Michigan Tech

CM3110
Transport I
Part I: Fluid Mechanics

More Complicated Flows III: Boundary-Layer Flow

(plus other applied topics)

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

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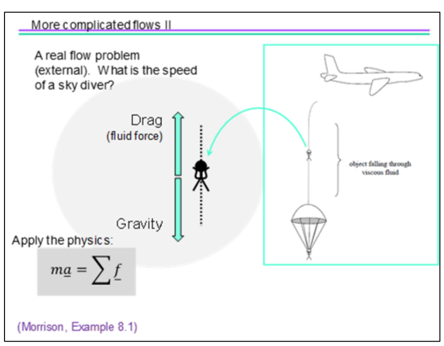
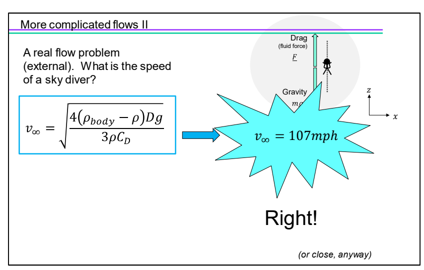
More complicated flows II

Powerful:





Solving <u>never-before-solved</u> problems.

With the right physics, and **dimensional analysis**

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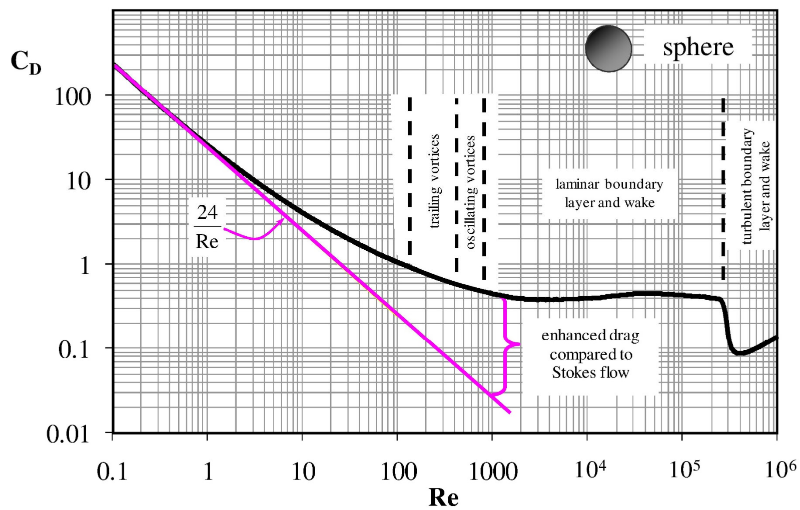
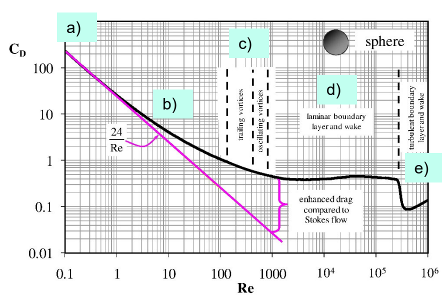
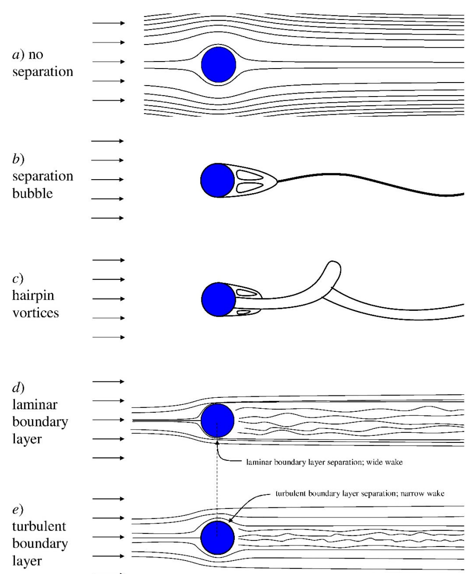
Fast flow around a sphere (dimensional analysis)



F. A. Morrison *An Introduction to Fluid Mechanics*, Cambridge 2013, Figure 8.23, p650

Fast flow around a sphere (dimensional analysis)





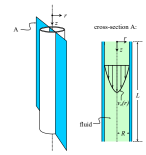
We have learned somethings that are very powerful.

1. External flows⇒ use drag coefficient for real external flows

More complicated flows II: From *Nice* to ***Powerful***

Nice:

Learning to solve one particular problem (or a group of related problems)



Powerful:

Solving never-before-solved problems.

?

2. In general, "simple" problems can lead to solutions to "complex" problems through dimensional analysis (and data correlation)

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More complicated flows II

Powerful:

Solving never-before-solved problems.

What's left?

Left to explore in fluid mechanics:

Videos: NCFMF (Drag parts 1-4)

- What is non-creeping flow like?
  (boundary layers)
- Viscosity dominates in creeping flow, what about the flow where inertia dominates?
  (potential flow)
- What about mixed flows (viscous+inertial)?
  (boundary layers)
- What about really complex flows (curly)?
  (vorticity, irrotational+circulation)

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More complicated flows II

Powerful:

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Left to explore in fluid mechanics:

- What is **non**-creeping flow like?
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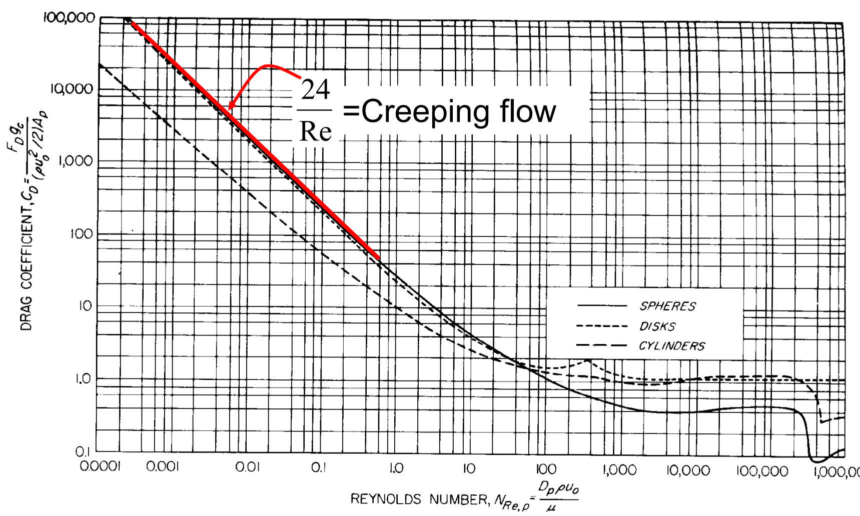
graphical correlation

Steady flow of an incompressible, Newtonian fluid around a sphere



McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

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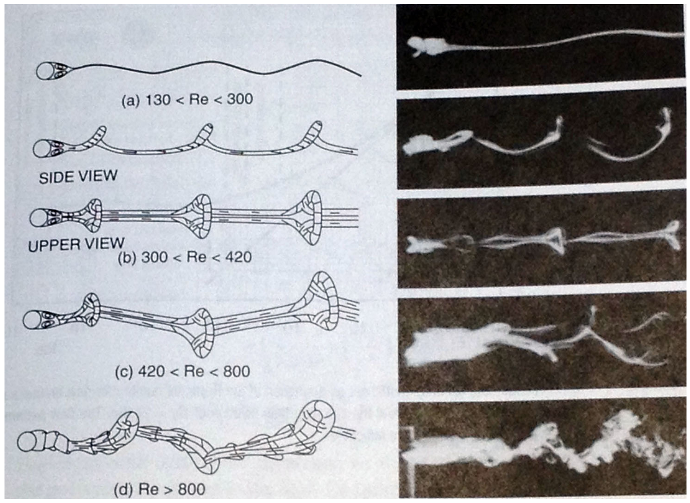
More complicated flows III

What does **non-creeping** flow look like?

(Let's look in a wind tunnel)

Can we predict these flows?



