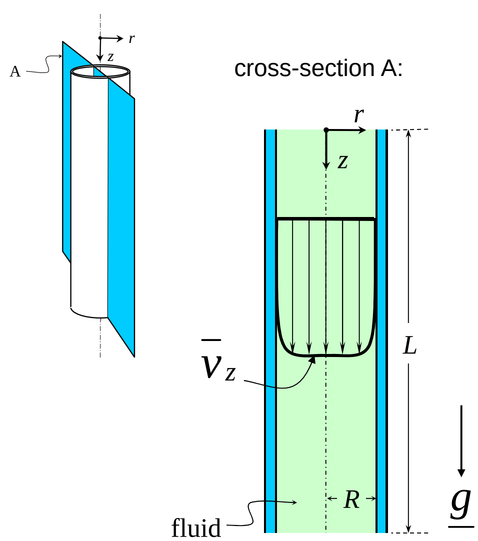
How did we nondimensionalized the Navier-Stokes before?



FORCED CONVECTION

EXAMPLE I: Pressure-driven flow of a Newtonian fluid in a tube:
- steady state
- well developed
- long tube

There was an average velocity used as the characteristic velocity

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FORCED CONVECTION FORCED CONVECTION FORCED CONVECTION

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

This velocity is an imposed (forced) average velocity

We do not have such an imposed velocity in natural convection

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FORCED CONVECTION FORCED CONVECTION FORCED CONVECTION

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

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FORCED CONVECTION ***FORCED CONVECTION*** ***FORCED CONVECTION***

z-component of the nondimensional Navier-Stokes Equation:

$$\frac{1}{\mathrm{Re}} \qquad \frac{1}{Fr}$$

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{\mu}{\rho V D}\left(\nabla^2 v_z\right)^* + \frac{gD}{V^2} g^*$$

$$\left(\nabla^2 v_z\right)^* \equiv \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial v_z^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^* \frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial v_z^*}{\partial \theta} + v_z \frac{\partial v_z^*}{\partial z^*}\right)$$

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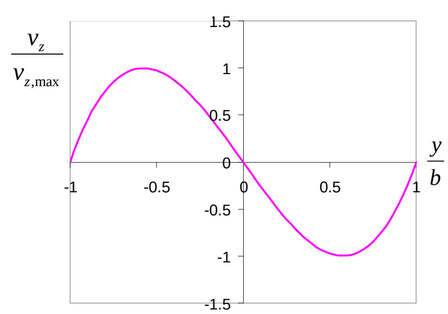
FREE CONVECTION ***FREE CONVECTION***

We do not have such an imposed velocity in natural convection

For free convection, what is the average velocity?

for forced convection we used: $\boxed{v_z^* = \frac{v_z}{V}}$ $\qquad V \equiv \langle v \rangle$



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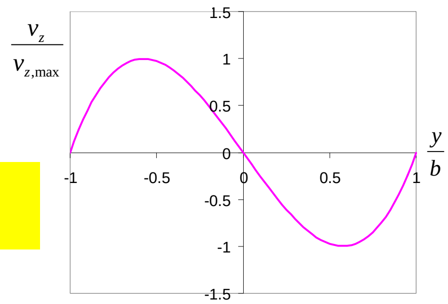
FREE CONVECTION *FREE CONVECTION*

We do not have such an imposed velocity in natural convection

For free convection, what is the average velocity?

for forced convection we used: $\boxed{v_z^* = \frac{v_z}{V}}$ $V \equiv \langle v \rangle$



ZERO

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FREE CONVECTION *FREE CONVECTION*

We do not have such an imposed velocity in natural convection

For free convection, what is the average velocity?
Answer: zero!

for forced convection we used: $\boxed{v_z^* = \frac{v_z}{V}}$ $V \equiv \langle v \rangle$

For free convection $\langle v \rangle = 0$; what V should we use for free convection?

Solution: use a Reynolds-number type expression so that no characteristic velocity imposes itself (we'll see now how that works):

$$v_z^* = \frac{v_z}{V} = \frac{\bar{\rho} v_z D}{\mu} \qquad \boxed{\Rightarrow V \equiv \frac{\rho}{D\bar{\rho}}}$$

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FREE CONVECTION **FREE CONVECTION** **FREE CONVECTION**

When non-dimensionalizing the Navier-Stokes, what do I use for ρ? (answer from idealized problem)

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$
$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

here we use $\bar{\rho}$ because the issue is volumetric flow rate

as before, for pressure gradient we use $-\bar{\rho}g$

here we use $\rho(T)$ because the issue is driving the flow by density differences affected by gravity

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FREE CONVECTION **FREE CONVECTION** **FREE CONVECTION**

non-dimensional variables:

time:

$$t^* \equiv \frac{t\mu}{D^2\bar{\rho}}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z D\bar{\rho}}{\mu}$$

$$v_r^* \equiv \frac{\bar{\rho}v_r D}{\mu}$$

$$v_\theta^* \equiv \frac{\bar{\rho}v_\theta D}{\mu}$$

driving force:

$$T^* = \frac{T-\bar{T}}{T_2-\bar{T}}$$

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FREE CONVECTION ***FREE CONVECTION*** ***FREE CONVECTION***

SOLUTION: *z*-component of the **nondimensional** Navier-Stokes Equation (free convection):

