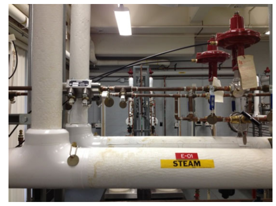
Michigan Tech

CM3110
Transport/Unit Ops I
Part II: Heat Transfer



Complex Heat Transfer – Dimensional Analysis

(Forced convection heat transfer)

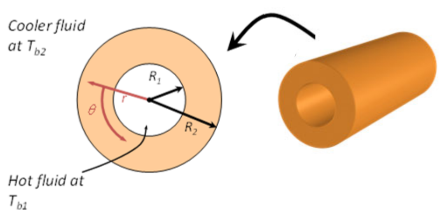
Professor Faith Morrison

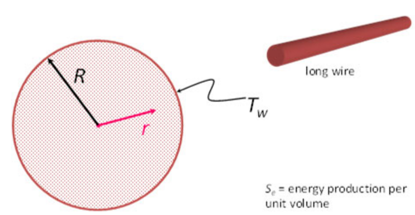
Department of Chemical Engineering
Michigan Technological University

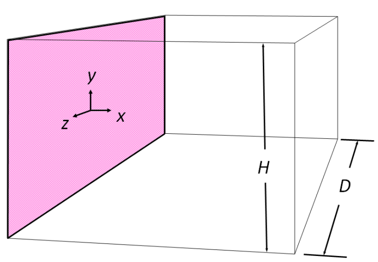
1D Heat Transfer

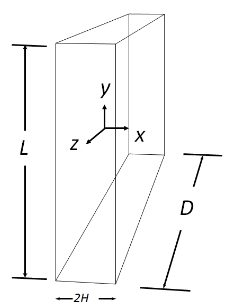
(what have we been up to?)

Examples of (simple, 1D) Heat Conduction



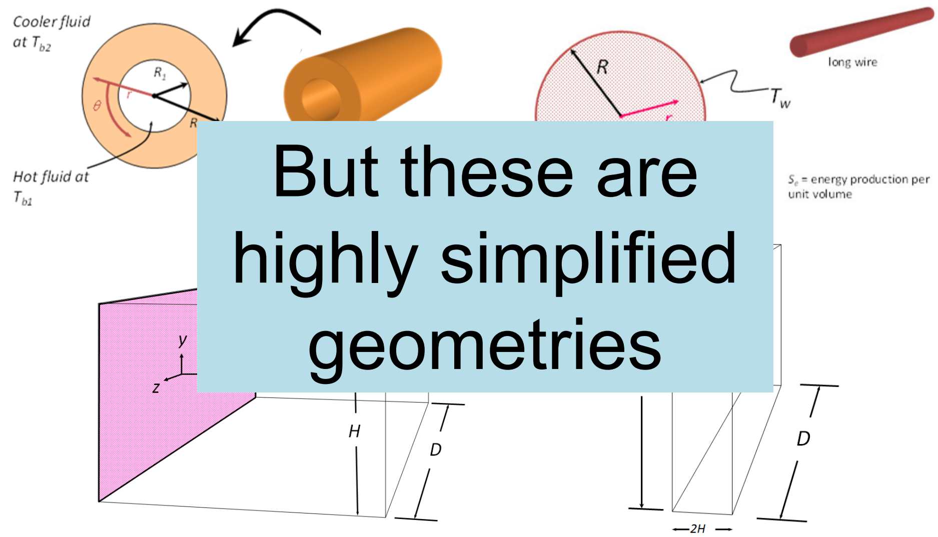






1D Heat Transfer

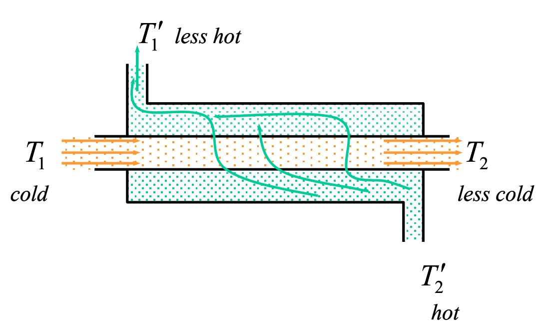
Examples of (simple, 1D) Heat Conduction

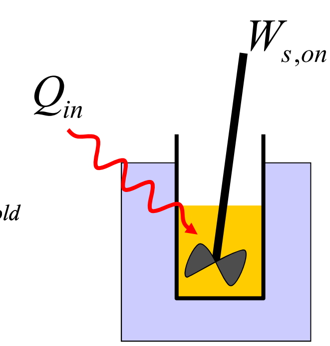


But these are highly simplified geometries

Complex Heat Transfer

How do we handle complex geometries, complex flows, complex machinery?