Text, Figure 8.22, p649, from Sakamoto and Haniu, 1990

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graphical correlation

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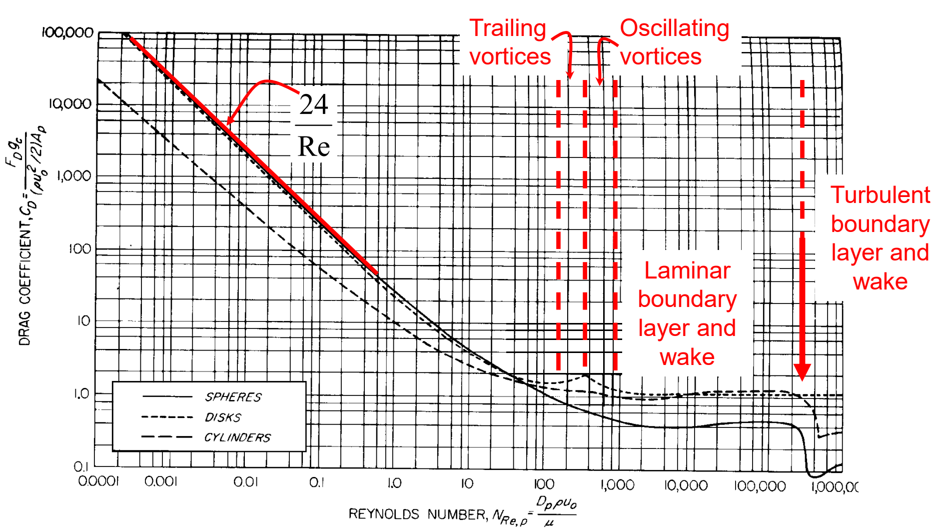

McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

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More complicated flows II

Powerful:

Solving never-before-solved problems.

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  (boundary layers)
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  (potential flow)
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  (boundary layers)
- What about really complex flows (curly)?
  (vorticity, irrotational+circulation)

Can we predict these flows?

Let's apply our methods

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Flow where **Viscosity** Dominates:

Nondimensional Navier-Stokes Equation:

$$\frac{\partial \underline{v}^*}{\partial t^*} + \left(\underline{v}\cdot\nabla\underline{v}\right)^* = -\nabla^* P + \frac{\mu}{\rho V D}\left(\nabla^2\underline{v}\right)^* + \frac{gD}{V^2}\underline{g}^*$$

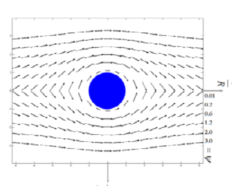
With the appropriate terms in spherical coordinates

No free surfaces

We considered the creeping flow limit:

$$\mathrm{Re}\left(\frac{\partial \underline{v}^*}{\partial t^*} + \left(\underline{v}\cdot\nabla\underline{v}\right)^*\right) = -\mathrm{Re}\nabla^* P + \left(\nabla^2\underline{v}\right)^*$$

small Re



Solve for a sphere,

$$\mathrm{Cd} = \frac{24}{\mathrm{Re}}$$

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Flow where **Inertia** Dominates (Re large):

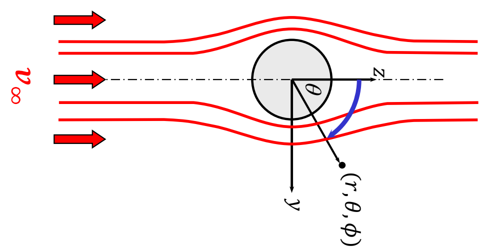
Let's predict these flows!

Consider the high Re limit:

$$\left(\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v} \cdot \nabla \underline{v})^*\right) = -\nabla^* P + \frac{1}{\text{Re}}(\nabla^2 \underline{v})^*$$

$\text{Re} \rightarrow \infty$

Now solve for flow around a sphere



Potential flow around a Sphere (high Re, no viscosity)

(equation 8.203)

Continuity: $\nabla^* \cdot \underline{v}^* = 0$

N-S:
$$\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v} \cdot \nabla \underline{v})^* = -\frac{\partial P^*}{\partial z^*}$$

drag:
$$C_D = \frac{2}{\pi}\int_0^{2\pi}\int_0^{\pi} \left[-P^* \cos\theta\right]_{r^*=\frac{1}{2}} \sin\theta \, d\theta \, d\phi$$

Predictions:

(the math requires specialized expertise)

Potential flow around a Sphere (high Re, no viscosity)

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(the math requires specialized expertise)

Solutions:

$$\underline{v} = \begin{pmatrix} v_\infty \left(1 - \left(\frac{R}{r}\right)^3\right)\cos\theta \\ -v_\infty \left(1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right)\sin\theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

$$P(r,\theta) = P_\infty + \frac{1}{2}\rho v_\infty^2 \left(2\left(\frac{R}{r}\right)^3\left(1 - \frac{3}{2}\sin^2\theta\right) - \left(\frac{R}{r}\right)^6\left(1 - \frac{3}{4}\sin^2\theta\right)\right)$$

(equation 8.238-9)

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drag: $$C_D = \frac{2}{\pi} \int_0^{2\pi} \int_0^{\pi} [-P^* \cos\theta]_{r^*=\frac{1}{2}} \sin\theta \, d\theta \, d\phi$$

How do these results compare to what we see at high Re?

Solution:

$$\underline{v} = \begin{pmatrix} v_\infty \left(1 - \left(\frac{R}{r}\right)^3\right)\cos\theta \\ -v_\infty \left(1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right)\sin\theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

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(equation 8.238-9)

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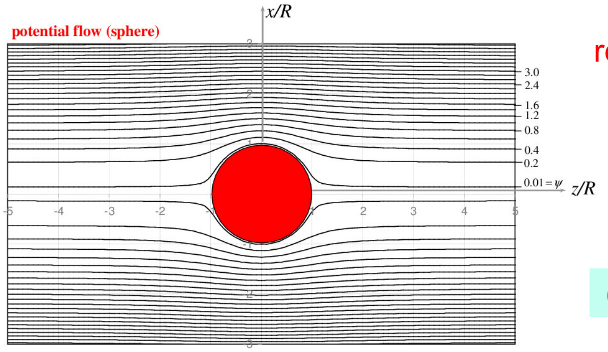
Potential flow around a Sphere (high Re, no viscosity)

Solution:



How do these results compare to what we see at high Re?

(does it match?)

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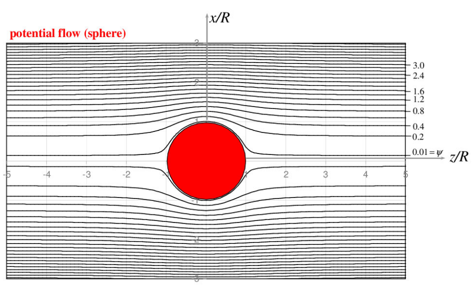
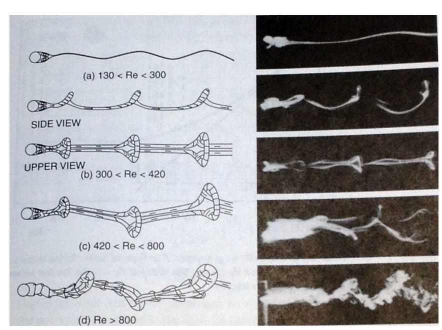
Potential flow around a Sphere (high Re, no viscosity)

(does it match?)

How does this compare to what we see at high Re?

Solution (all high Re):





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Potential flow around a Sphere (high Re ... viscosity)

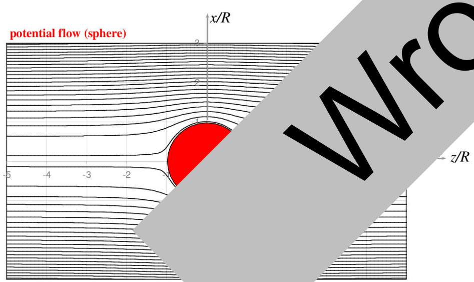
(does it match?)

