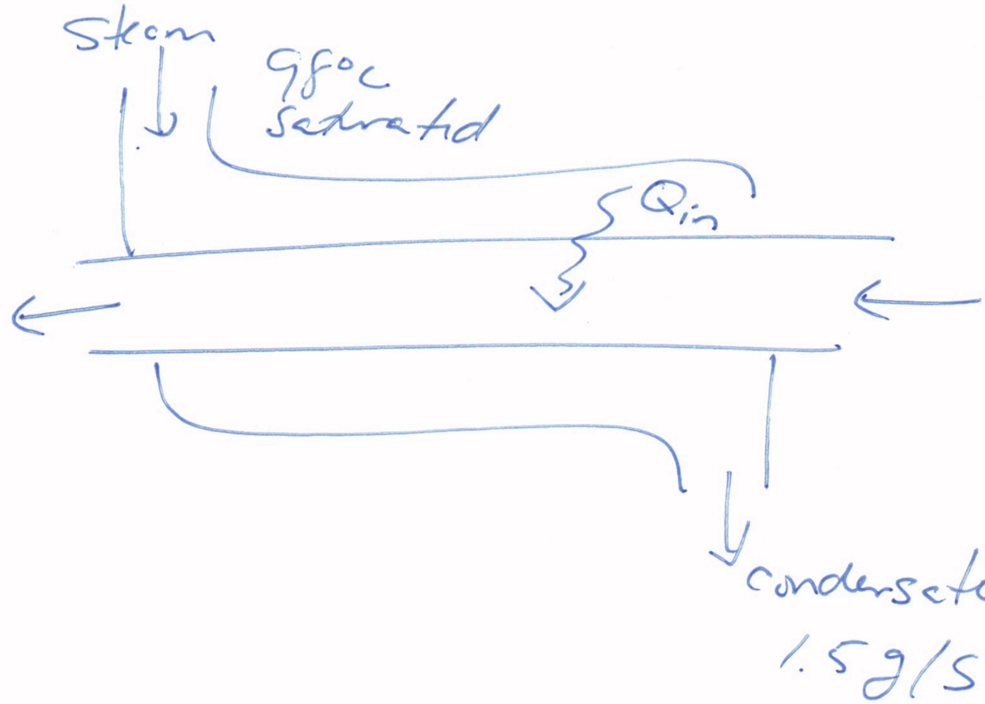
SOLUTION

$\boxed{\text{CM3120}}$ EXAM 1

Spring 2021

1.



MACROSCOPIC OPEN SYSTEM E-BAL:

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

negligible

no shaft

$$Q_{in} = \Delta H$$

$$= \sum_{out} m_i \hat{H}_i - \sum_{in} m_i H_i$$

$$Q_{in} = \dot{m}\left(\underbrace{\hat{H}_{out} - \hat{H}_{in}}_{-\Delta\hat{H}_{vap}}\right)$$

$$= \dot{m}\left(-\Delta\hat{H}_{vap}\right)$$

need to obtain from steam tables

T (°C)	LIQ kJ/kg	VAP kJ/kg
95	397.96	2668.1
98	X_1	X_2
100	419.04	2676.1

$$\frac{3}{5} = \frac{98-95}{100-95} = \frac{X_1 - 397.96}{419.04 - 397.96} = \frac{X_2 - 2668.1}{2676.1 - 2668.1}$$

$\Rightarrow X_1 = 410.608$

$\Rightarrow X_2 = 2672.90$

$$\boxed{\Delta\hat{H}_{vap} = X_2 - X_1 = 2262.292 \ \frac{kJ}{kg}}$$

$$\dot{Q}_{in} = -\dot{m}\,\Delta\hat{H}_{vap}$$

$$= \frac{1.5\,g}{s}\,\frac{\cancel{kg}}{10^3\,g}\cdot -2262.292\,\frac{kJ}{\cancel{kg}}$$

$$= -3.393\ kJ/s$$

$$\dot{Q}_{in} = \boxed{-3.4\ kW}$$ in to steam stream

$\boxed{3.4\,kW}$ out, i.e. into inner stream

2.) $k \left(\frac{W}{mK}\right)$ = transport coefficient that indicates the thermal conductivity of a mat'l

$$\frac{q_x}{A} = -k \frac{dT}{dx}$$

It is a material property that indicates the material's heat transfer response to a temperature gradient.

