CM3110
Transport Processes and Unit Operations I

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

CM3110 - Momentum and Heat Transport
CM3120 – Heat and Mass Transport

www.chem.mtu.edu/~fmorriso/cm310/cm310.html

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EMERGENCY EVACUATION PROCEDURES

Important: The Michigan Bureau of Fire Services has adopted new rules for colleges and universities effective 2015

1. Only residence halls are required to hold fire and tornado drills.
2. In lieu of fire drills in other university buildings all faculty and instructional staff are required to do the following on the first day of class:
   - Explain the university fire evacuation procedures to the class (see below).
   - Explain the locations of the primary and secondary exit routes for your class location.
   - Explain your designated safe location where the class will meet after evacuating the building.
3. The class instructor is responsible for directing the class during a building evacuation.

General evacuation procedure:

- Use the nearest safe exit route to exit the building. **The nearest safe exit from room 15-139 is the front (south) entrance that is close to highway 41. The secondary exit is the campus (north) exit, that connects to the main path through campus.**
- Close all doors on the way out to prevent the spread of smoke and fire.
- After exiting, immediately proceed to a safe location at least 100 feet from the building. **Our designated safe location is east of Fisher, in the parking lot of the Center for Diversity and Inclusion.**
- Do not re-enter the building until the all-clear is given by Public Safety or the fire department.

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Why study fluid mechanics?

3

Why study fluid mechanics?

- It’s required for my degree

4

Why study fluid mechanics?

~~•It's required for my degree~~ (too literal)

5

Why study fluid mechanics?

~~•It's required for my degree~~ (too literal)

•Fluids are involved in engineered systems

Image from: newegg.com

Image from: money.cnn.com

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Why study fluid mechanics?

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~~•It's required for my degree~~ (too literal)

~~•Fluids are involved in engineering systems~~ (many devices that employ fluids can be operated and maintained and sometimes designed without detailed mathematical analysis)

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Why study fluid mechanics?

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•Modern engineering systems are complex and often cannot be operated and maintained without analytical understanding

•Design of new systems will come from high-tech innovation, which can only come from detailed, analytical understanding of how physics/nature works

Image: wikipedia.org

Image: planetforward.ca

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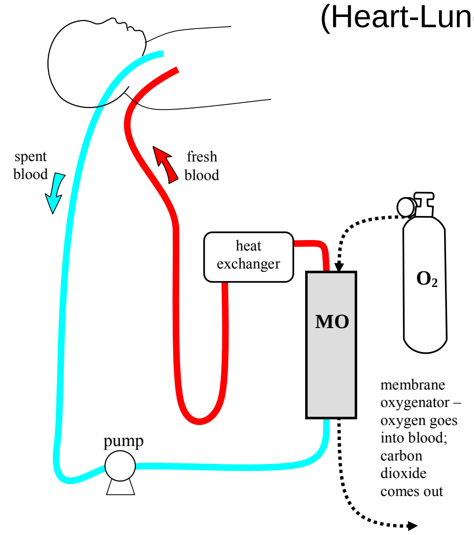
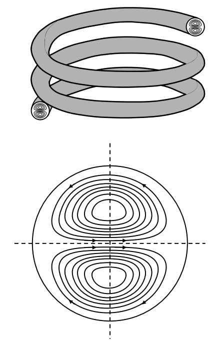
Membrane Oxygenator, MO (Heart-Lung Machine)





Secondary flow drives the O_2 mass transfer

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Artificial Heart

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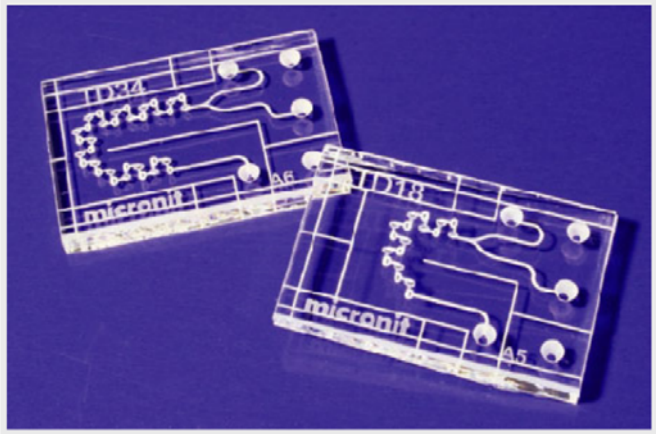
Image: health.howstuffworks.com/medicine/modern/artificial-heart.htm

Microfluidics – Lab on a Chip

Sensors, diagnostics



An example of a passive mixer in which fluids are mixed by chaotic advection. (Courtesy of Micronit Microfluidics.)

www.nature.com/nmeth/journal/v4/n8/full/nmeth0807-665.html

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And more. . . .

Image from: en.wikipedia.org

- Helicopters
- Airplanes
- Quieter fans
- Flexible body armor
- Undersea oil drilling
- Surgery
- Food processing
- Plastics
- 2D and 3D printing
- Battery manufacture
- Celestial exploration
- Volcanos
- Biomedical devices (stents, artificial organs, prosthetics)
- Sensor development
- …

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Where to start?