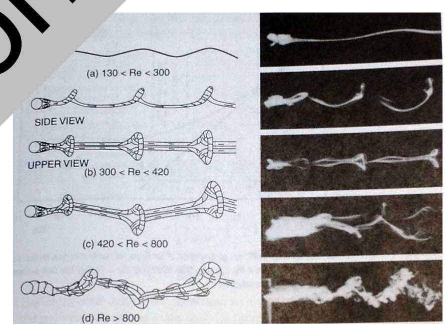
No.

Solution (all high Re):

...does this ...pare to what we see at high Re?

Wrong!





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Potential flow around a Sphere (high Re, no viscosity)

$$C_D = \frac{2}{\pi}\int_0^{2\pi}\int_0^{\pi}\left[-P^*\cos\theta\right]_{r^*=\frac{1}{2}}\sin\theta\, d\theta\, d\phi$$

Solution:

$$P(r,\theta) = P_\infty + \frac{1}{2}\rho v_\infty^2\left(2\left(\frac{R}{r}\right)^3\left(1-\frac{3}{2}\sin^2\theta\right)-\left(\frac{R}{r}\right)^6\left(1-\frac{3}{4}\sin^2\theta\right)\right)$$

$$P^* = \frac{P}{\rho v_\infty^2}$$

$$C_D = \cdots$$

(equation 8.238-9)

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Potential flow around a Sphere (high Re, no viscosity)

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$$P^* = \frac{P}{\rho v_\infty^2}$$

…

$C_D = 0$

What?

(d'Alembert's paradox)

(Example 8.10)

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Potential flow around a Sphere (high Re … viscosity)

$$C_D = \frac{2}{\pi}\int_0^{2\pi}\int_0^{\pi}\left[-P^*\cos\theta\right]_{r^*=\frac{1}{2}}\sin\theta\, d …$$

Solution:

$P(r,\theta) = P$ … $\frac{}{2}\sin^2\theta\Big) - \left(\frac{R}{r}\right)^6\left(1-\frac{3}{4}\sin^2\theta\right)\Big)$

P^* …

…

$C_D = 0$

…bert's paradox)

Wrong!

(Example 8.10)

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Potential flow around a Sphere (high Re viscosity)

(equation 8.20

Continuity: $\nabla^* \cdot \underline{v}^* = 0$

N-S: $\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v} \cdot \nabla \underline{v})^* = -\frac{\partial P^*}{}$

drag: $C_D = \frac{2}{\pi}\int_0^{2\pi}\int_0^{\pi}[-P^* \text{co}$ ... ϕ

Wrong!

Solution:

$$\underline{v} = \begin{pmatrix} v_\infty\left(1-\left(\frac{R}{r}\right)^3\right)\cos\theta \\ -v_\infty\left(1+\frac{1}{2}\left(\frac{R}{r}\right)^3\right)\sin\theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

$$P \ldots \frac{}{2}\rho v_\infty^2\left(2\left(\frac{R}{r}\right)^3\left(1-\frac{3}{2}\sin^2\theta\right)-\left(\frac{R}{r}\right)^6\left(1-\frac{3}{4}\sin^2\theta\right)\right)$$

(equation 8.238-9)

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Potential flow around a Sphere (high Re, no viscosity)

Wrong!

$\frac{\partial \underline{v}^*}{} + (\underline{v} \cdot \nabla \underline{v})^* = -\frac{\partial P^*}{}$

Predicts:

- No drag (d'Alembert's paradox)
- Slip at the wall
- *Approximately* right pressure profile (near the wall)
- *Correct* velocity field away from the wall

$$P(r,\theta) = P_\infty + \frac{1}{2}\rho v_\infty^2\left(2\left(\frac{R}{r}\right)^3\left(1-\frac{3}{2}\sin^2\theta\right)-\left(\frac{R}{r}\right)^6\left(1-\frac{3}{4}\sin^2\theta\right)\right)$$

(equation 8.238-9)

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Potential flow around a Sphere (high Re, no viscosity)

~~Wrong!~~

$\frac{\partial \underline{v}^*}{} + (v \cdot \nabla v)^* = -\frac{\partial P^*}{}$

Predicts:

- No drag (d'Alembert's paradox)
- Slip at the wall
- *Approximately right pressure profile (near the wall)*
- *Correct velocity field away from the wall*

partially right.

$$P(r,\theta) = P_\infty + \frac{1}{2}\rho v_\infty^2 \left(2\left(\frac{R}{r}\right)^3 \left(1 - \frac{3}{2}\sin^2\theta\right) - \left(\frac{R}{r}\right)^6 \left(1 - \frac{3}{4}\sin^2\theta\right) \right)$$

(equation 8.238-9)

25

Potential flow around a Sphere (high Re, no viscosity)

Wrong!

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Predicts:

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(equation 8.238-9)

?

What now?

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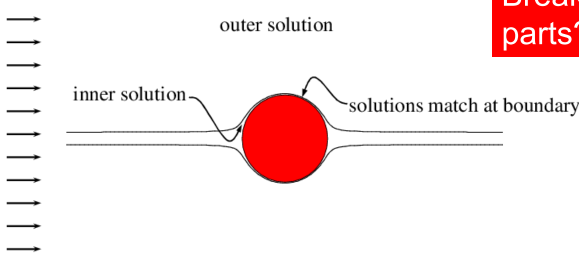
More complicated flows III

Predicts: **partially right.**

- No drag (d'Alembert's paradox)
- Slip at the wall
- *Approximately right pressure profile (near the wall)*
- *Right velocity field away from the wall*

Break into two parts?



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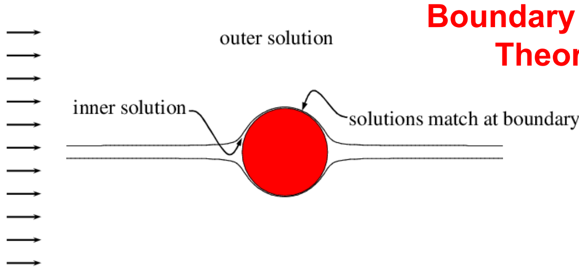
More complicated flows III

Prandtl's Great Idea (1904):

- Keep the good parts of the potential flow solution: $\underline{v}$ in free stream, $p(r,\theta)$ near the surface
- **Throw away the bad parts**: slip at the wall
- Solve a new no-slip problem near the wall, with $p(r,\theta)$ from the potential-flow solution, imposed in the free stream

Boundary Layer Theory



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What can we do to understand Boundary Layers?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Compare, adjust
6. Solve real problems

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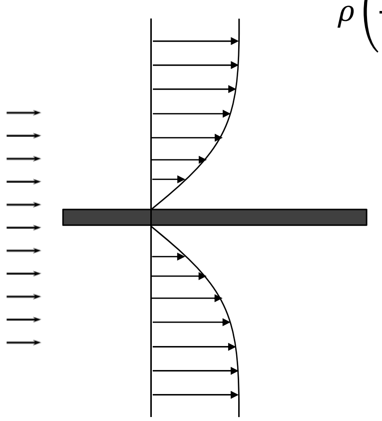
More complicated flows III

(Section 8.2)

Boundary Layer Theory

- Choose simplest boundary layer (flat plate)
- Nondimensionalize Navier-Stokes
- Eliminate small terms
- Solve

$$\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla p+\mu\nabla^2\underline{v}+\rho\underline{g}$$



<u>Characteristic values:</u>

U in principal flow direction v_1

V in direction perpendicular to wall, v_2

L length of plate for x_1

δ boundary layer thickness for x_2

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More complicated flows III

(Section 8.2)

Boundary Layer Theory

- Choose simplest boundary layer (flat plate)
- Nondimensionalize Navier-Stokes
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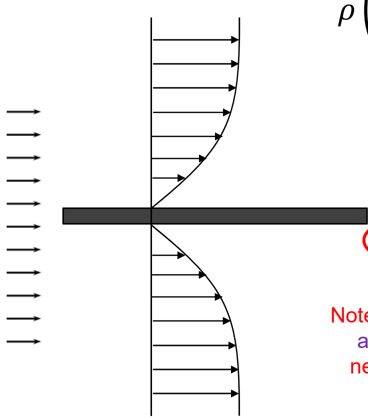

Characteristic values:

U in principal flow direction v_1

V in direction perpendicular to wall, v_2

L length of plate for x_1

δ boundary layer thickness for x_2

Note that for this flow, **two length** scales and **two velocities** are found to be needed for the dimensional analysis.