$\hat{C}_p \left(\frac{kJ}{kg\,K}\right)$ = represents the material's capability to store thermal energy

$$\Delta H = \int m C_p \, dT$$

$$C_p = \frac{\partial H}{\partial T}$$

3.) a.) $\left|\frac{q_x}{A}\right| = h \left| T_{bulk} - T_{wall} \right|$ ⑤

b.) $T(r) = C_1 \ln r + C_2$

Newton's law of cooling at R_1, R_2:

$$-k \left.\frac{dT}{dr}\right|_{r=R_1} = \frac{q_r}{A} = h_1 \left(T_{b1} - T(R_1) \right)$$

$$-k \left.\frac{dT}{dr}\right|_{r=R_2} = \frac{q_r}{A} = h_2 \left(T(R_2) - T_{b2} \right)$$

$$\frac{dT}{dr} = C_1 \, d\ln r = \frac{C_1}{r}$$

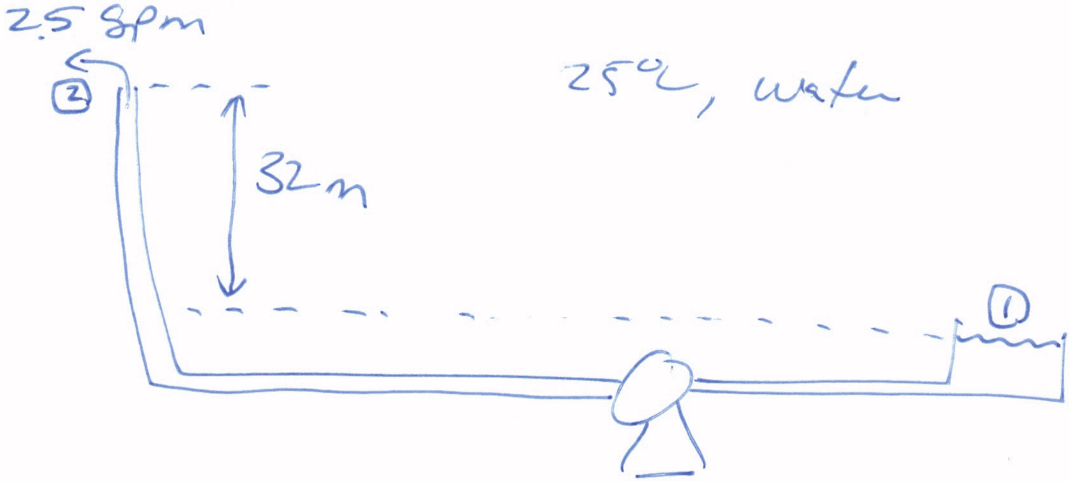
$$\left.\frac{dT}{dr}\right|_{r=R_1} = \frac{C_1}{R_1}$$

$$\left.\frac{dT}{dr}\right|_{r=R_2} = \frac{C_1}{R_2}$$

$$\begin{cases} \dfrac{-kC_1}{R_1} = h_1\left(T_{b_1} - [C_1 \ln R_1 + C_2]\right) \\ \dfrac{-kC_1}{R_2} = h_2\left([C_1 \ln R_2 + C_2] - T_{b_2}\right) \end{cases}$$

2 equations,
2 unknowns

4.)



Mechanical Energy Balance

$$\frac{\cancel{P_2 - P_1}}{\rho} + \frac{V_2^2 - \cancel{V_1^2}}{2\alpha} + g\underbrace{(z_2 - z_1)}_{32.\,m} + F_{21} = \frac{W_{s,on,21}}{\dot{m}}$$

both atmospheric

$\alpha = 1$ (assume)

calc V_2:

$$V_2 = \frac{Q}{\pi R^2} = \frac{(2.5\ gpm)}{\pi}\ \frac{m^3/s}{15{,}850.2\ gpm}\left(\frac{2}{1.2\times10^{-2}\,m}\right)^2$$

$$\boxed{V_2 = 1.39461\ \frac{m}{s}}$$

calc $\dot{m}$:

$$(2.5\ gpm)\left(\frac{m^3/s}{15850.2\ gpm}\right)\frac{997.08\ kg}{m^3} = \boxed{\dot{m} = 0.157266\ \frac{kg}{s}}$$