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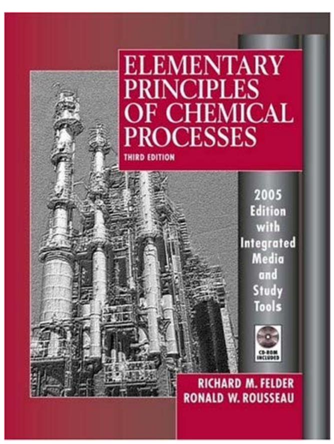
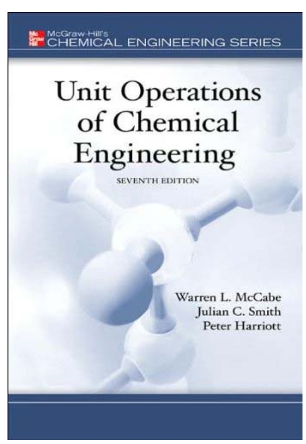
Where to start?

We've already started.





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We've already started.

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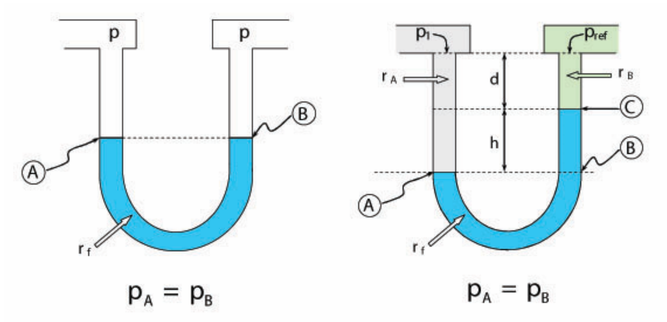
1. We've learned fluid **statics**.



DrMorrisonMTU on **YouTube**: *Introduction to Manometers: Two Essential Rules*

On 4Sept17 #views >126,000!

www.youtube.com/watch?v=zeNQOqr63cc

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We've already started.

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2. There are **flow** problems that can be addressed with a macroscopic **energy** balance:

The Mechanical Energy Balance

$$\frac{\Delta p}{\rho}+\frac{\Delta\langle v\rangle^2}{2\alpha}+g\Delta z+F=\frac{W_{s,on}}{\dot m}$$

$$\frac{p_2-p_1}{\rho}+\frac{\langle v\rangle_2^2-\langle v\rangle_1^2}{2\alpha}+g(z_2-z_1)+F_{21}=\frac{W_{s,on,21}}{\dot m}$$

F = friction

Assumptions:

1. single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical rxn
6. Insignificant amounts of heat transferred

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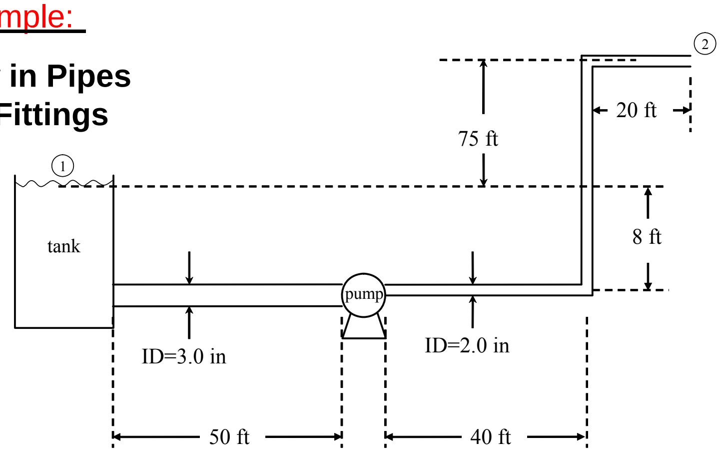
For example:

Flow in Pipes and Fittings



1. Single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical rxn
6. Insignificant amounts of heat transferred

→ Mechanical Energy Balance

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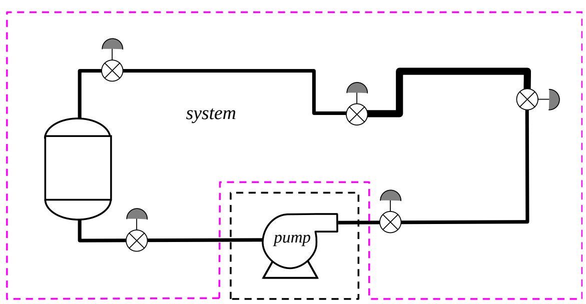
For example:

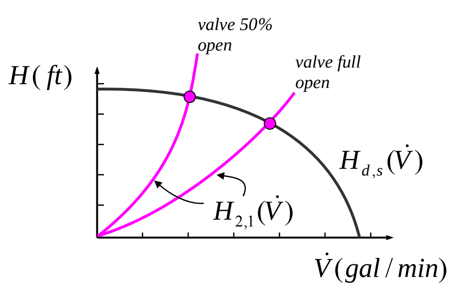
Centrifugal Pumps

What flow rate does a centrifugal pump produce?

Answer: Depends on how much work it is asked to do.