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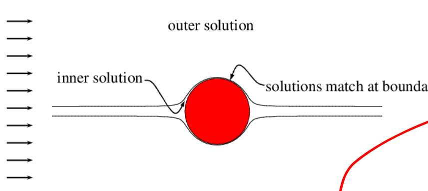
More complicated flows III

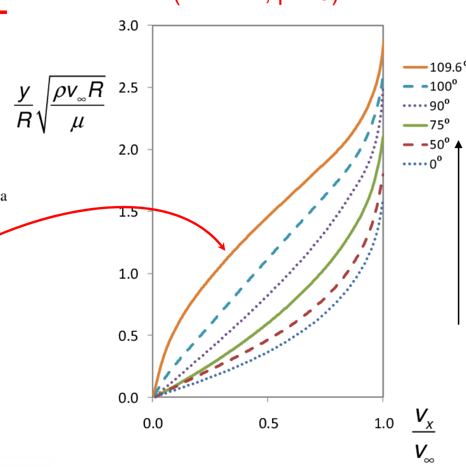
It works!

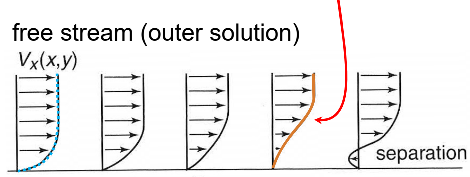
Boundary Layer Theory

(see text, p710)

- Apply to uniform flow approaching a sphere







Boundary layer velocity profiles as you progress from the stagnation point (0^o) to the top of the sphere (90^o) and beyond.

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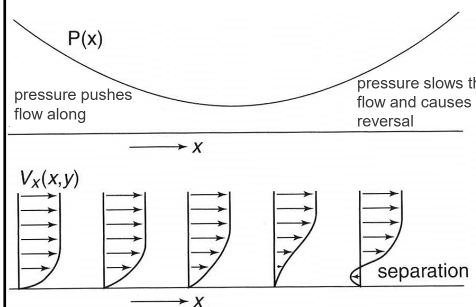
More complicated flows III

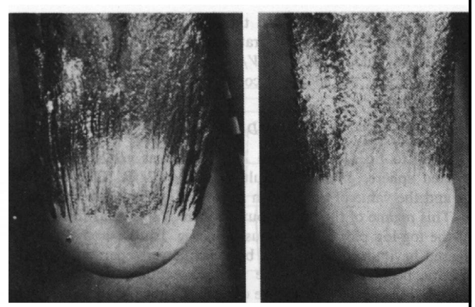
Boundary Layer Theory

- Explains boundary-layer separation
- Golf ball problem
- **BL separation caused by adverse pressure gradient**



It works!

smooth ball rough ball



H. Schlichting, Boundary Layer Theory (McGraw-Hill, NY 1955.

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More complicated flows III

Boundary Layer Theory

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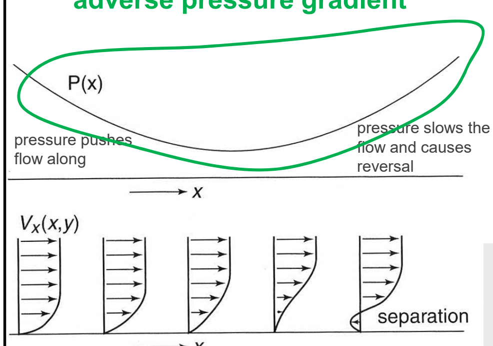

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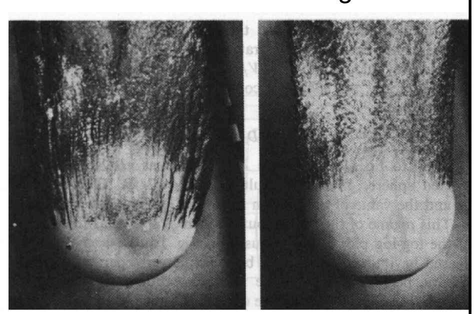
smooth ball rough ball



H. Schlichting, Boundary Layer Theory (McGraw-Hill, NY 1955.

The pressure distribution is like a storage mechanism for momentum in the flow; as other momentum sources die out, the pressure drives the flow.

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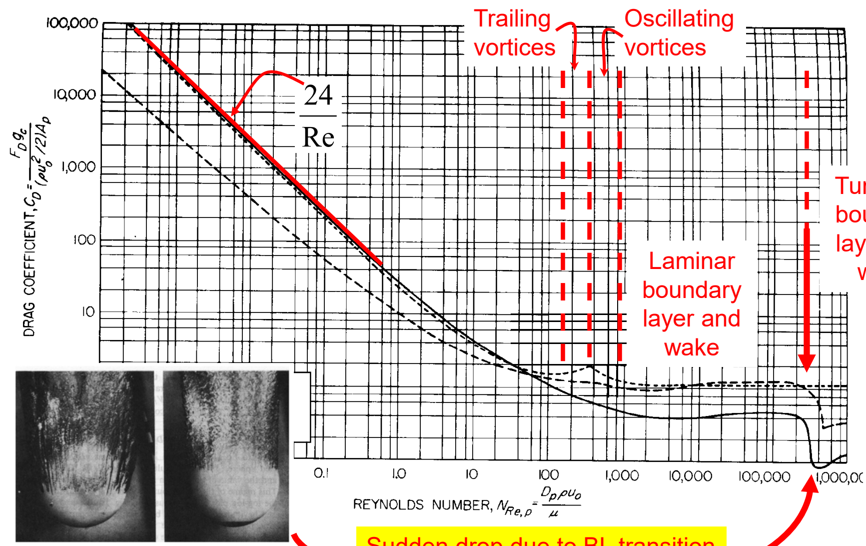
graphical correlation

Steady flow of an incompressible, Newtonian fluid around a sphere



McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

What do we do to understand complex flows?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles
- **Boundary Layers**

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

Solve Real Problems.

Powerful.

More complicated flows II

Powerful:

Solving never-before-solved problems.

Left to explore in fluid mechanics:

- What is non-creeping flow like?
  (boundary layers)
- Viscosity dominates in creeping flow, what about the flow where inertia dominates?
  (potential flow)

See text:

- What about mixed flows (viscous+inertial)?
  (boundary layers)
- What about really complex flows (curly)?
  (vorticity, irrotational+circulation)

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What do we do to understand complex flows?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles
- Boundary Layers
- Curvy flows, ….

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

Solve Real Problems.

Powerful.

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CM3110
Transport I
Part I: Fluid Mechanics

MichiganTech

Applied Topics:
Fluidized Beds

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

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ChemE Application of Ergun Equation

Fluidized beds

- •ion exchange columns
- •packed bed reactors
- •packed distillation columns
- •filtration

- •flow through soil (environmental issues, enhanced oil recovery)
- •fluidized bed reactors

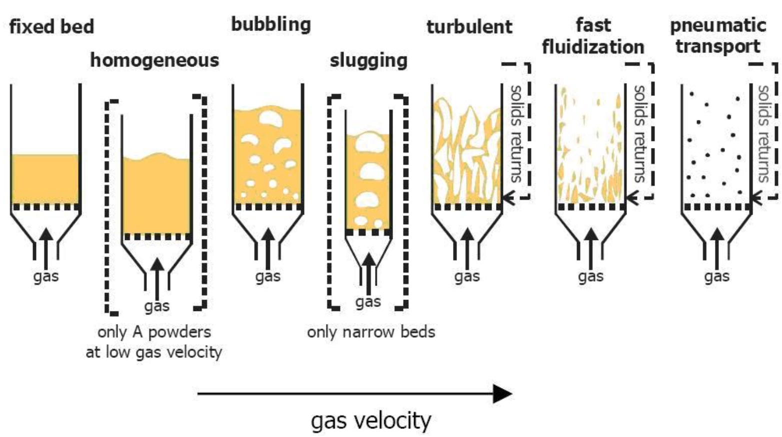

Image from: fluidizedbed2008.webs.com

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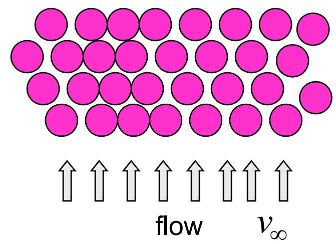
ChemE Application of Ergun Equation

Calculate the minimum superficial velocity at which a bed becomes fluidized.