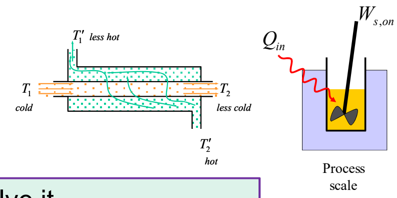




Process scale

Complex Heat Transfer – Dimensional Analysis

(***Answer***: *Use the same techniques we have been using in fluid mechanics)*



Engineering Modeling

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis thus far:

- Flow in pipes at all flow rates (laminar and turbulent)
  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\mathrm{Re}, L/D)$
- Rough pipes — ***Solution:*** add additional length scale; then nondimensionalize
- Non-circular conduits — ***Solution:*** Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes
  Solution: Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\mathrm{Re})$
- Boundary layers — ***Solution:*** Two components of velocity need independent lengthscales

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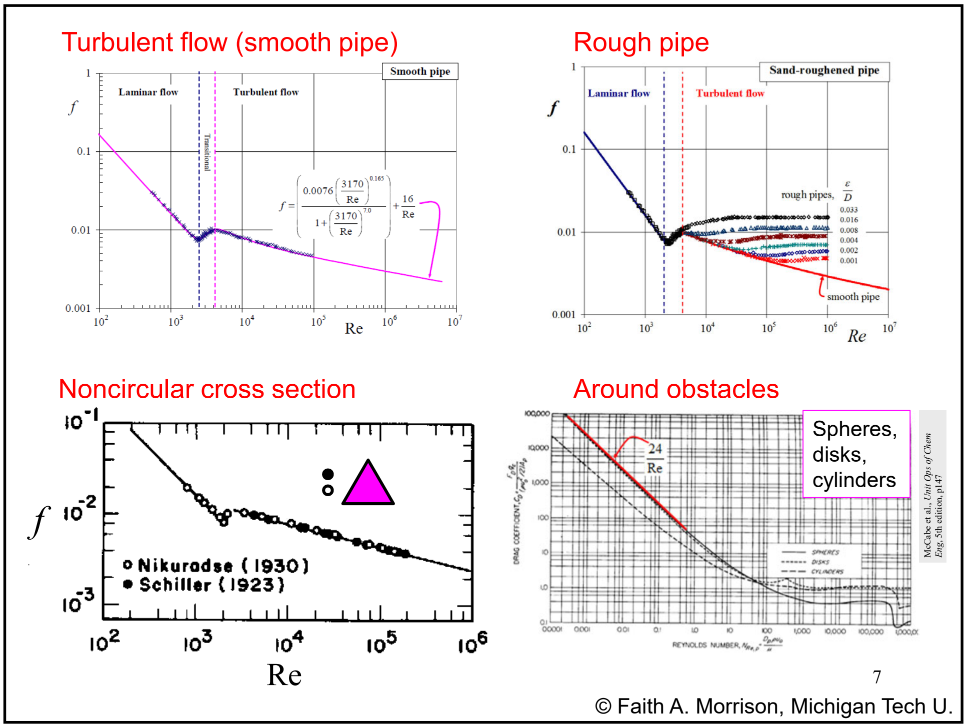
Turbulent flow (smooth pipe)
Smooth pipe
Laminar flow
Turbulent flow
Transitional
f
Re
Rough pipe
Sand-roughened pipe
Laminar flow
Turbulent flow
rough pipes, ε/D
0.033
0.016
0.008
0.004
0.002
0.001
smooth pipe
Re
Noncircular cross section
Nikuradse (1930)
Schiller (1923)
f
Re
Around obstacles
24/Re
Spheres, disks, cylinders
McCabe et al., Unit Ops of Chem Eng, 5th edition, p147
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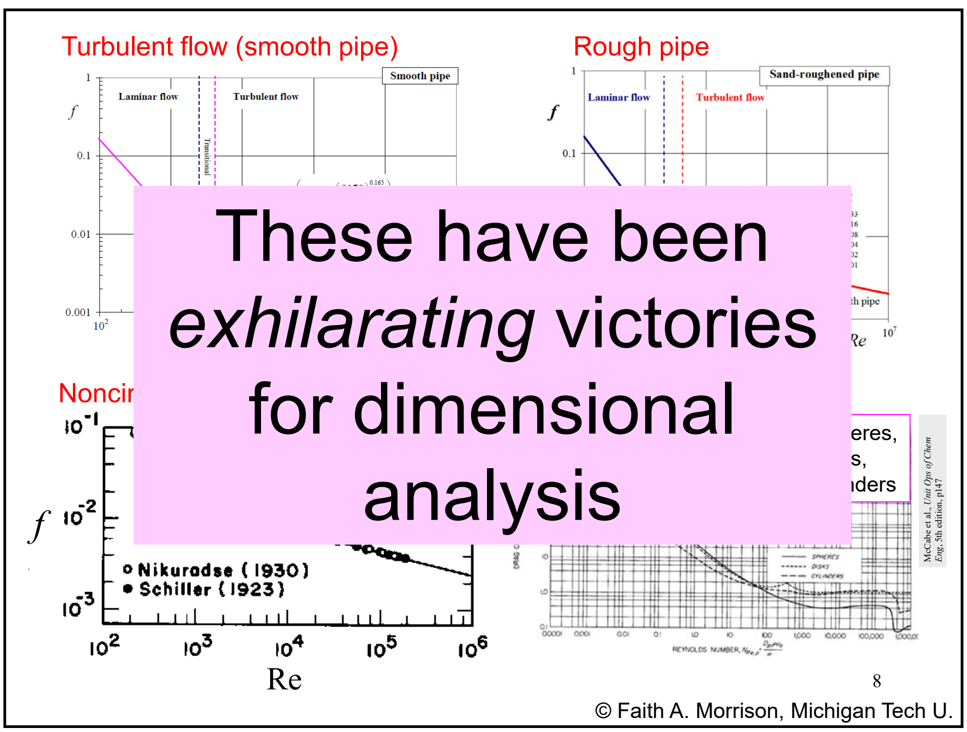

Turbulent flow (smooth pipe)
Rough pipe
These have been *exhilarating* victories for dimensional analysis
Nikuradse (1930)
Schiller (1923)
Re
8
© Faith A. Morrison, Michigan Tech U.