Calculate with the Mechanical Energy Balance (CM2110, CM2120, CM3215)



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We can apply the MEB to many important engineering systems

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Image from: www.directindustry,com

Image from: www.processindustryforum.com

Image from: www.directindustry.com

Image from: www.epa.gov

MEB Assumptions:

1. single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical rxn
6. Insignificant amounts of heat transferred

Calculate: **Work, pressures, flows**

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The Mechanical Energy Balance

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(Review)

$$\frac{\Delta p}{\rho} + \frac{\Delta \langle v \rangle^2}{2\alpha} + g\Delta z + F = \frac{W_{s,on}}{\dot{m}}$$

$$\frac{p_2 - p_1}{\rho} + \frac{\langle v \rangle_2^2 - \langle v \rangle_1^2}{2\alpha} + g(z_2 - z_1) + F_{21} = \frac{W_{s,on,21}}{\dot{m}}$$

F = friction

Where do we get this?

This is the friction due to wall drag (straight pipes) and fittings and valves.

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The Mechanical Energy Balance – Friction Term

(Review)

The friction has been measured and published in this form:

Straight pipes:

$$F_{straight\ pipes} = \left(4f\frac{L}{D}\right)\frac{\langle v\rangle^2}{2}$$

Use literature plot of *f* as a function of Reynolds Number

Fittings and Valves:

$$F_{fittings,\ valves} = K_f \frac{\langle v\rangle^2}{2}$$

Use literature tables of K_f for laminar and turbulent flow

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F = friction

Friction term in Mechanical Energy Balance

(see McCabe et al., or Morrison Chapter 1, or Perry's Chem Eng Handbook)

(Review)

$$F_{friction} = \left(4f\frac{L}{D} + \sum_i K_{f_i} n_i\right)\frac{\langle v\rangle^2}{2}$$

length of straight pipe

number of each type of fitting

Note *f* is a function of velocity)

(from literature; the Moody chart)

friction-loss coefficients

(from literature; see McCabe et al., Geankoplis, or Morrison Chapter 1)

Note that friction overall is directly a function of velocity)

If the velocity changes within the system (e.g. pipe diameter changes), then we need different friction terms for each velocity

(Review)

Data are organized in terms of two dimensionless parameters:

Flow rate

Reynolds Number

$$\text{Re} = \frac{\rho \langle v_z \rangle D}{\mu}$$

Pressure Drop

Fanning Friction Factor

$$f = \frac{\frac{1}{4}\left(P_0 - P_L\right)}{\left(\frac{L}{D}\right)\left(\frac{1}{2}\rho\langle v_z\rangle^2\right)}$$

ρ – density
$\langle v_z \rangle$ – average velocity
D – pipe diameter
μ – viscosity
$P_0 - P_L$ – pressure drop
L – pipe length

Moody Chart: Data Correlation for Friction in Straight Pipes

(Review)

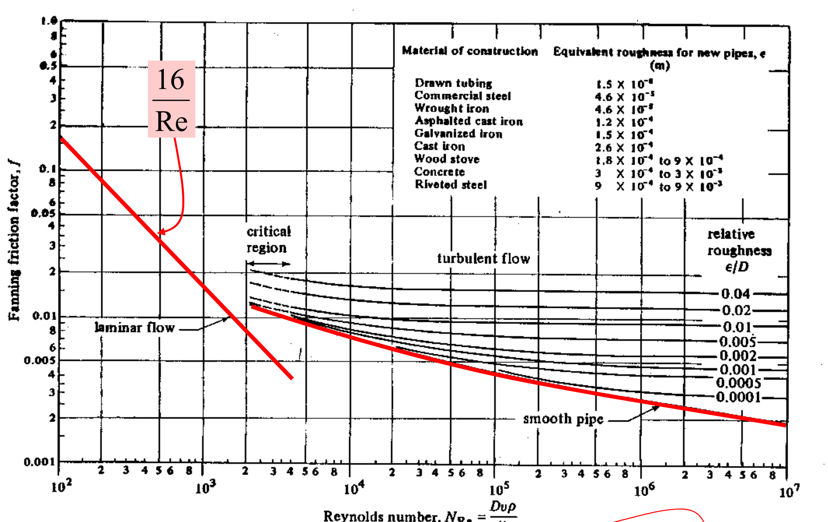

FIGURE 2.10-3. Friction factors for fluids inside pipes. [Based on L. F. Moody, Trans. A.S.M.E., 66, 671, (1944); Mech. Eng. 69, 1005 (1947). With permission.]

(image from: Geankoplis)

Friction Loss from Fittings K_f

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Table 1.4. Published friction-loss factors for turbulent flow through valves, fittings, expansions, and contractions

Fitting	Friction-loss factor, K_f
Standard elbow, 45°	0.35
Standard elbow, 90°	0.75
Tee used as ell	1.0
Tee, branch blanked off	0.4
Return bend	1.5
Coupling	0.04
Union	0.04
Gate valve, wide open	0.17
Gate valve, half open	4.5
Globe valve, bevel seat, wide open	6.0
Globe valve, bevel seat, half open	9.5
Check valve, ball	70.0
Check valve, swing	2.0
Water meter, disk	7.0
Expansion from A_1 to A_2	$\left(1-\frac{A_1}{A_2}\right)^2$
Contraction from A_1 to A_2	$0.55\left(1-\frac{A_2}{A_1}\right)$

Source: Perry's Handbook [132]