In a fluidized bed reactor, the flow rate of the gas is adjusted to overcome the force of gravity and fluidize a bed of particles; in this state heat and mass transfer is good due to the chaotic motion.

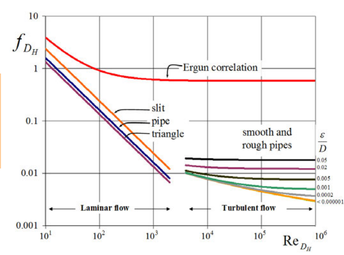


The Δp vs Q relationship can come from the **Ergun eqn** at *small* Re_{D_H}

$$\frac{100/3}{Re_{D_H}} + \frac{1.75}{3} = f_{D_H}$$

dominates

neglect



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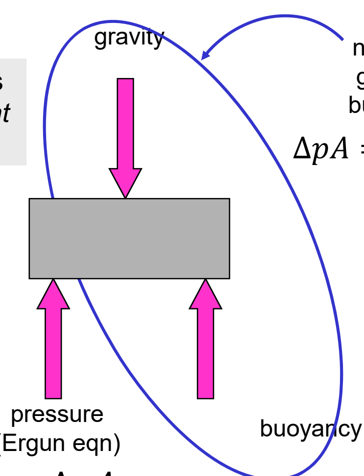
More Complex Applications II: Fluidized beds

Now we perform a force balance on the bed:

$$m\underline{a} = \sum \underline{f}$$

bed volume $= (1-\varepsilon)AL$

When the forces balance, *incipient fluidization*



net effect of gravity and buoyancy is:

$$\Delta pA = \underbrace{(\rho_p - \rho)}_{\frac{mass}{volume}}\underbrace{(1-\varepsilon)AL}_{volume}g$$

net force down $= \Delta$

force up $= \Delta pA$

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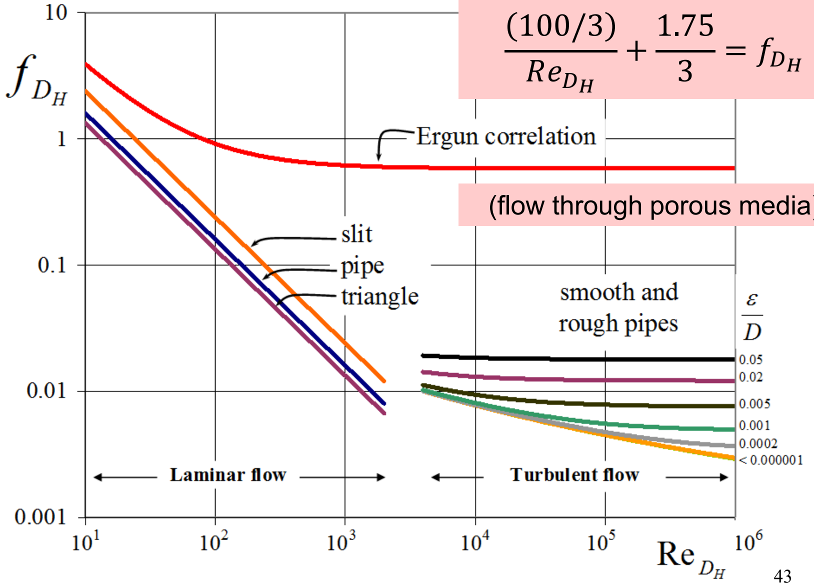
Flow Through Noncircular Conduits – All Flow Regimes

Ergun equation:

$$\frac{(100/3)}{Re_{D_H}} + \frac{1.75}{3} = f_{D_H}$$

(flow through porous media)



note: Re_p vs Re_{D_H}

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More Complex Applications II: Fluidized beds

When the forces balance, *incipient fluidization*

eliminate Δp;
solve for v_0

$$\begin{cases} \Delta p A = (\rho_p - \rho)(1-\varepsilon) A L g \\ \dfrac{100/3}{Re_{D_H}} = f_{D_H} = \dfrac{\Delta p}{L} \dfrac{D_H \varepsilon^2}{2\rho v_0^2} \end{cases}$$

Note:

$$Re_{D_H} = \frac{\rho (v_0/\varepsilon) D_H}{\mu}$$

$$D_H = \frac{2\varepsilon D_p}{3(1-\varepsilon)}$$

$$v_0 = \frac{(\rho_p - \rho) g D_p^2 \varepsilon^3}{150 \mu (1-\varepsilon)}$$

velocity at the point of *incipient fluidization*

Complete solution steps in Denn, Process Fluid Mechanics (Prentice Hall, 1980)

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Fluidized beds?

What do we do to understand ~~complex flows?~~

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles
- Boundary Layers

1. Find a simple problem that allows us to identify the physics (flow through packed bed)
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate (Ergun equation)
5. Solve real problems

Model the slow flow; calculate incipient fluidization criteria; take data on the more complex cases

Solve Real Problems.

Powerful.

See Perry's Handbook for more on Fluidized Beds

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CM3110
Transport I
Part I: Fluid Mechanics

MichiganTech

Applied Topics: Compressible Flow

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

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Compressible Fluids

- most fluids are somewhat compressible
- in chemical-engineering processes, compressibility is unimportant at most operating pressures
- even gases may be modeled as incompressible if $\Delta p < p_{mean}$

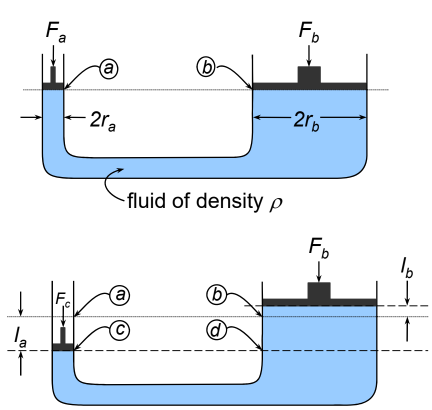
EXCEPT:
When the fluid velocity approaches the speed of sound

Compressible Fluids

How is pressure information transmitted in liquids and gases?

The Hydraulic Lift operates on Pascal's principle

Pressure exerted on an enclosed liquid is transmitted equally to every part of the liquid and to the walls of the container.



Compressible Fluids

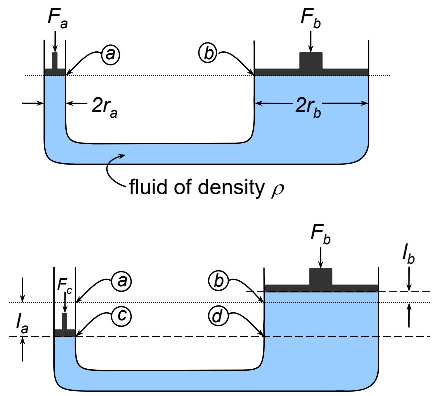
For static incompressible liquids,

The Hydraulic Lift operates on Pascal's principle

Pressure exerted on an enclosed liquid ***is transmitted equally*** *to every part of the liquid and to the walls of the container.*

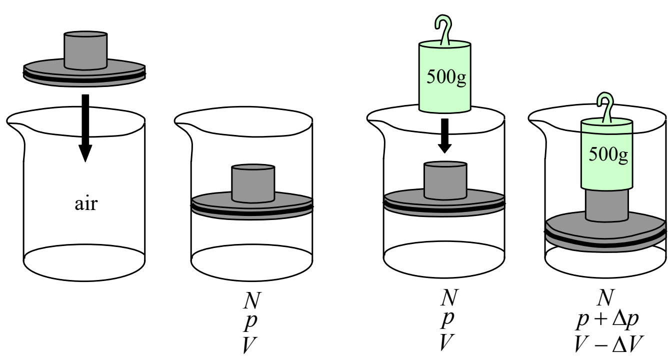
and essentially, instantaneously

(speed of sound)



Compressible Fluids

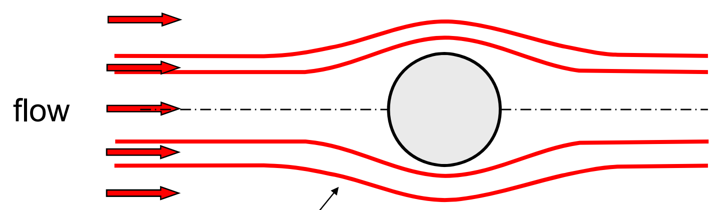
For static compressible fluids (gases), pressure causes volume change.



