Thermal conductivity $k$ and heat transfer coefficient $h$ may be thought of as sources of resistance to heat transfer.

These resistances stack up in a logical way, allowing us to quickly and accurately determine the effect of adding insulating layers, encountering pipe fouling, and other applications.
Using the solution: Composite Door:

For an outside door, a metal is used \((k_1)\) for strength, and a cork \(k_2\) is used for insulation. Both are the same thickness \(B/2\). What is the temperature profile in the door at steady state? What is the flux? The inside temperature of the metal is \(T_1\) and the outside temperature of the cork is \(T_3\).

Let’s try.

\[ k_1 \gg k_2 \]

Note: in the hand notes the temperatures from left to right are \(T_1, T_3, T_2\).

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html
Example 1b: Composite Door (two equal width layers)

**SOLUTION:**

\[
q_x = \frac{(T_1 - T_3)}{A} = \frac{B/2}{k_1 + k_2} (T_2 - T_1)
\]

k₁ material: \(0 \leq x \leq B/2\)

\[T(x) = \frac{(T_2 - T_1)}{B/2} x + T_1\]

k₂ material: \(B/2 \leq x \leq B\)

\[T(x) = \frac{(T_3 - T_2)}{B/2} x + (2T_2 - T_3)\]

Each of the layers contributes a resistance, added in series (like in electricity).
1D Heat Transfer

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

What is the steady state temperature profile in a rectangular slab if the fluid on one side is held at $T_{b1}$ and the fluid on the other side is held at $T_{b2}$?

Assumptions:
• wide, tall slab
• steady state
• $h_1$ and $h_2$ are the heat transfer coefficients of the left and right walls

$T_{b1} > T_{b2}$

See handwritten notes (in class, also on web).

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html
https://pages.mtu.edu/~fmorriso/cm310/algebra_details_N_law_cooling.pdf

© Faith A. Morrison, Michigan Tech U.
Example 2: Heat flux in a rectangular solid – Newton’s law of cooling BC

Solution: (temp profile, flux)

Temperature profile:
\[ \frac{T_{b1} - T}{T_{b1} - T_{b2}} = \frac{x + \frac{1}{k}}{\frac{1}{h_1} + \left( \frac{B}{k} + \frac{1}{h_2} \right)} \]

Flux:
\[ q_x = \frac{T_{b1} - T_{b2}}{\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}} \]

Rectangular slab with Newton’s law of cooling BCs

© Faith A. Morrison, Michigan Tech U.
Example 3: Heat flux in a cylindrical shell – Temp BC

Assumptions:
• long pipe
• steady state
• $k =$ thermal conductivity of wall

What is the steady state temperature profile in a cylindrical shell (pipe) if the inner wall is at $T_1$ and the outer wall is at $T_2$? ($T_1>T_2$)

Cooler wall at $T_2$
Hot wall at $T_1$
Material of thermal conductivity $k$

See handwritten notes in class.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html
Example 3: Heat flux in a **cylindrical shell** – Temp BC

**Solution for Cylindrical Shell:**

\[
\frac{q_r}{A} = \frac{1}{\ln \left( \frac{R_2}{R_1} \right)} \frac{1}{r} \left( T_1 - T_2 \right)
\]

The heat flux \( \frac{q_r}{A} \) **DOES depend** on \( k \); also \( \frac{q_r}{A} \) decreases as \( 1/r \)

\[
\frac{T_2 - T}{T_2 - T_1} = \frac{\ln \left( \frac{R_2}{R_1} \right)}{r} \frac{1}{\ln \left( \frac{R_2}{R_1} \right)}
\]

Note that \( T(r) \) does not depend on the thermal conductivity, \( k \) (steady state)

Pipe with temperature BCs

---

**Example 3:** Heat flux in a **cylindrical shell** – Temp BC

**Solution for Cylindrical Shell:**

\[
\frac{q_r}{A} = \frac{\left( T_1 - T_2 \right)}{\ln \left( \frac{R_2}{R_1} \right)} \frac{1}{r}
\]

Let: \( R_i \equiv \frac{k_i R_{i+1}}{\ln \frac{R_{i+1}}{R_i}} \)

\[
\frac{q_r}{A} = \left( \frac{T_1 - T_2}{R_1} \right) \frac{1}{R_i} \frac{1}{\ln \left( \frac{R_2}{R_1} \right)} \frac{1}{r} = \text{driving force} \frac{1}{\text{resistance}}
\]