Table 1.5. Friction-loss factors K_f for laminar flow through selected valves, fittings, expansions and contractions

	K_f					
Fitting	$Re_l = 50$	100	200	400	1,000	Turbulent
Elbow, 90°	17	7	2.5	1.2	0.85	0.75
Tee	9	4.8	3.0	2.0	1.4	1.0
Globe valve	28	22	17	14	10	6.0
Check valve, swing	55	17	9	5.8	3.2	2.0
Expansion from A_1 to A_2	$2\left(1-\frac{A_1}{A_2}\right)^2$					$\left(1-\frac{A_1}{A_2}\right)^2$
Contraction from A_1 to A_2	$\frac{0.55}{0.5}\left(1-\frac{A_2}{A_1}\right)$					$0.55\left(1-\frac{A_2}{A_1}\right)$

Source: Perry's Handbook [132]

(source: Morrison, Chapter 1; originally from Perry's Handbook)

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Example 1

What is the pressure change over 50 meters of 1/2 inch inner-diameter straight pipe? The average velocity is 5.2 ft/s and the pipe is smooth.

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Example 1

What is the pressure change over 50 meters of 1/2 inch inner-diameter straight pipe? The average velocity is 5.2 ft/s and the pipe is smooth.

ANSWER: 18 psi

(our TA has the solution: HW/Example help session Sunday 6:30-7:30)

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Example 2

What is the volumetric flow rate at the drain from a constant-head tank with a fluid level h? You may neglect frictional losses.

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Example 2

What is the volumetric flow rate at the drain from a constant-head tank with a fluid level *h*? You may neglect frictional losses.

ANSWER: $\pi R^2 \sqrt{2hg}$

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(Review)

For more examples: see CM2110/20 notes; HW1; Prerequisite review readings

Homework 1				
area	Topics	Chap.	Assigned Problems	Stretch Problems
prereq	problem solving	1	2	
prereq	friction in pipes	1	8	
prereq	flow rate	1	10	
prereq	friction in pipes	1	13	
prereq	Re	1		14
prereq	Re	1	16	
prereq	flow rate	1		19
prereq	siphon	1		25
prereq	fluid statics	1	26	
prereq	mech energy bal (MEB)	1	30	
prereq	friction in pipes	1		31
prereq	pumps	1	32	
prereq	pumps	1		34
prereq	math - vectors	1	41	
prereq	math - matrix	1	44	
prereq	math - dot product	1	45	
prereq	math - cyl coords	1		48
prereq	math - plot profile	1		58
prereq	math - coordinate system	1		65
prereq	fluid statics/manometer	4	13	

Reading Assignments CM3110
Fall Semester
Dr. Faith Morrison

Texts:

- Faith A. Morrison, *An Introduction to Fluid Mechanics*, Cambridge, New York (2013).
- Christie J. Geankoplis, *Transport Processes and Unit Operations*, 4th Edition, Prentice Hall, New York (2003).
- Richard Felder and Ronald Rousseau, Elementary Principles of Chemical Processes, 3rd Edition, Wiley, New York (2005).
- Warren McCabe, Julian Smith, Peter Harriott, Unit Operations of Chemical Engineering McGraw-Hill Professional (2004).

Prerequisite topic review suggested readings: CM3110 Transport I Morrison			
Mechanical energy balance	Felder and Rousseau	Ch 7.7	pp 333-337
	McCabe, Smith, Harriott	Ch 4	pp 86-94
	Morrison	Ch 1	pp 8-93
	Morrison	Ch 9	pp 766-800
	Geankoplis	Ch 2.7ff	pp 67-71
Fluid statics	Felder and Rousseau	Ch 3	pp 54-59
	McCabe, Smith, Harriott	Ch 2	pp 31-44
	Morrison	Ch 4.2	pp 236-277
	Geankoplis	Ch 2.1-2.2	pp 34-42
Pumps	McCabe, Smith, Harriott	Ch 8	pp 202-206
	Morrison	Ch 9	pp 800-823
	Geankoplis	Ch 3.3	pp 144-149

Note that there are many references offered. You do not need to use them all, just use the ones that explain it in a way that you can understand and do the homework problems. You may also seek out your own references on the web or in the library.

1

Exam 1: Next Tues 6:30-8:00pm
Last year's exam and solution is on the web. TA help session is Sunday night
Exam topics: vectors, linear algebra, integration, MEB, fluid statics

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$$\frac{p_2 - p_1}{\rho} + \frac{\langle v\rangle_2^2 - \langle v\rangle_1^2}{2\alpha} + g(z_2 - z_1) + F_{21} = \frac{W_{s,on,21}}{\dot{m}}$$

F = friction

The *Mechanical Energy Balance* (MEB) is a macroscopic analysis.

- **It is limited in application:**
  1. single-input, single output
  2. Steady state
  3. Constant density (incompressible fluid)
  4. Temperature approximately constant
  5. No phase change, no chemical rxn
  6. Insignificant amounts of heat transferred
- **It cannot determine flow patterns**
- **It does not model momentum exchanges**
- **It cannot be adapted to systems other than those for which it was designed** **(see list above)**

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Energy balances (the MEB) can only take us so far with fluids modeling (due to assumptions).