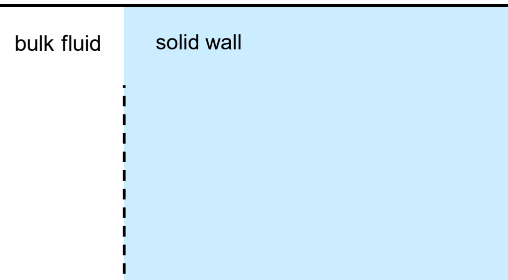
Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?



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Complex Heat Transfer – Dimensional Analysis

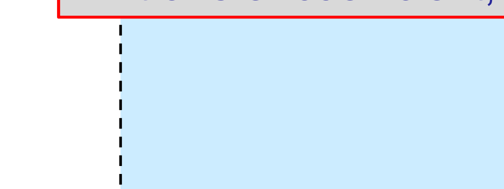
Now, move to heat transfer:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection hea
Solut

•Radiation heat transfer
Solut

We have already started using the <u>results</u>/techniques of dimensional analysis through defining the heat transfer coefficient, h



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Complex Heat Transfer – Dimensional Analysis

Now, move to heat transfer:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection hea…
Solut…

•Radiation heat transfer…
Solut…

bulk

We have already started using the results/techniques of dimensional analysis through defining the heat transfer coefficient, h

(recall that we did this in fluids too: we used the $f(\mathrm{Re})$ correlation (Moody chart) long before we knew where that all came from)

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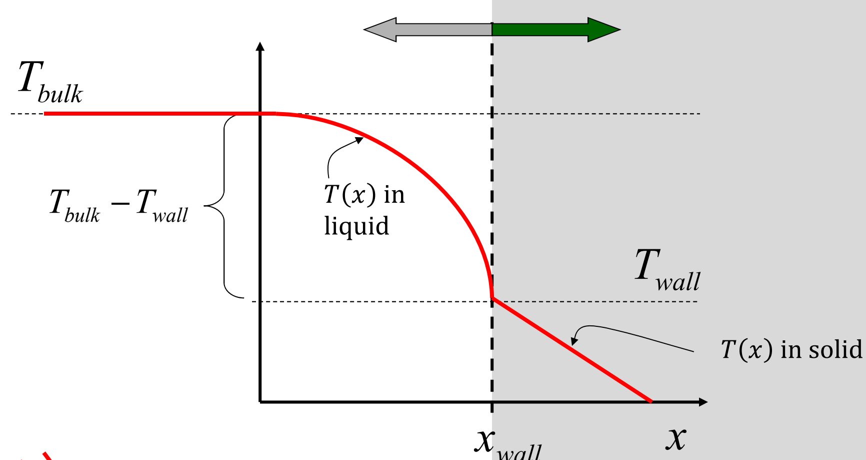
Handy tool:
Heat Transfer Coefficient

bulk fluid solid wall



The temperature variation near-wall region is caused by complex phenomena that are lumped together into the heat transfer coefficient, *h*

Complex Heat Transfer – Dimensional Analysis

The flux at the wall is given by the empirical expression known as
Newton's Law of Cooling

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| \equiv h \left|T_{bulk} - T_{wall}\right|$$

***h* depends on:**
- geometry
- fluid velocity
- fluid properties
- temperature difference

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Complex Heat Transfer – Dimensional Analysis

The flux at the wall is given by the empirical expression known as
Newton's Law of Cooling

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| \equiv h \left|T_{bulk} - T_{wall}\right|$$

To get values of h for various situations, we need to measure data and create data correlations (**dimensional analysis**)

***h* depends on:**
- geometry
- fluid velocity
- fluid properties
- temperature difference

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Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall

Solution: ?

•Natural convection heat transfer from fluid to wall

Solution: ?

•Radiation heat transfer from solid to fluid

Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

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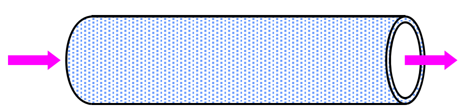
Chosen problem: Forced Convection Heat Transfer
Solution: Dimensional Analysis



Following procedure familiar from pipe flow,

- **What are governing equations?**
- **Scale factors (dimensionless numbers)?**
- **Quantity of interest?**

  Answer: Heat flux at the wall

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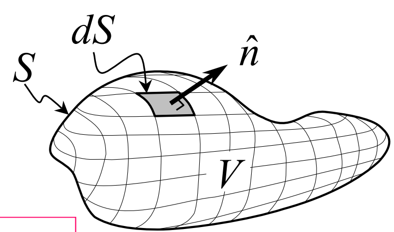
General Energy Transport Equation

(microscopic energy balance)

As for the derivation of the microscopic momentum balance, the microscopic energy balance is derived on an arbitrary volume, *V*, enclosed by a surface, *S*.



Gibbs notation:

$$\boxed{\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S}$$

see handout for component notation

General Energy Transport Equation

(microscopic energy balance; **<u>in the fluid</u>**)

$$\rho \hat{C}_p \left(\underbrace{\frac{\partial T}{\partial t}}_{\text{rate of change}} + \overbrace{\underline{v} \cdot \nabla T}^{\text{convection}} \right) = \underbrace{k \nabla^2 T}_{\text{conduction (all directions)}} + \overbrace{S}^{\text{source}}$$

