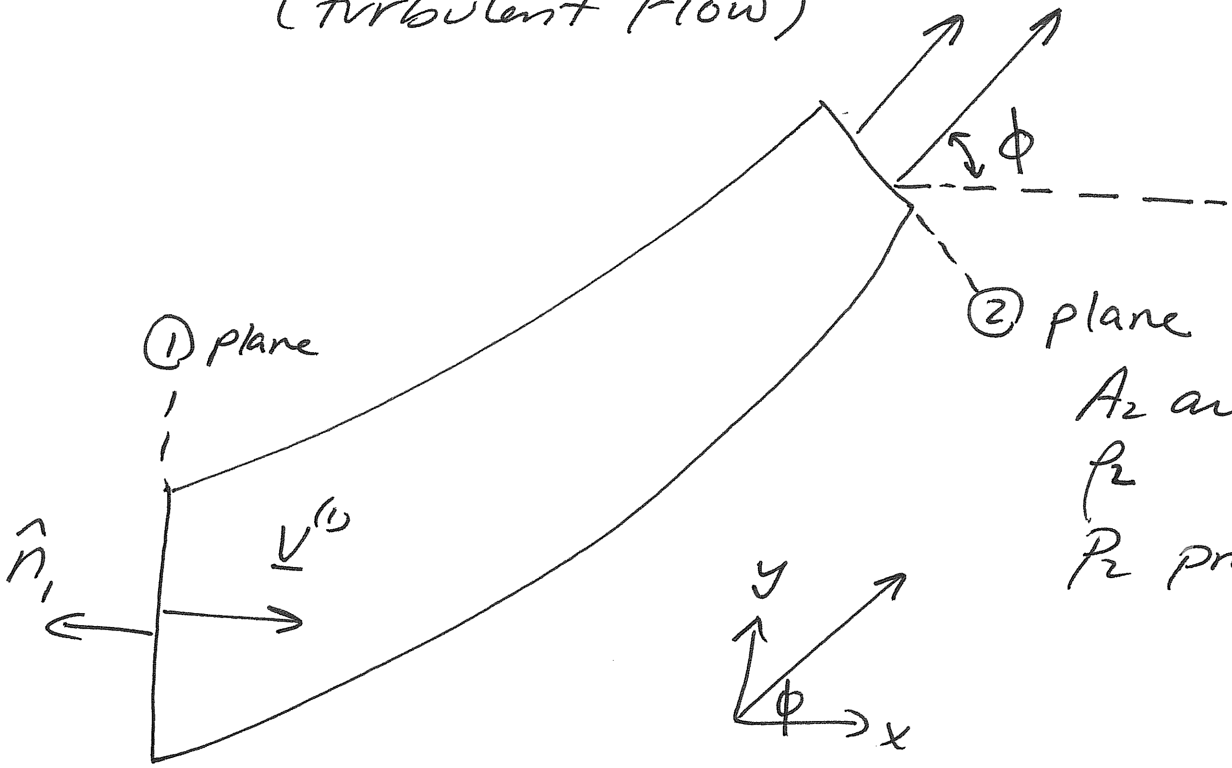
12 SEPT 2011
FAM

CALCULATE THE FORCE ON A REDUCING BEND

(turbulent Flow)



A_2 area
ρ_2 density
P_2 pressure

A_1 area
ρ_1 density
P_1 pressure

We seek the force on the walls $=$

$-\underline{R}$, the force on the fluid

MACROSCOPIC MASS BALANCE

$$\langle V^{(1)}\rangle \rho_1 A_1 = \langle V^{(2)}\rangle \rho_2 A_2$$

MACROSCOPIC MOMENTUM BAL (STEADY)

$$\left[\frac{\rho A \cos\theta \langle V\rangle^2}{\beta}\hat{u}\right]_1 + \left[\frac{\rho A \cos\theta \langle V\rangle^2}{\beta}\hat{u}\right]_2$$

$$= \left[-pA\hat{n}\right]_1 + \left[-pA\hat{n}\right]_2$$

$$+ \underline{R} + M_{cv}\,\underline{g}$$

$\beta_1 = \beta_2 = 1$ turbulent flow

$\theta_1 = 180°$ $\cos\theta_1 = -1$

$\theta_2 = 0°$ $\cos\theta_1 = 1$

$\hat{u}^{(1)} = \hat{e}_x$

$$\hat{u}^{(2)} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}_{xyz}$$

$$\underline{g} = -g\hat{e}_y = \begin{pmatrix} 0 \\ -g \\ 0 \end{pmatrix}_{xyz}$$

$\hat{n}_1 = -\hat{e}_x$

$$\hat{n}_2 = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}_{xyz}$$

$$-\rho_1 A_1 \langle V\rangle_1^2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz} + \rho_2 A_2 \langle V\rangle_2^2 \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}_{xyz}$$

$$= -P_1 A_1 \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}_{xyz} - P_2 A_2 \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}_{xyz}$$

$$+ \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}_{xyz} + M_{cv} \begin{pmatrix} 0 \\ -g \\ 0 \end{pmatrix}_{xyz}$$

$$-\underline{R} = \begin{pmatrix} \rho_1 A_1 \langle V\rangle_1^2 - \rho_2 A_2 \langle V\rangle_2^2 \cos\phi + P_1 A_1 - P_2 A_2 \cos\phi \\ -\rho_2 A_2 \langle V\rangle_2^2 \sin\phi - P_2 A_2 \sin\phi - g M_{cv} \\ 0 \end{pmatrix}_{xyz}$$