To understand complex flows, we must use the **MOMENTUM** balance.

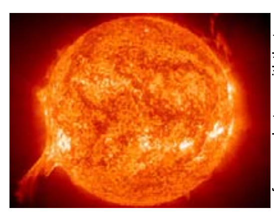

Image from: whatsupwiththat.com

Image from: www-math.mtu.edu

Image from: www.123rf.com

Naruto Whirlpools, Japan

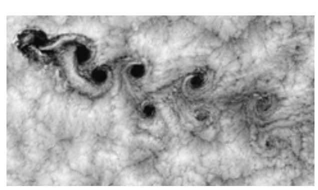

Image from: commons.wikipedia.org

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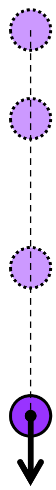
Momentum Balance: Newton's 2nd Law of Motion

$$\underline{f} = m\underline{a}$$



PH 2100: apply to individual bodies

CM 3110: apply to a continuum

Image from: www.texturex.com

See also: http://youtu.be/6KKNnjFpGto

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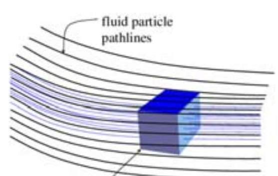
Fluid Mechanics



$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz}$$

$$\rho(x, y, z) \qquad \underline{\underline{\tau}} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}_{xyz}$$

- Continuum (density, velocity, stress fields) *(calc3)*
- Control volume
- Stress in a fluid at a point (stress tensor)
- Stress and deformation (Newtonian constitutive equation)
- Microscopic and macroscopic momentum balances
- Internal flows – pipes, conduits
- External flows – drag, boundary layers
- Advanced fluid mechanics – complex shapes

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Momentum . . . is a vector

$$\rho \underline{v} = \begin{pmatrix} \rho v_x \\ \rho v_y \\ \rho v_z \end{pmatrix}_{xyz}$$

Microscopic momentum balance

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla P + \mu\nabla^2\underline{v} + \rho\underline{g}$$ Ch 6

Macroscopic momentum balance

$$\frac{d\underline{\mathbf{P}}}{dt} + \sum_{i=1}^{\#streams}\left[\frac{\rho A\cos\theta\langle v\rangle^2}{\beta}\hat{v}\right]\Bigg|_{A_i} = \sum_{i=1}^{\#streams}\left[-pA\hat{n}\right]\Big|_{A_i} + \underline{R} + M_{CV}\underline{g}$$ Ch 9

So we need vector math.

$(Calc\ 3)$

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Vectors

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z}$$

Note:
$v_x \neq v_1 \neq v_r$
(usually)

Same vector,
different coordinate systems,
different components.

$$|\underline{v}| = v = vector\ magnitude$$

$$\frac{(\underline{v})}{v} = \hat{v} = unit\ vector$$

We choose coordinate systems for convenience.

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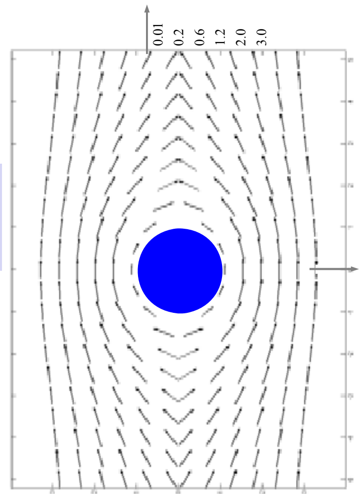
Fluid velocity is a vector field

$$\underline{v} = v(x, y, z)$$

Vector plot of the velocity field in slow flow around a sphere



creeping flow
(sphere)

The flow is a steady upward flow; the length and direction of the vector indicates the velocity at that location.

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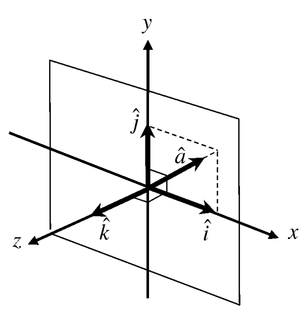
Vectors – Cartesian coordinate system

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz} = v_x\hat{e}_x + v_y\hat{e}_y + v_z\hat{e}_z$$

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = v_1\hat{e}_1 + v_2\hat{e}_2 + v_3\hat{e}_3$$

$$= v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k}$$



(three ways of writing the same thing, the Cartesian basis vectors)

- We do algebra with the basis vectors the same way as with other quantities
- The Cartesian basis vectors are constant (do not change with position)

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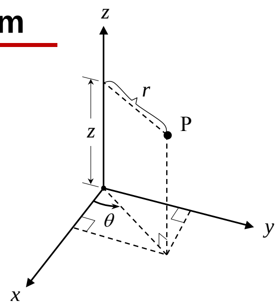
Vectors – Cylindrical coordinate system

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z} = v_r \hat{e}_r + v_\theta \, \hat{e}_\theta + v_z \hat{e}_z$$



•The cylindrical basis vectors are variable (depend on position)

$$x = r\cos\theta \qquad \hat{e}_r = \cos\theta \hat{e}_x + \sin\theta \hat{e}_y$$
$$y = r\sin\theta \qquad \hat{e}_\theta = -\sin\theta \hat{e}_x + \cos\theta \hat{e}_y$$
$$z = z \qquad \hat{e}_z = \hat{e}_z$$