Compressible Fluids

For moving incompressible liquids and gases,

The presence of the obstacle is felt by the upstream fluid (pressure) and that information is transmitted very rapidly throughout the fluid.



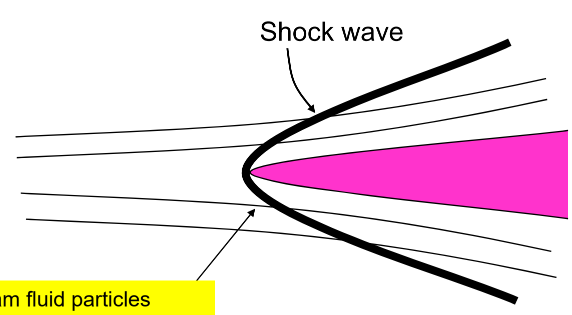
The streamlines adjust according to momentum conservation.

These fluid particles are not blocked by the sphere, but they feel its presence due to the **pressure field**.

Compressible Fluids

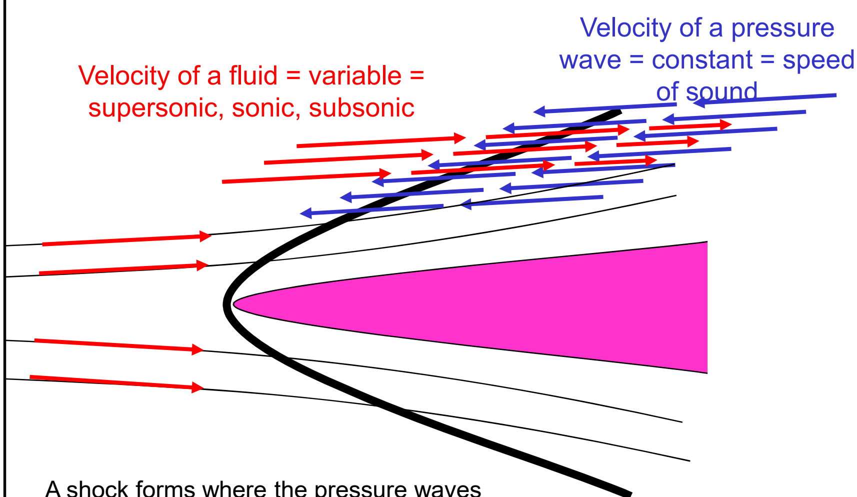
For **compressible fluids** moving near sonic speeds, information (pressure) and the gas itself are moving at comparable speeds.

Pressure piles up at the shock wave



The upstream fluid particles cannot feel the presence of the object because the object is **outrunning** the pressure field.

Compressible Fluids



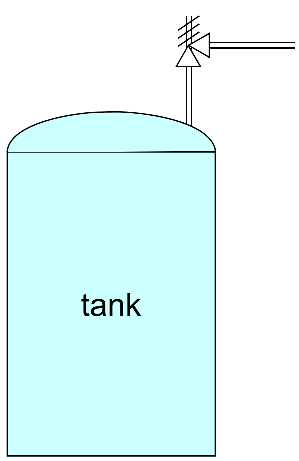
A shock forms where the pressure waves from the obstacle stack up, and the speed of the pressure wave traveling upstream equals the speed of the fluid traveling downstream.

Compressible Fluids

Super-sonic flows in Chem Eng:

Relief Valves (Safety Valves)

The rapid flows in relief valves can become sonic.



For supersonic flow, the flow rate is constant <u>no matter what the pressure drop is.</u>

(pressure waves pile up)

Choked Flow

Choked flow can be understood from basic equations of compressible fluid mechanics.

Compressible Fluids

Momentum and Energy in Compressible Fluids

incompressible

$\underline{\underline{\tilde{\tau}}} = \mu(\nabla \underline{v} + (\nabla \underline{v})^T)$

Microscopic momentum balance:

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \nabla\cdot\underline{\underline{\tilde{\tau}}} + \rho\underline{g}$$

Mechanical energy balance:

incompressible

$$\frac{\Delta p}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\delta z + F = \frac{W_{s,on}}{\dot{m}}$$

compressible?

Compressible Fluids

Momentum and Energy in Compressible Fluids

incompressible

$\underline{\underline{\tilde{\tau}}} = \mu(\nabla \underline{v} + (\nabla \underline{v})^T)$

Microscopic momentum balance:

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \nabla\cdot\underline{\underline{\tilde{\tau}}} + \rho\underline{g}$$

compressible

$$\underline{\underline{\tilde{\tau}}} = \mu\left(\nabla\underline{v} + (\nabla\underline{v})^T\right) - \left(\frac{2}{3}\mu - \kappa\right)\nabla\cdot\underline{v}$$

κ = bulk viscosity

Mechanical energy balance:

incompressible

$$\frac{\Delta p}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\delta z + F = \frac{W_{s,on}}{\dot{m}}$$

MEB for compressible?

Compressible Fluids

Mechanical energy balance (compressible)

Back up one step in the derivation and reintegrate without constant ρ assumption.

$$\frac{dp}{\rho} + VdV + gdz + dF = \frac{dW_{s,on}}{\dot{m}}$$

Assume:
- constant cross section
- constant mass flow $\rho VA = GA$
- neglect gravity
- no shaft work

$$G \equiv \rho V = \text{mass velocity}$$

Compressible Fluids
(need some thermo)

Mechanical energy balance (compressible)

Ideal Gas Law

$$pV = NRT$$

$$\frac{V}{N} = \frac{RT}{p}$$

$$\frac{V}{MN} = \frac{RT}{pM}$$

$$\frac{1}{\rho} = \frac{RT}{pM}$$

For isothermal flow:

$$p_1V_1 = NRT$$

$$p_2V_2 = NRT$$

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Also,

$$\frac{\rho_{av}}{p_{av}} = \frac{M}{RT}$$

$$\frac{2\rho_{av}}{p_1 + p_2} = \frac{M}{RT}$$

Compressible Fluids

Mechanical energy balance (compressible)

$G \equiv \rho V = \text{mass velocity}$

$$\left(p_2 - p_1\right) + \frac{G^2}{\rho_{av}} \ln \frac{p_1}{p_2} + \frac{2 f G^2}{\rho_{av} D}\left(L_2 - L_1\right) = 0$$

The compressible MEB predicts that there is a maximum velocity at

(see book)

$$V_{\max} = \sqrt{\frac{p_2}{\rho_2}} = \sqrt{\frac{RT}{M}} = \text{isothermal speed of sound}$$

Compressible Fluids

A better assumption than isothermal flow is adiabatic flow (no heat transferred). For this case,

$$V_{\max} = \sqrt{\frac{\gamma p_2}{\rho_2}} = \sqrt{\frac{\gamma RT}{M}} = \text{adiabatic speed of sound}$$

$$\gamma = \frac{C_p}{C_v}$$

(see book)

CM3110
Transport I
Part I: Fluid Mechanics

MichiganTech

More Complicated Flows III: Boundary-Layer Flow

(plus other applied topics)

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

Done!

Just one more thing

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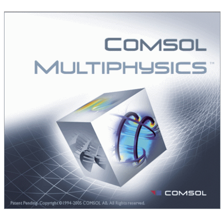
Numerical PDE Solving with Comsol 5.3a

Michigan Tech



www.comsol.com

Finite-element numerical differential equation solver. Applications include fluid mechanics and heat transfer.