(energy generated per unit volume per time)

fluid velocity must satisfy equation of motion, equation of continuity

see handout for component notation

Complex Heat Transfer – Dimensional Analysis

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S$$

Note: this handout is also on the web

https://pages.mtu.edu/~fmorriso/cm310/energy.pdf

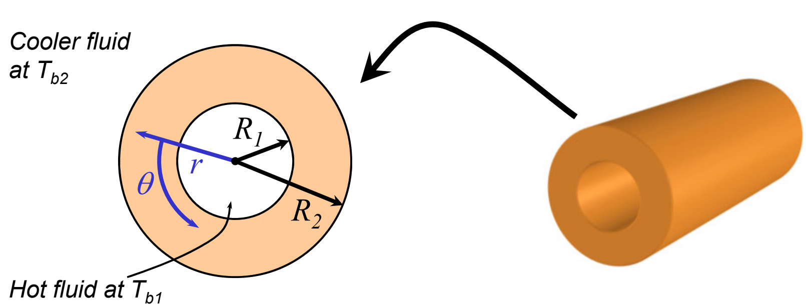
** REVIEW ** REVIEW **

Forced-convection heat transfer

Example: Heat flux in a cylindrical shell

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **fluid on the inside** is at T_{b1} and the **fluid on the outside** is at T_{b2}? (T_{b1}>T_{b2})*



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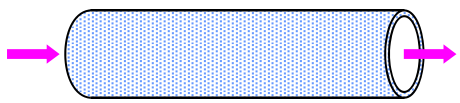
Now: How do develop correlations for *h*?

Consider: Heat-transfer to from flowing fluid inside of a tube – forced-convection heat transfer



T_1= core bulk temperature
T_o= wall temperature
$T(r,\theta,z)$ = temp distribution in the fluid

In principle, with the right math/computer tools, we could calculate the complete temperature and velocity profiles in the moving fluid.

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Complex Heat Transfer – Dimensional Analysis

What are governing equations?

Microscopic energy balance plus Navier-Stokes, continuity

Scale factors?

Re, Fr, L/D plus whatever comes from the rest of the analysis

Quantity of interest (like wall force, drag)?

Heat transfer coefficient

The quantity of interest in forced-convection heat transfer is *h*

How is the heat transfer coefficient related to the full solution for $T(r,\theta,z)$ in the fluid?

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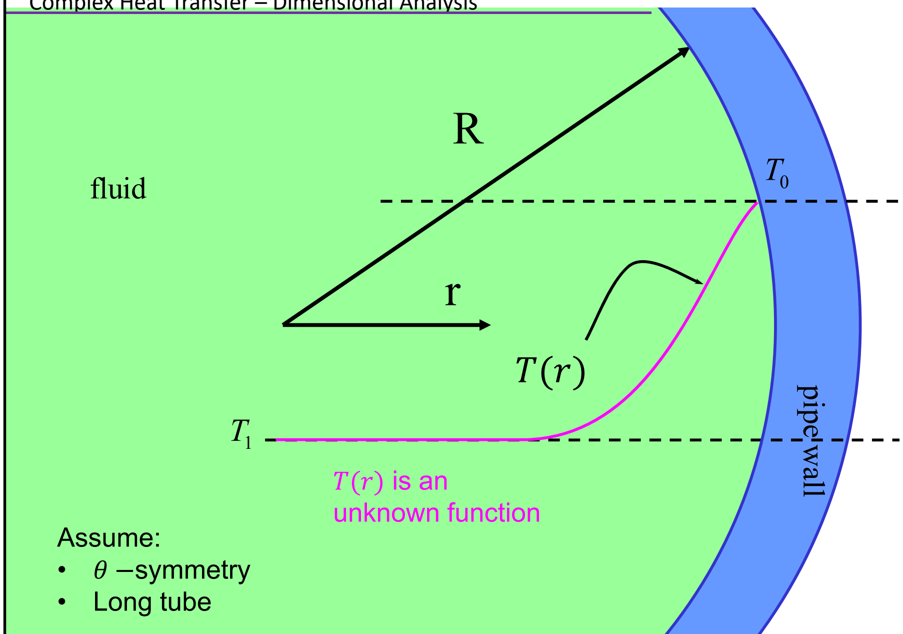
Complex Heat Transfer – Dimensional Analysis



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Complex Heat Transfer – Dimensional Analysis

At the boundary, (Newton's Law of Cooling is the **boundary condition**)

Total heat flow through (at) the wall in terms of *h*

$$\left|\frac{q_r}{A}\right| = h|T_1 - T_0|$$

$$Q = (2\pi RL)(h)(T_1 - T_0)$$

We can calculate the total heat transferred from $T(r)$ in the fluid:

Total heat conducted to the wall from the fluid

$$Q = \iint_S [\hat{n} \cdot \underline{\tilde{q}}]_{surface} \, dS$$

$$\tilde{q} = \frac{q_r}{A} = -k\frac{\partial T}{\partial r}$$

We need $T(r)$ ***in the fluid***

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Complex Heat Transfer – Dimensional Analysis

Equate these two: Total heat flow through the wall

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S [\hat{e}_r \cdot \underline{\tilde{q}}]_{surface}\, dS$$

$$\underbrace{(2\pi RL)(h)(T_1 - T_0)}_{\text{Total heat flow at the wall in terms of } h} = Q = \underbrace{\int_0^{2\pi}\int_0^{L} -k \left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta}_{\text{Total heat conducted to the wall from the fluid}}$$

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Complex Heat Transfer – Dimensional Analysis

Equate these two: Total heat flow through the wall

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S [\hat{e}_r \cdot \underline{\tilde{q}}]_{surface}\, dS$$

$$(2\pi RL)(h)(T_1 - T_0) = Q = \int_0^{2\pi}\int_0^{L} -k \left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta$$

Now, non-dimensionalize this expression

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Complex Heat Transfer – Dimensional Analysis