(see inside back cover of text; also, supplemental handouts)

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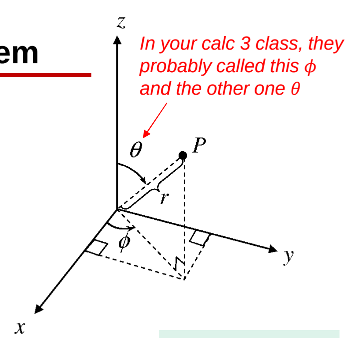
Vectors – Spherical coordinate system

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_\phi \end{pmatrix}_{r\theta\phi} = v_r \hat{e}_r + v_\theta \, \hat{e}_\theta + v_z \hat{e}_\phi$$



Note: spherical coordinate system in use by the fluid mechanics community uses $0 < \theta < \pi$ as the angle from the z-axis to the point.

•The spherical basis vectors are variable (with position)

(see inside back cover of text; also, supplemental handouts)

$$x = r\sin\theta\cos\phi \qquad \hat{e}_r = \sin\theta\cos\phi\hat{e}_x + \sin\theta\sin\phi\hat{e}_y + \cos\theta\hat{e}_z$$
$$y = r\sin\theta\sin\phi \qquad \hat{e}_\theta = \cos\theta\cos\phi\hat{e}_x + \cos\theta\sin\phi\hat{e}_y + (-\sin\theta)\hat{e}_z$$
$$z = r\cos\theta \qquad \hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\hat{e}_y$$

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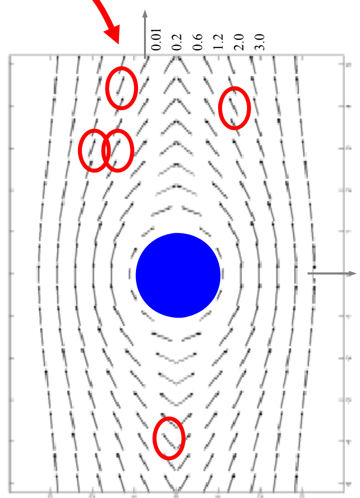
Fluid Velocity is a Vector Field

Velocity magnitude and direction vary with position

$$\underline{v} = v(x, y, z)$$



creeping flow (sphere)

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Example 3: At positions (1,45º,0) and (1,90º,0) in the r, θ, z coordinate system, the velocity vector of a fluid is given by

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{r\theta z}$$

What is this vector in the usual xyz coordinate system?

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Example 3: At positions (1,45°,0) and (1,90°,0) in the r, θ, z coordinate system, the velocity vector of a fluid is given by

$$\underline{v} = \begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{r\theta z}$$

What is this vector in the usual xyz coordinate system?

ANSWERS:

$$\underline{v}_{45^o} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}_{xyz}$$

$$\underline{v}_{90^o} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$hint: \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{r\theta z} = \hat{e}_\theta$$

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We use Calculus in Fluid Mechanics to:

1. Calculate flow rate, Q
2. Calculate average velocity, $\langle v \rangle$
3. Express forces on surfaces due to fluids (vectors)
4. Express torques on surfaces due to fluids (vectors)

These are quantities of interest.
These items are what we are learning to calculate.

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1. Calculate Flow rate: Q or $\dot{V}$

General:

$$Q = \iint_{area} (\underline{v} \cdot \hat{n})\, d(area)$$

Tube flow:

$$Q = \int_0^{2\pi} \int_0^R v_z(r)\, r dr d\theta$$

$(\underline{v} \cdot \hat{n})$ is the component of $\underline{v}$ in the direction normal to the area

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Common surface shapes in the standard coordinate systems:

$$\begin{aligned}
&\textit{rectangular}: && d(area) = dxdy \\
&\textit{circular}: && d(area) = r\, drd\theta \\
&\textit{surface of cylinder}: && d(area) = Rd\theta dz \\
&\textit{spherical}: && d(area) = (rd\theta)(r \sin\theta\, d\phi) = r^2 \sin\theta\, d\theta d\phi
\end{aligned}$$

(see inside back cover of text; also, supplemental handouts)

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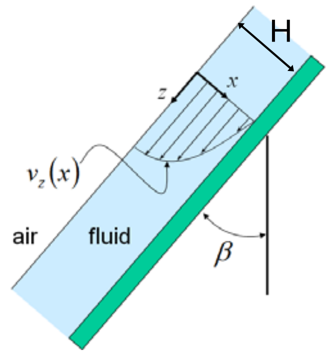
Example 4: *Calculate the flow rate in flow down an incline plane of width W.*

Momentum balance calculation gives:

$$v_z(x) = \frac{\rho g \cos(\beta)}{2\mu}\left(H^2 - x^2\right)$$

(we will learn how to get this equation for $v_z(x)$; here it is given)



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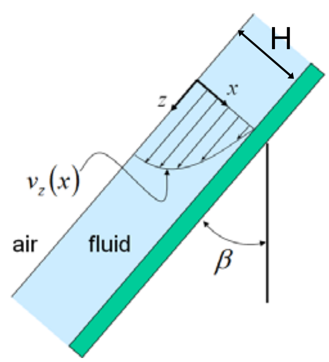