Or any appropriate characteristic ΔT

$$\frac{Dv_z^*}{Dt} = \left(\nabla^2 v_z\right)^* + \left[\frac{gD^3\bar{\rho}^2\bar{\beta}\overbrace{\left(T_2 - \bar{T}\right)}}{\mu^2}\right]T^*$$

≡Grashof number

$$\left(\nabla^2 v_z\right)^* \equiv \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial v_z^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left(\frac{\partial v_z^*}{\partial t^*} + v_r^*\frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial v_z^*}{\partial \theta} + v_z\frac{\partial v_z^*}{\partial z^*}\right)$$

FREE CONVECTION ***FREE CONVECTION***

$$\mathrm{Gr} \equiv \frac{gD^3\bar{\rho}^2\bar{\beta}\Delta T}{\mu^2}$$

Dimensionless Equation of Motion (free convection)

$$\frac{Dv_z^*}{Dt^*} = \left(\nabla^2 v_z\right)^* + \mathrm{Gr}T^*$$

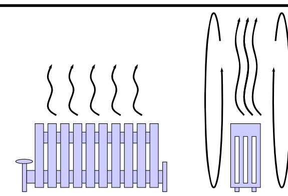
Dimensionless Energy Equation (free convection; Re = 1)

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{Pr}}\nabla^{*2}T^*$$

No Pe
No Re

$$\boxed{\mathrm{Nu} = \mathrm{Nu}\left(T^*, \frac{L}{D}\right) \Rightarrow \mathrm{Nu} = \mathrm{Nu}\left(\mathrm{Pr}, \mathrm{Gr}, \frac{L}{D}\right)}$$

Free Convection i.e. hot air rises



Engineering Modeling

- ✓ •Choose an idealized problem and solve it
- ✓ •From insight obtained from **ideal** problem, identify governing equations of **real** problem
- ✓ •Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- **•Design experiments to test modeling thus far**
- •Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- •Design additional experiments
- •Iterate until useful correlations result

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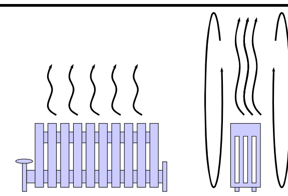
Free Convection i.e. hot air rises



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Done (see literature)

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Literature Results:

$$\mathrm{Gr} \equiv \frac{g D^3 \bar{\rho}^2 \bar{\beta} \Delta T}{\mu^2}$$

Example: Natural convection from vertical planes and cylinders

$$\boxed{\mathrm{Nu} = \frac{hL}{k} = a\mathrm{Gr}^m \,\mathrm{Pr}^m}$$

•*a*,*m* are given in Table 4.7-1, page 255 Geankoplis for several cases
•*L* is the height of the plate
•all physical properties evaluated at the **film temperature**, T_f

Free convection correlations use the film temperature for calculating the physical properties

$$T_f = \frac{T_w + T_b}{2}$$

Free convection correlations use the film temperature for calculating the physical properties

Complex Heat Transfer – Correlations for Nu

Natural convection
Vertical planes and cylinders

$$\mathrm{Nu} = \frac{hL}{k} = a\mathrm{Gr}^m\mathrm{Pr}^m$$

Physical Properties evaluated at:

$$T_f = \frac{T_w + T_b}{2}$$

•all physical properties evaluated at the **film temperature**, T_f

compare with:

Forced convection
Heat Transfer in Laminar flow in pipes

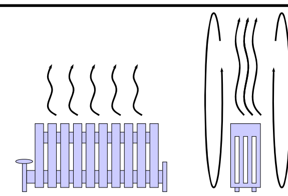
$$\mathrm{Nu}_a = \frac{h_a L}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Physical Properties evaluated at:

$$\frac{T_{b,in} + T_{b,out}}{2}$$

•all physical properties (except μ_w) evaluated at the **bulk mean temperature**
•(true also for turbulent flow correlation)

Free Convection i.e. hot air rises



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Success!

(Dimensional Analysis wins again)

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Practice Heat-Transfer Problems:

Forced Convection
Free Convection

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Practice 1: A wide, deep rectangular oven ($1.0\ ft$ tall) is used for baking loaves of bread. During the baking process the temperature of the air in the oven reaches a stable value of 100^oF. The oven side-wall temperature is measured at this time to be a stable 450^oF. Please estimate the heat flux from the wall per unit width.

Reference: Geankoplis Ex. 4.7-1 page 279

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Practice 2: A hydrocarbon oil enters a pipe ($0.0303\ ft$ inner diameter; $15.0\ ft$ long) at a flow rate of $80\ lb_m/h$. Steam condenses on the outside of the pipe, keeping the inside pipe surface at a constant 350^oF. If the temperature of the entering oil is 150^oF, what is temperature of the oil at the outlet of the pipe?