$$F_{21} = 4f \frac{L}{D} \frac{V^2}{2}$$

need Re

$$Re = \frac{\rho V D}{\mu} = \frac{\left(997.08 \frac{\cancel{kg}}{\cancel{m^3}}\right)\left(1.39461 \frac{\cancel{m}}{\cancel{s}}\right)\left(1.2 \times 10^{-2} \cancel{m}\right)}{8.937 \times 10^{-4} \frac{\cancel{kg}}{\cancel{m\,s}}}$$

$$\boxed{Re = 18{,}671 \text{ (turbulent)}}$$

f ? from Data Correlation (simplified)

$$f = \frac{1.02}{4} \left[\log(Re)\right]^{-2.5}$$

$$\boxed{f = 0.0067635}$$

$$F_{21} = \cancel{4}^{2} f \frac{L}{D} \frac{V^2}{\cancel{2}}$$

$$= 2(0.0067635)\left(\frac{105\,\cancel{m}}{1.2\times10^{-2}\,\cancel{m}}\right)\left(1.39461\,\frac{m}{s}\right)^2$$

$$\boxed{F_{21} = 230.206\ \frac{m^2}{s^2}}$$

combine

$$\frac{W_{s,on,21}}{\dot{m}} = F_{21} + g\Delta z + \frac{V_2^{\,2}}{2}$$

$$= \left(230.206\,\frac{m^2}{s^2}\right) + \left(9.8066\,\frac{m}{s^2}\right)(32\,m) + \frac{\left(1.39461\,\frac{m}{s}\right)^2}{2}$$

$$= 230.206 + 313.811 + 0.972$$

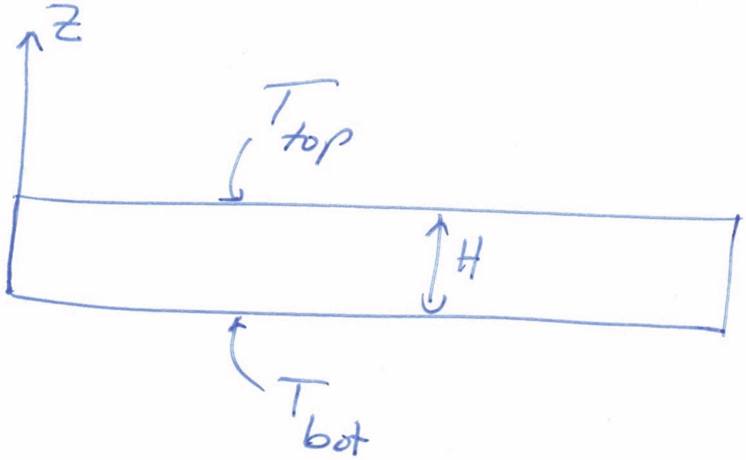
$$= 544.989\ \frac{m^2}{s^2}$$

$$W_{s,on,21} = \left(544.989\,\frac{\cancel{m^2}}{\cancel{s^2}}\right)\left(0.157266\,\frac{\cancel{kg}}{s}\right)\frac{\cancel{N}\,\cancel{s^2}}{\cancel{kg}\,\cancel{m}}\,\frac{J}{\cancel{Nm}}$$

$$= 85.71\ W$$

$$= \boxed{86\ W}$$ //

5.)



steady

Long $\frac{\partial T}{\partial x} \sim 0$

wide $\frac{\partial T}{\partial y} \sim 0$

$\underline{v} = 0$ (solid)

$S \sim 0$ no current
no reaction

$$\Rightarrow \quad 0 = k \frac{d^2 T}{dz^2}$$

$$\frac{d}{dz}\underbrace{\left(\frac{dT}{dz}\right)}_{= \Phi} = 0$$

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

no current, no reaction

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

long wide

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

steady

$\underline{v} = 0$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

integrate: $\frac{d\Phi}{dz} = 0$

$$\Rightarrow \quad \Phi = c_1 = \frac{dT}{dz}$$

integrate: $\boxed{T = c_1 z + c_2}$

Boundary conditions:

$z = 0 \quad T = T_{bot} \quad \Rightarrow \boxed{c_2 = T_{bot}}$

$z = H \quad T = T_{top}$

$$\Rightarrow \quad T_{top} = c_1 H + T_{bot}$$

$$\boxed{c_1 = \frac{T_{top} - T_{bot}}{H}}$$

$$\boxed{T = (T_{top} - T_{bot})\frac{z}{H} + T_{bot}}$$

check: $z = 0 \quad T = T_{bot}$ ✓

$z = H \quad T = T_{top}$ ✓