**Resistance due to finite thermal conductivity, radial**

© Faith A. Morrison, Michigan Tech U.
Using the solution: Insulated Pipe (Composite, radial conduction)

For a metal pipe carrying a hot liquid ($k_1$) an insulation layer is added with thermal conductivity $k_2$. What is the temperature profile in the composite pipe at steady state? What is the flux? The inside temperature of the metal pipe is $T_1$ and the outside temperature of the insulation is $T_3$.

$$k_1 \gg k_2$$

Example 3b: Insulated Pipe (Composite, radial conduction)

**SOLUTION:**

$$\frac{q_r}{A} = -k_1 \left( \frac{dT}{dr} \right) = \text{(constant)} \frac{1}{r}$$

- $k_1$ material: ($R_1 \leq r \leq R_2$)
  
  $$T(r) = a_1 \ln r + b_1$$

- $k_2$ material: ($R_2 \leq r \leq R_3$)
  
  $$T(r) = a_2 \ln r + b_2$$

© Faith A. Morrison, Michigan Tech U.
See Lecture 16 Slides

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

1D Heat Transfer – Radial

Example 3b: Insulated Pipe
(Composite, radial conduction)

SOLUTION:

\[
\frac{q_r}{A} = \left( \frac{T_1 - T_3}{\frac{1}{k_1} \ln \frac{R_2}{R_1} + \frac{1}{k_2} \ln \frac{R_3}{R_2}} \right) \frac{1}{r}
\]

Let: \( \mathcal{R}_i \equiv \frac{1}{k_i} \ln \frac{R_{i+1}}{R_i} \)

\[
\frac{q_r}{A} = \left( \frac{T_1 - T_3}{\mathcal{R}_1 + \mathcal{R}_2} \right) \frac{1}{r} = \text{driving force} \quad \text{resistance}
\]

Each of the layers contributes a resistance, added in series (like in electricity).
Example 4: Heat flux in a cylindrical shell – Newton’s law of cooling

Assumptions:
• long pipe
• steady state
• $k$ = thermal conductivity of wall
• $h_1$, $h_2$ = heat transfer coefficients

What is the steady state temperature profile in a cylindrical shell (pipe) if the fluid on the inside is at $T_{b1}$ and the fluid on the outside is at $T_{b2}$? ($T_{b1} > T_{b2}$)

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html
1D Heat Transfer – Radial

Solution: Radial Heat Flux in an Annulus

\[ T(r) = \frac{(T_{b1} - T_{b2})}{k} \left( \ln \left( \frac{R_2}{r} \right) + \frac{k}{h_2R_2} \right) - \frac{1}{h_2R_2} \ln \left( \frac{R_2}{R_1} \right) + \frac{k}{h_1R_1} \]

\[ q_r(r) = \frac{(T_{b1} - T_{b2})}{A} \left( \frac{1}{h_2R_2} + \frac{1}{k} \ln \left( \frac{R_2}{R_1} \right) + \frac{1}{h_1R_1} \right) \]

Note that we can continue to add layers in terms of resistance.

Resistance \( \mathcal{R} \) due to heat transfer coefficients, radial
Resistance \( \mathcal{R} \) due to finite thermal conductivity, radial

© Faith A. Morrison, Michigan Tech U.
1D Heat Transfer – Composite Structures

Let: $R_i \equiv \frac{\Delta x}{k_i}$

$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{R_1 + R_2} = \text{driving force} \div \text{resistance}$$

Note: Geankoplis uses a different resistance. For rectangular heat flux:

$$R_{\text{Geankoplis}} = \frac{R}{LW}$$

Let: $R_i \equiv \frac{1}{k_i} \ln \frac{R_{i+1}}{R_i}$

$$\frac{q_r}{A} = \left( \frac{T_1 - T_3}{R_1 + R_2} \right) \frac{1}{r} = \text{driving force} \div \text{resistance}$$

Note: Geankoplis uses a different resistance. For radial heat flux:

$$R_{\text{Geankoplis}} = \frac{R}{2\pi L}$$

© Faith A. Morrison, Michigan Tech U.