1. Choose the physics (2D, 2D axisymmetric, laminar, steady/unsteady, etc.)
2. Choose flow geometry and fluid (shape of the flow domain)
3. Define boundary conditions
4. Design and generate mesh
5. Solve the problem
6. Calculate and plot engineering quantities of interest.

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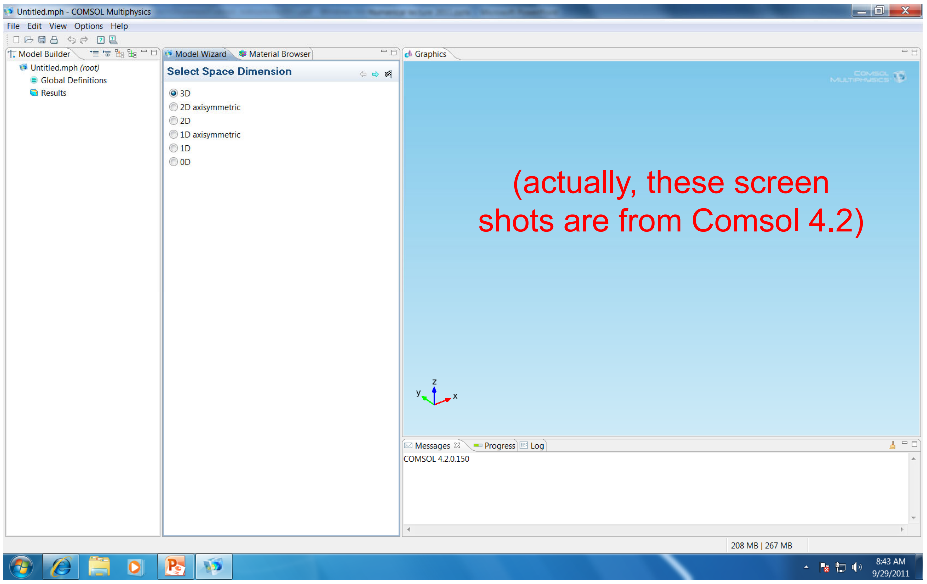
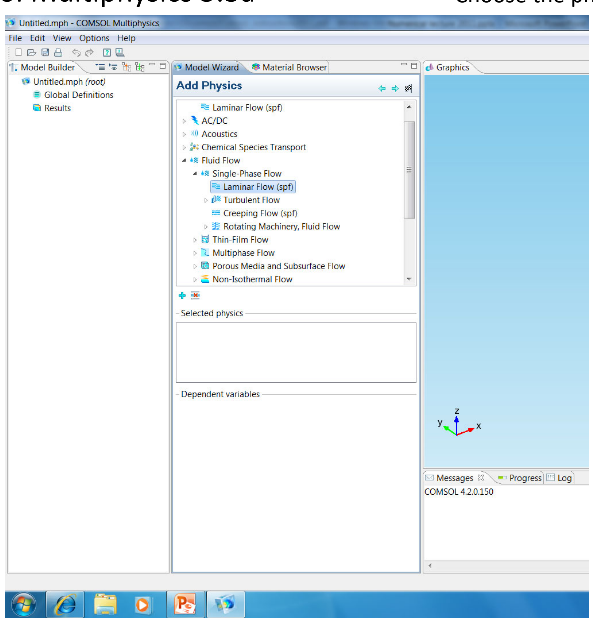
Comsol Multiphysics 5.3a

Launch the program **0**



Comsol Multiphysics 5.3a

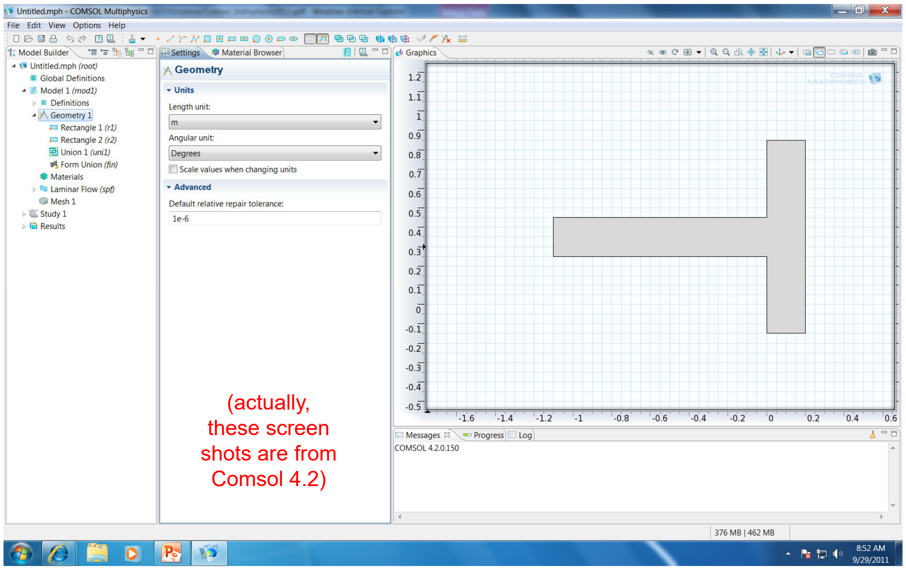
Choose the physics **1**



(actually, these screen shots are from Comsol 4.2)