ANSWER:

$$Q = \frac{H^2 \rho g cos\beta}{3\mu}$$

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2. Calculate Average velocity: $\langle v \rangle$

General:

$$\langle v \rangle = \frac{Q}{area}$$

Tube flow:

$$\langle v \rangle = \frac{Q}{\pi R^2}$$

"*area*" is the cross-sectional area normal to flow

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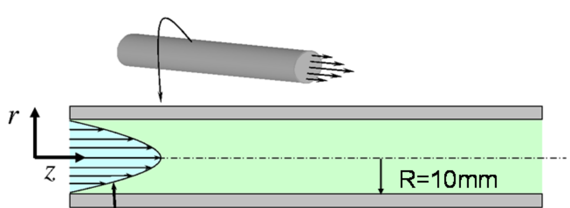
Example 5: *The shape of the velocity profile for a steady flow in a tube is found to be given by the function below. Over the range 0 <r<10 mm, (R=10mm), what is the average value of the velocity?*



$$\frac{v_z}{v_{\max}} = f(r) = 1 - \left(\frac{r}{10}\right)^2$$

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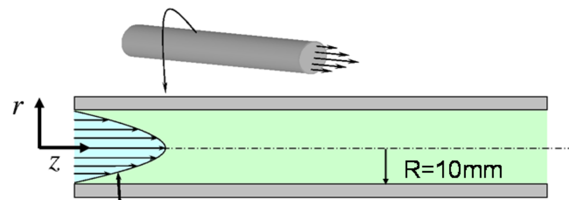
Example 5: *The shape of the velocity profile for a steady flow in a tube is found to be given by the function below. Over the range 0 <r<10 mm, (R=10mm), what is the average value of the velocity?*



$$\frac{v_z}{v_{max}} = f(r) = 1 - \left(\frac{r}{10}\right)^2$$

ANSWER: $\frac{v_{max}}{2}$

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3. Express forces on surfaces due to fluids

Total fluid force on a surface $= \underline{F} = \iint [\hat{n} \cdot \underline{\underline{\Pi}}]_{surface} dS$

$\underline{\underline{\Pi}} \equiv \underline{\underline{\tau}} - p\underline{\underline{I}} =$ Total stress tensor

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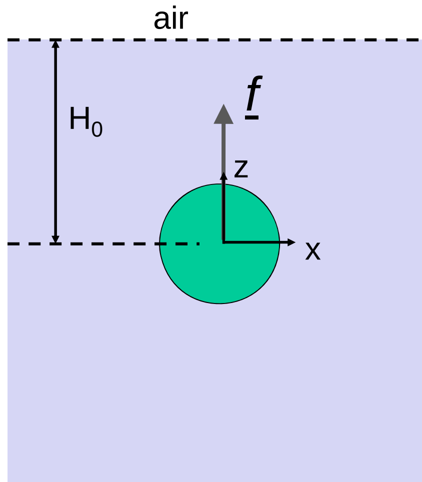
(p81)

Example 6: In a liquid of density ρ, what is the net fluid force on a submerged sphere (a ball or a balloon)? What is the direction of the force and how does the magnitude of the fluid force vary with fluid density?



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Solution: We will be able to do this in this course (Ch4, p257).

From expression for force due to fluid, obtain:
(in spherical coordinates)

Total fluid force on a surface $= \underline{F} = \iint [\hat{n} \cdot \underline{\underline{\Pi}}]_{surface} dS$

$$\underline{F} = -\rho g R^2 \int_0^{2\pi} \int_0^{\pi} (H_0 - R\cos\theta)\hat{e}_r \sin\theta \, d\theta d\phi$$

We can do the math from here. (Calc 3)

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Solution: We will be able to do this in this course (Ch4, p257).

From expression for force due to fluid, obtain:
(in spherical coordinates)

Total fluid force on a surface $= \underline{F} = \iint [\hat{n} \cdot \underline{\underline{\Pi}}]_{surface} dS$

$$\underline{F} = -\rho g R^2 \int_0^{2\pi} \int_0^{\pi} (H_0 - R cos\theta) \hat{e}_r \sin\theta \, d\theta d\phi$$

ANSWER: (see p83)

$$\underline{f} = \begin{pmatrix} 0 \\ 0 \\ \frac{4\pi R^3 \rho g}{3} \end{pmatrix}_{xyz}$$

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4. Express torques on surfaces due to fluids

total fluid torque on a surface $= \underline{T} = \iint_S \left[\underline{R} \times \left[\hat{n} \cdot \underline{\underline{\Pi}} \right] \right]_{at\ surface} dS$

$\underline{R} = lever\ arm$ (Points from axis of rotation to position where torque is applied)

$\underline{\underline{\Pi}} = \underline{\underline{\tau}} - p\underline{\underline{I}} = total\ stress\ tensor$

We will learn to write the stress tensor for our systems; then we can calculate stresses, torques.