Non-dimensionalize

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$$

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Complex Heat Transfer – Dimensional Analysis

$$h(\pi \cancel{D} L)\cancel{(T_1 - T_o)} = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{(T_1 - T_o)}}{\cancel{D}} \frac{\cancel{D^2}}{2} dz^* d\theta$$

$$2\pi \underbrace{\left(\frac{hD}{k}\right)}_{}\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} - \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$\boxed{Nu = Nu\left(T^*, \frac{L}{D}\right)}$$

one additional dimensionless group

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Complex Heat Transfer – Dimensional Analysis

$$h\left(\pi \cancel{D} L\right)\cancel{\left(T_1 - T_o\right)} = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{\left(T_1 - T_o\right)}}{\cancel{D}}\frac{\cancel{D^2}}{2}\, dz^*\, d\theta$$

This is a function of Re through fluid $\underline{v}$ distribution

$$2\pi\underbrace{\left(\frac{hD}{k}\right)}\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^*\, d\theta$$

Nusselt number, Nu
(dimensionless heat-transfer coefficient)

$$\boxed{Nu = Nu\left(T^*, \frac{L}{D}\right)}$$

one additional dimensionless group

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Complex Heat Transfer – Dimensional Analysis

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \frac{1}{\mathrm{Pe}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\boxed{\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr}g^*}$$

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Non-dimensional Continuity Equation

$$\boxed{\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0}$$

Quantity of interest

$$\boxed{Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^*\, d\theta}$$

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Complex Heat Transfer – Dimensional Analysis

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ three dimensionless groups:

no free surfaces

$$Nu = Nu\left(\text{Re}, \text{Pr}, \cancel{\text{Fr}}, \frac{L}{D} \right)$$

Peclet number

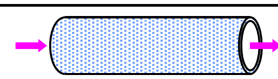
$$\text{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k} \frac{\rho V D}{\mu}$$

Prandtl number

$$\text{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Now, do the experiments.

Complex Heat Transfer – Dimensional Analysis



Now, do the experiments.

Forced Convection Heat Transfer

- Build apparatus (*several* actually, with different D, L)
- Run fluid through the inside (at different $\underline{v}$; for different fluids $\rho, \mu, \hat{C}_p, k$)
- Measure T_{bulk} on inside; T_{wall} on inside
- Measure rate of heat transfer, Q
- Calculate h: $|Q| = hA|T_{bulk} - T_{wall}|$
- Report h values in terms of dimensionless correlation:

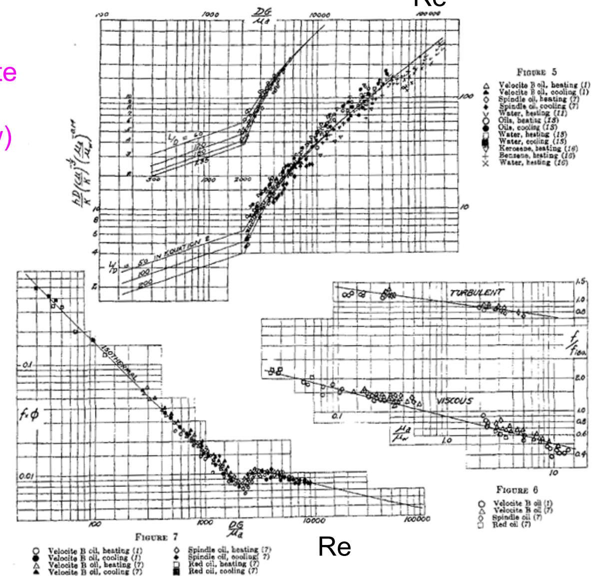
$$\text{Nu} = \frac{hD}{k} = f\left(\text{Re}, \text{Pr}, \frac{L}{D} \right)$$

It should only be a function of these dimensionless numbers (***if*** our Dimensional Analysis is correct…..)

Complex Heat Transfer – Dimensional Analysis

Sieder and Tate equation (turbulent flow)

Sieder and Tate equation (laminar flow)



Effect of viscosity ratio

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)

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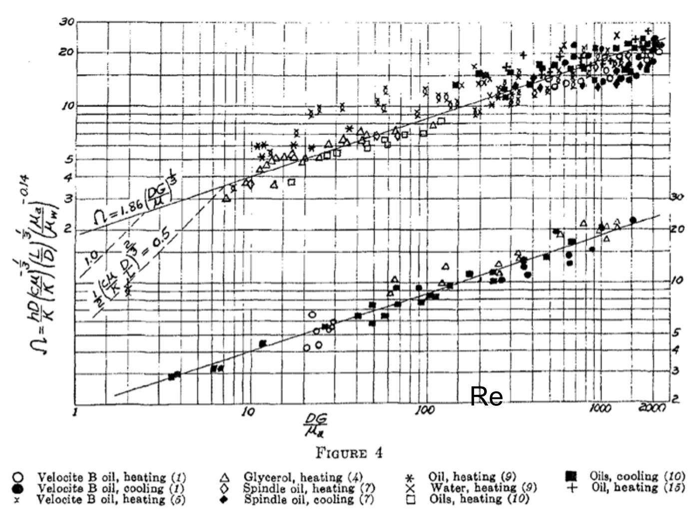
Complex Heat Transfer – Dimensional Analysis

Sieder and Tate equation (laminar flow)



Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)

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Complex Heat Transfer – Dimensional Analysis