Hydrocarbon oil properties:

Mean heat capacity $= 0.50 \frac{BTU}{lb_m{}^oF}$

Thermal conductivity $= 0.083 \frac{BTU}{h\ ft\ {}^oF}$

Viscosity =

$6.50\ cp, 150^oF$
$5.05\ cp\ 200^oF$
$3.80 cp\ 250^oF$
$2.82\ cp\ 300^oF$
$1.95\ cp\ 350^oF$

Reference: Geankoplis Ex. 4.5-5 page 269

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Practice 3: Air flows through a tube ($25.4\ mm$ inside diameter, long tube) at $7.62\ m/s$. Steam condenses on the outside of the tube such that the inside surface temperature of the tube is $488.7\ K$. If the air pressure is $206.8\ kPa$ and the mean bulk temperature of the air is $(\mathrm{T_{out}}+\mathrm{T_{in}})/2 = 477.6\ K$, what is the steady-state heat flux to the air?

Reference: Geankoplis Ex. 4.5-1 page 262

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Practice 4: Hard rubber tubing ($\mathrm{inside\ radius} = 5.0mm$; $\mathrm{outside\ radius} = 20.0mm$) is used as a cooling coil in a reaction bath. Cold water is flowing rapidly inside the tubing; the inside wall temperature is $274.9\ K$ and the outside wall temperature is $297.1\ K$. To keep the reaction in the bath under control, the required cooling rate is $14.65\ W$. What is the minimum length of tubing needed to accomplish this cooling rate? What length would be needed if the coil were copper?

<u>Hard rubber properties:</u>

Density $= 1198\frac{kg}{m^3}$

Thermal conductivity $(0^oC) = 0.151\frac{W}{mK}$

Reference: Geankoplis Ex. 4.2-1 page 243, but don't do it his way—follow class methods.

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Practice 5: A cold-storage room is constructed of an inner layer of pine (thickness $= 12.7\ mm$), a middle layer of cork board (thickness $= 101.6\ mm$), and an outer layer of concrete (thickness $= 76.2\ mm$). The inside wall surface temperature is $255.4\ K$ and the outside wall surface temperature is $297.1\ K$. What is the heat loss per square meter through the walls and what is the temperature at the interface between the wood and the cork board?

Material properties:

Thermal conductivity pine $= 0.151 \frac{W}{mK}$

Thermal conductivity cork board $= 0.0433 \frac{W}{mK}$

Thermal conductivity concrete $= 0.762 \frac{W}{mK}$

Reference: Geankoplis Ex. 4.3-1 page 245, but don't do it his way—follow class methods.

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Practice 6: A thick-walled tube (stainless steel; $0.0254\ m$ inner diameter; $0.0508\ m$ outer diameter; length $0.305\ m$) is covered with a $0.0254\ m$ thickness of insulation. The inside-wall temperature of the pipe is $811.0\ K$ and the outside surface temperature of the insulation is $310.8\ K$. What is the heat loss and the temperature at the interace between the steel and the insulation?

Material properties of stainless steel:

Thermal conductivity $= 21.63 \frac{W}{mK}$

Density $= 7861 \frac{kg}{m^3}$

Heat Capacity $= 490 \frac{J}{kg\ K}$

Material properties of insulation:

Thermal conductivity $= 0.2423 \frac{W}{mK}$

Reference: Geankoplis Ex. 4.3-2 page 247, but don't do it his way—follow class methods.

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Experience with Dimensional Analysis thus far:

✓ •Flow in pipes at all flow rates (laminar and turbulent)

Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force = f; f=f(Re, L/D)

✓ •Flow around obstacles (spheres, other complex shapes

Solution: Navier-Stokes, Re, dimensionless drag= C_D; $C_D = C_D(Re)$

✓ •Forced convection heat transfer from fluid to wall

Solution: Microscopic energy, Navier-Stokes, Re, Pr, L/D, heat transfer coefficient=h; h = h(Re,Pr,L/D)

✓ •Natural convection heat transfer from fluid to wall

Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)

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Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)

Now, move to last heat-transfer mechanism:

•Radiation heat transfer from solid to fluid?

Solution: ?

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Experience with Dimensional Analysis thus far:

- Flow in pipes at all flow rates (laminar and turbulent)

  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force = f; f=f(Re, L/D)

Actually, we'll hold off on radiation and spend some time on heat exchangers and other practical concerns

L/D,

- Natural convection heat transfer from fluid to wall

  Solution: Microscopic energy, Navier-Stokes, Gr, Pr, L/D, heat transfer coefficient=h; h = h(Gr,Pr,L/D)

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Next:

CM3110
Transport I
Part II: Heat Transfer

Michigan Tech

Applied Heat Transfer:
Heat Exchanger Modeling, Sizing, and Design

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

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