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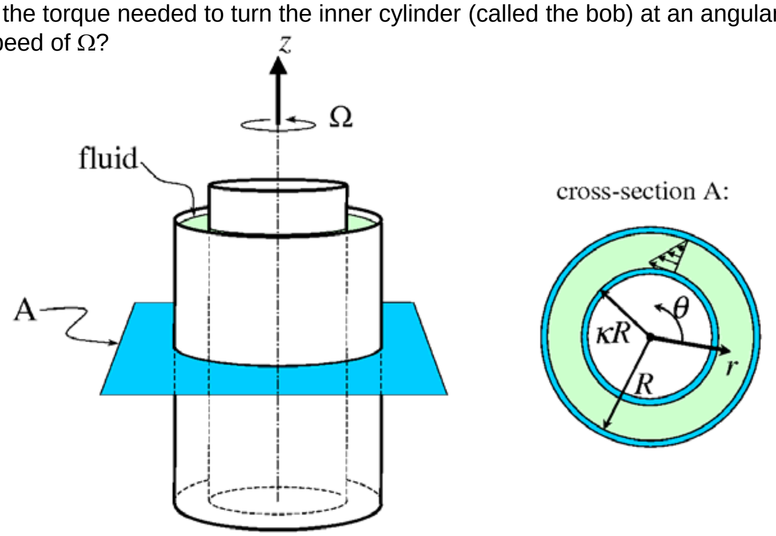
Example 7, Torque in Couette Flow: A cup-and-bob apparatus is widely used to measure viscosities for fluids. For the apparatus below, what is the torque needed to turn the inner cylinder (called the bob) at an angular speed of Ω?



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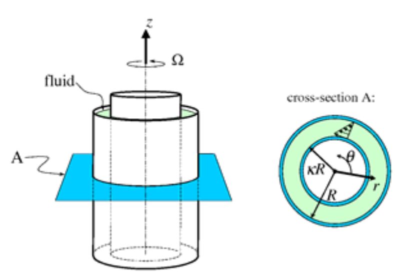
Torque in Couette Flow
Solution:

1. Solve for velocity field (microscopic momentum balance)
2. Calculate stress tensor
3. Formulate equation for torque (an integral)
4. Integrate
5. Apply boundary conditions



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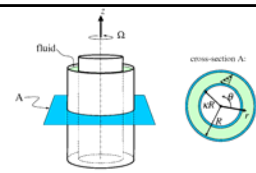
See problem 6.22 p487

Torque in Couette Flow Solution:



Velocity solution:

$$\underline{v} = \begin{pmatrix} 0 \\ \left(\frac{\kappa^2 \Omega R}{\kappa^2 - 1}\right)\left(\frac{r}{R} - \frac{R}{r}\right) \\ 0 \end{pmatrix}_{r\theta z}$$

$$\underline{\underline{\tau}} = \mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$

$$\underline{\underline{\Pi}} = \underline{\underline{\tau}} - p\underline{\underline{I}}$$

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{pmatrix}_{r\theta z}$$

$$\begin{matrix}\textit{total fluid torque} \\ \textit{on a surface}\end{matrix} = \underline{T} = \iint_S \left[\underline{R} \times \left[\hat{n} \cdot \underline{\underline{\Pi}}\right]\right]_{\text{at surface}} dS$$

What is *lever arm*, R?

etc…

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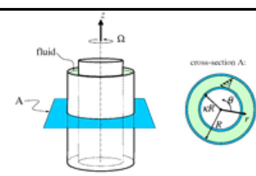
See problem 6.22 p487

Torque in Couette Flow Solution:



$$\begin{matrix}\textit{total fluid torque} \\ \textit{on a surface}\end{matrix} = \underline{T} = \iint_S \left[\underline{R} \times \left[\hat{n} \cdot \underline{\underline{\Pi}}\right]\right]_{\text{at surface}} dS$$

ANSWER: (see p308)

$$\underline{T} = \frac{4\pi R^2 \kappa^2 L \mu \Omega}{(\kappa^2 - 1)} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{r\theta z}$$

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F = friction

$$\frac{\Delta p}{\rho}+\frac{\Delta\langle v\rangle^2}{2\alpha}+g\Delta z+F=\frac{W_{s,on}}{\dot{m}}$$

$$\frac{p_2-p_1}{\rho}+\frac{\langle v\rangle_2^2-\langle v\rangle_1^2}{2\alpha}+g(z_2-z_1)+F_{21}=\frac{W_{s,on,21}}{\dot{m}}$$

Summary of Quick Start

A: Mechanical Energy Balance

1. SI-SO, steady, incompressible, no rxn, no ΔT, no Q
2. Macroscopic
3. Choose points 1 and 2 wisely
4. Solve for F or $W_{s,on}$ or p, velocity, elevation

B): Use Calculus in Fluid Mechanics to

1. Calculate flow rate
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End of **Quick Start**.

We have reviewed:

- MEB (energy bal)
- Math tools

Now, on to **Fluid Mechanics**, i.e. momentum transport.

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NEXT: Fluid Behavior

CM3110
Transport Processes and Unit Operations I

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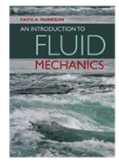
How do fluids behave?

1. Viscosity
2. Drag
3. Boundary Layers
4. Laminar versus Turbulent Flow
5. Lift
6. Supersonic
7. Surface Tension
8. Curved Streamlines
9. Magnetohydrodynamics



(Ch2)

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