Sieder and Tate equation
effect of viscosity ratio

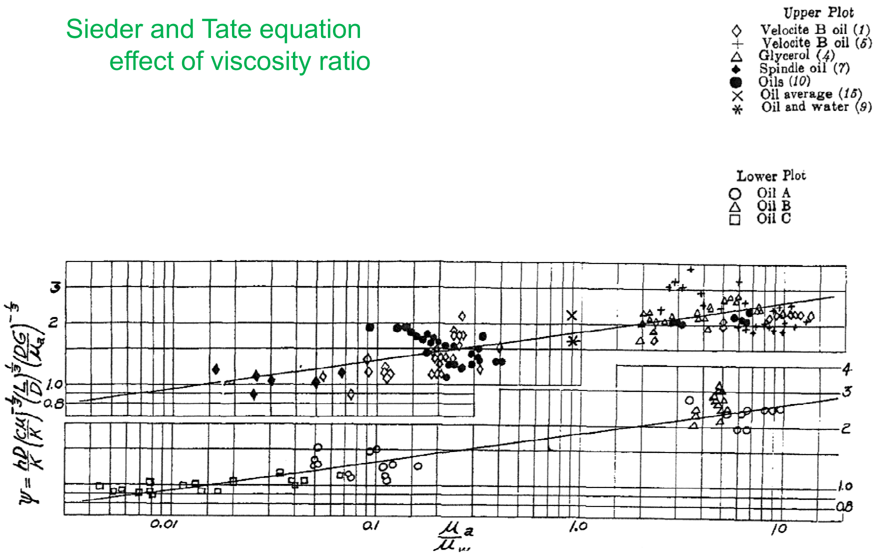

FIGURE 3

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)

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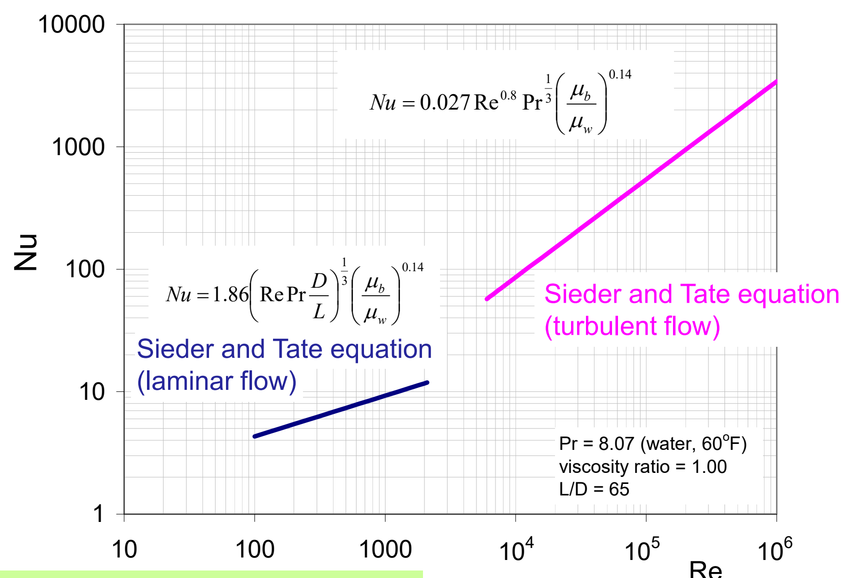
Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients



Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)
Geankoplis, 4th ed. eqn 4.5-4, page 260

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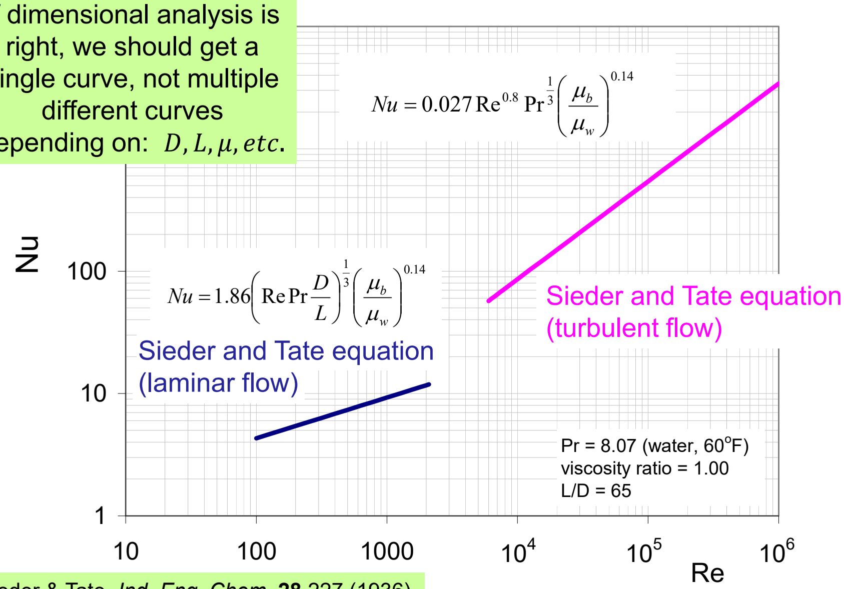
Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients

If dimensional analysis is right, we should get a single curve, not multiple different curves depending on: $D, L, \mu, etc.$



Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)
Geankoplis, 4th ed. eqn 4.5-4, page 260