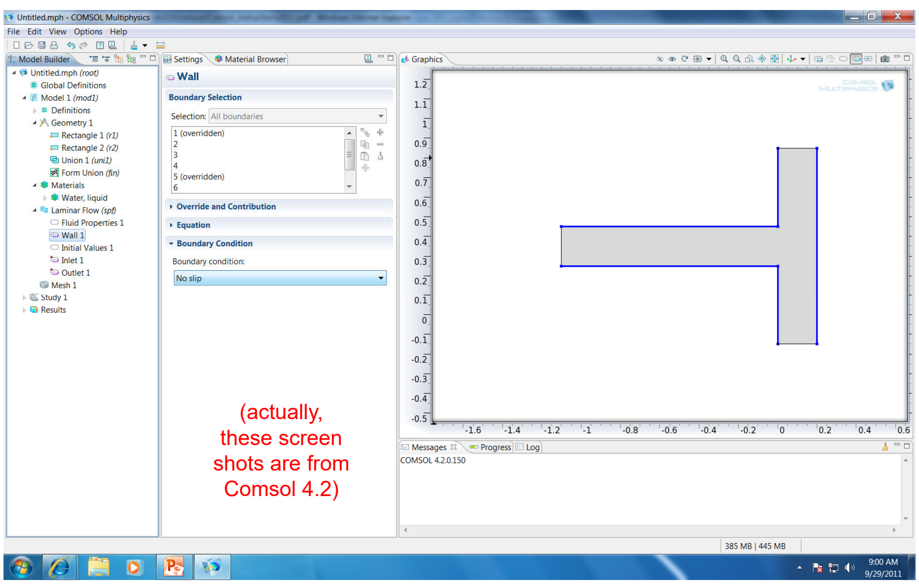
Comsol Multiphysics 5.3a

Choose flow geometry and fluid **2**



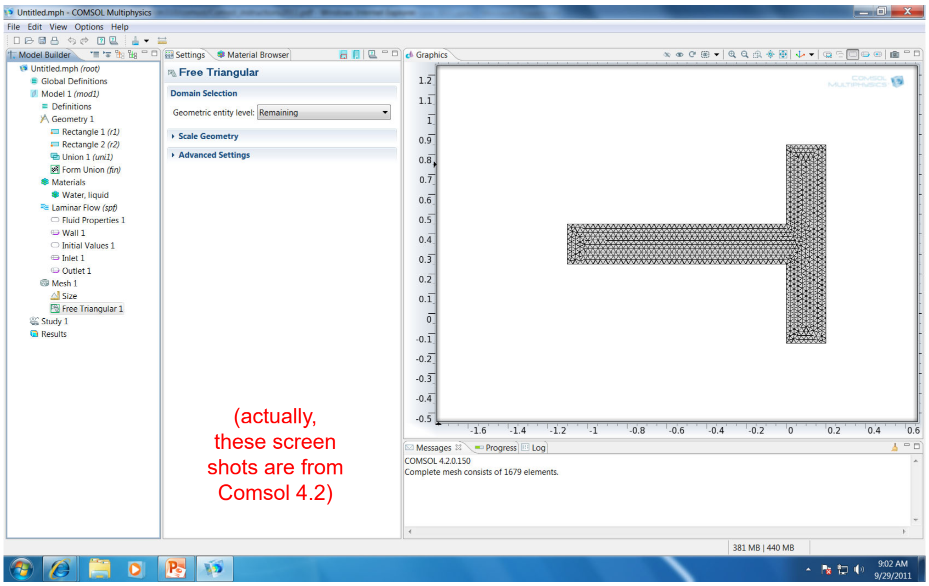
Comsol Multiphysics 5.3a

Define boundary conditions **3**



Comsol Multiphysics 5.3a

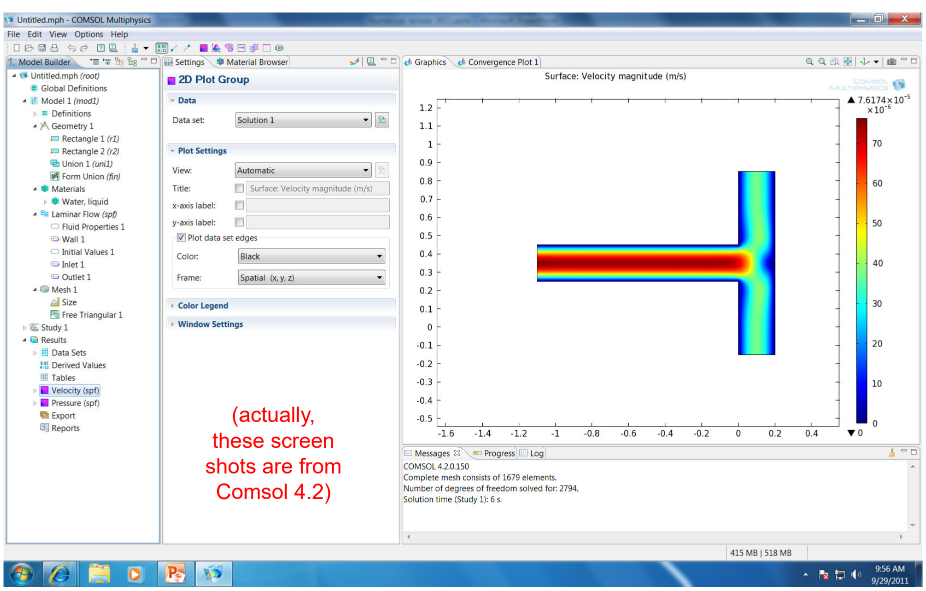
Design and generate mesh **4**



(actually, these screen shots are from Comsol 4.2)

Comsol Multiphysics 5.3a

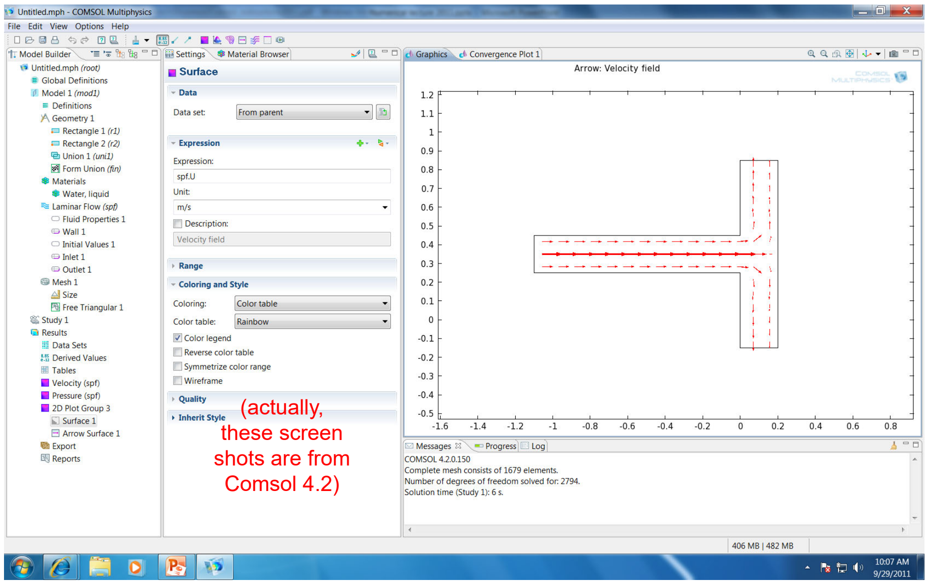
Solve the problem **5**



(actually, these screen shots are from Comsol 4.2)

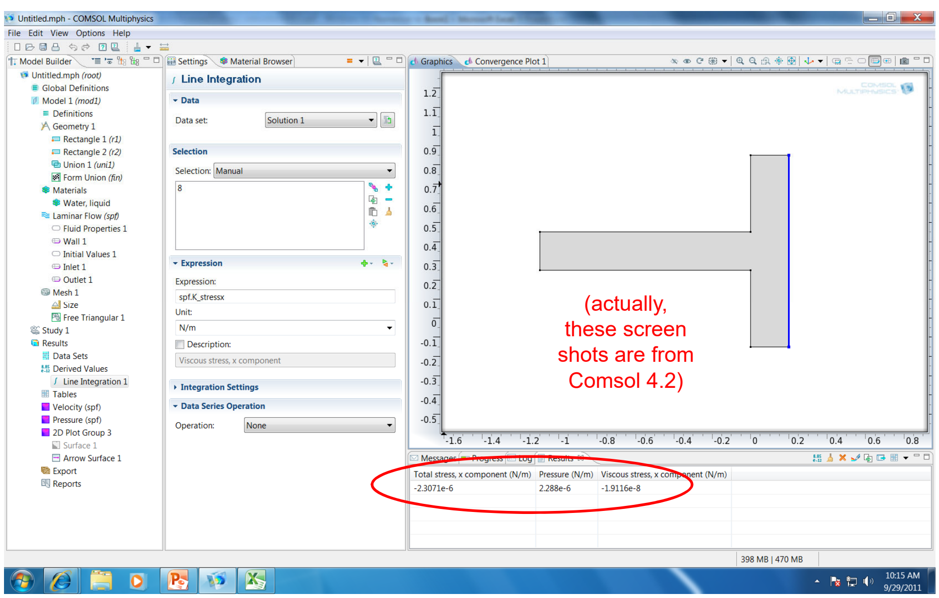
Comsol Multiphysics 5.3a

View the solution **5**

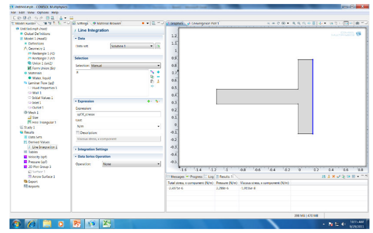


Comsol Multiphysics 5.3a

Calculate engineering problems of interest **6**



Comsol Multiphysics

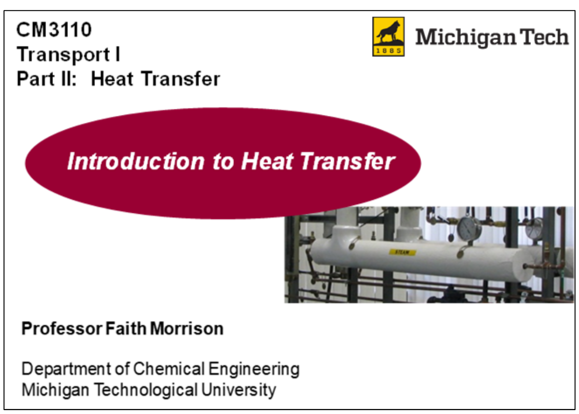


Comsol project:

- Due last day of classes
- **Individual** work
- 2 points for part 1 (instructions given)
- 3 points for part 2 (no instructions)
- Coming soon

Points applied to your course grade

Part II: Heat Transfer



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