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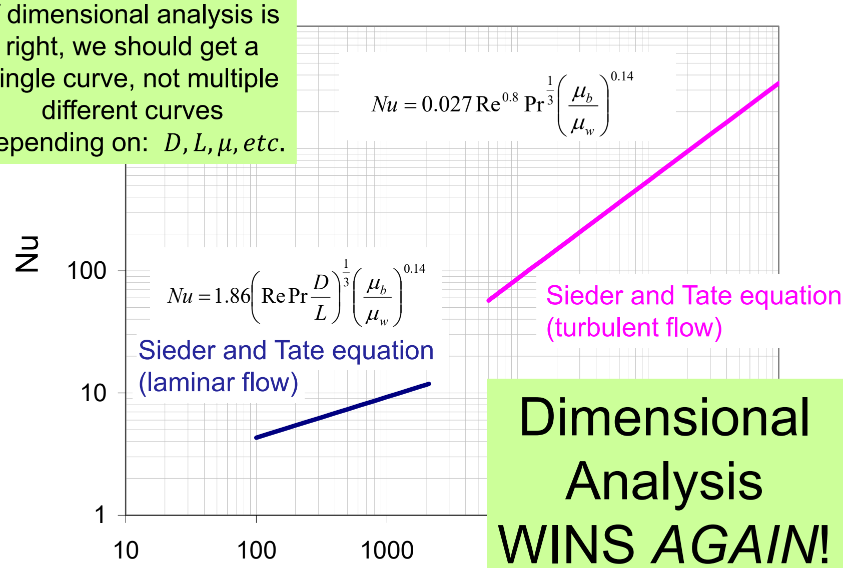
Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients

If dimensional analysis is right, we should get a single curve, not multiple different curves depending on: $D, L, \mu, etc.$



Dimensional Analysis WINS ***AGAIN***!

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)
Geankoplis, 4th ed. eqn 4.5-4, page 260

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Complex Heat Transfer – Dimensional Analysis

Heat Transfer in **Laminar** flow in pipes:

data correlation for *forced convection heat transfer coefficients*

$$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (laminar flow)

Sieder & Tate, Ind. Eng. Chem. ***28*** *227 (1936)*

the subscript "*a*" refers to *the type of average temperature* used in calculating the heat flow, *q*

$$q = h_a A \Delta T_a$$

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

Geankoplis, 4th ed. eqn 4.5-4, page 260

$Re < 2100$, $(RePr\frac{D}{L}) > 100$, horizontal pipes; all physical properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the (constant) wall temperature.

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Complex Heat Transfer – Dimensional Analysis

Heat Transfer in **Turbulent** flow in pipes:

data correlation for *forced convection heat transfer coefficients*

$$\mathrm{Nu}_{lm} = \frac{h_{lm} D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (turbulent flow)

Sieder & Tate, Ind. Eng. Chem. ***28*** *227 (1936)*

the subscript "*lm*" refers to *the type of average temperature* used in calculating the heat flow, *q*

$$q = h_{lm} A \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_{w-bi} - \Delta T_{w-bo}}{\ln\left(\frac{\Delta T_{w-bi}}{\Delta T_{w-bo}}\right)}$$

Geankoplis, 4th ed. section 4.5

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Complex Heat Transfer – Dimensional Analysis

Forced convection
Heat Transfer in Laminar flow in pipes

$$\mathrm{Nu}_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (laminar flow)

Forced convection
Heat Transfer in Turbulent flow in pipes

$$\mathrm{Nu}_{lm} = \frac{h_{lm} D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (turbulent flow)

Physical Properties evaluated at:

$$\frac{T_{b,in} + T_{b,out}}{2}$$

May have to be estimated

$$\frac{T_{b,in} + T_{b,out}}{2}$$

bulk mean temperature

Fine print matters!

•all physical properties (except μ_w) evaluated at the **bulk mean temperature**
•Laminar or turbulent flow

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Complex Heat Transfer – Dimensional Analysis

Forced convection Heat Transfer in Laminar flow in pipes

? In our dimensional analysis, we assumed constant ρ, k, μ, etc. Therefore we did not predict a viscosity-temperature dependence. If viscosity is not assumed constant, the dimensionless group shown below is predicted to appear in correlations.

$$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re\,Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (laminar flow)

Sieder & Tate, Ind. Eng. Chem. ***28*** *227 (1936)*

(reminiscent of pipe wall roughness; needed to modify dimensional analysis to correlate on roughness)

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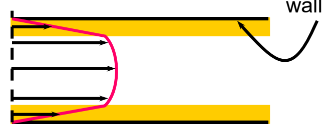
Complex Heat Transfer – Dimensional Analysis

Viscous fluids with large ΔT

heating



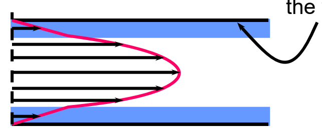
lower viscosity fluid layer speeds flow near the wall => higher h

$\mu_b > \mu_w$

empirical result:

$$\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

cooling



higher viscosity fluid layer retards flow near the wall => lower h

$\mu_b < \mu_w$

ref: McCabe, Smith, Harriott, 5th ed, p339
Sieder & Tate, Ind. Eng. Chem. 28 227 (1936)

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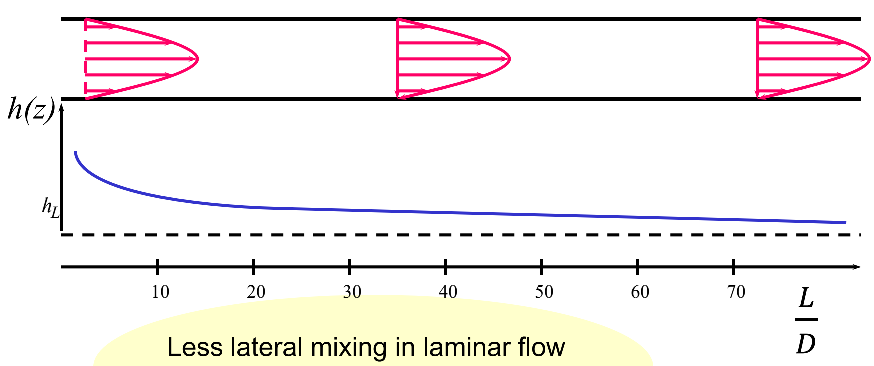
Complex Heat Transfer – Dimensional Analysis

Forced convection
Heat Transfer in Laminar flow in pipes

$$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Why does $\frac{L}{D}$ appear in laminar flow correlations and not in the turbulent flow correlations?

LAMINAR



Less lateral mixing in laminar flow means more variation in $h(x)$.

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Complex Heat Transfer – Dimensional Analysis

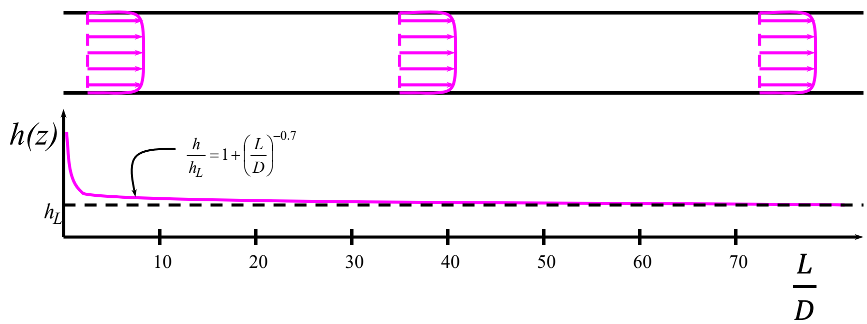
Forced convection

Heat Transfer in Turbulent flow in pipes

$$Nu_{lm} = \frac{h_{lm}D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder & Tate eqn, turbulent flow

TURBULENT



In turbulent flow, good lateral mixing reduces the variation in h along the pipe length.

(Exam 4 2016)

Example:

Water flows at $0.0522\ kg/s$ (turbulent) in the inside of a double pipe heat exchanger (inside steel pipe, inner diameter= $0.545\ inches$, length unknown, physical properties given on page 1); the water enters at 30.0^oC and exits at 65.6^oC. In the shell of the heat exchanger, steam condenses at an unknown saturation pressure. What is the heat transfer coefficient, h_{lm} (based on log mean temperature driving force) in the water flowing in the pipe? You may neglect the effect on heat-transfer coefficient of the temperature-dependence of viscosity. Please give your answer in W/m^2K.

Physical properties of steel:
thermal conductivity = $16.3\ W/mK$
heat capacity = $0.49\ kJ/kg\ K$
density = $8050\ kg/m^3$

Complex Heat Transfer – Dimensional Analysis

Example of *partial* solution to Homework 6 (bring to final exam)

laminar flow in pipes	$Nu_a = \frac{h_a D}{k} = 1.86\left(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$ Sieder-Tate equation (laminar flow)	Re<2100, (RePrD/L)>100, horizontal pipes, eqn 4.5-4, page 238; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.
turbulent flow in smooth tubes	$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$ Sieder-Tate equation (turbulent flow)	Re>6000, 0.7 <Pr <16,000, L/D>60 , eqn 4.5-8, page 239; all properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the wall temperature. The mean is the average of the inlet and outlet bulk temperatures; not valid for liquid metals.
air at 1atm in turbulent flow in pipes	$h_{lm} = \frac{3.52V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = \frac{0.5V(ft/s)^{0.8}}{D(ft)^{0.2}}$	equation 4.5-9, page 239
water in turbulent flow in pipes	$h_{lm} = 1429\left(1+0.0146T(^{o}C)\right)\frac{V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = 150\left(1+0.011T(^{o}F)\right)\frac{V(ft/s)^{0.8}}{D(ft)^{0.2}}$	4 < T(°C)<105, equation 4.5-10, page 239

(T_{bulk} mean)

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Complex Heat Transfer – Dimensional Analysis

Complex Heat transfer Problems to Solve:

- Forced convection heat transfer from fluid to wall
  Solution: ?
- Natural convection heat transfer from fluid to wall
  Solution: ?
- Radiation heat transfer from solid to fluid
  Solution: ?

We started with a forced-convection pipe problem, did dimensional analysis, and found the dimensionless numbers.

To do a situation with different physics, we must start with a different starting problem.

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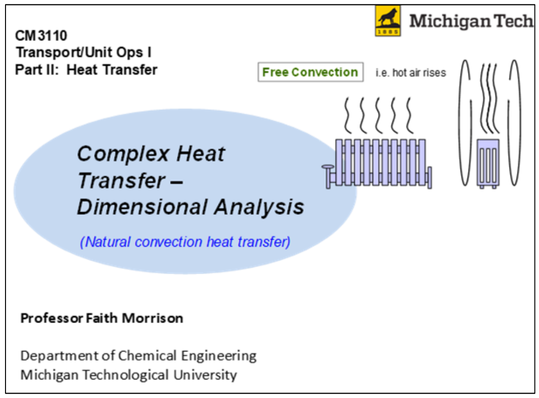
Next:


Michigan Tech
CM3110
Transport/Unit Ops I
Part II: Heat Transfer
Free Convection
i.e. hot air rises
Complex Heat Transfer – Dimensional Analysis
(Natural convection heat transfer)
Professor Faith Morrison
Department of Chemical Engineering
Michigan Technological University

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