17 April 2019 FAM 1

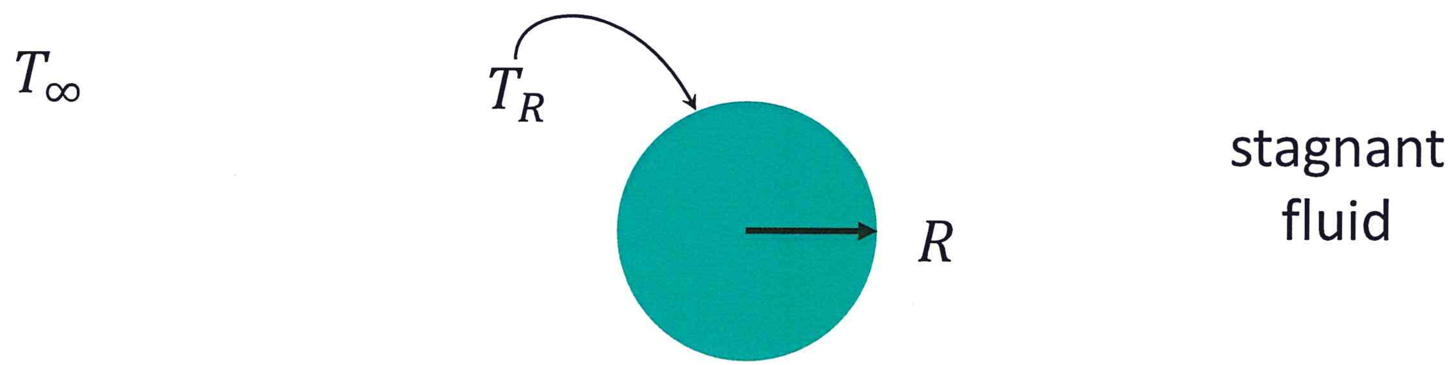
Routes to Mass Transfer Correlations

Theoretical

- Based on a model of the situation; can be solved for flux, and thus for k_x
- All models have assumptions; how good the assumptions are determine how good the correlations are
- The heat-transfer analogy counts as a theoretical pathway

heat

Example: A spherical pellet of reacting solid slowly emits heats steadily into a stagnant fluid such that the surface temperature is constant. What is the heat transfer coefficient h? What is the Nusselt number for this situation?



Solve.

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The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\cancel{\frac{\partial T}{\partial t}}+\cancel{v_r\frac{\partial T}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}}+\cancel{\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)}+\cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}}\right)+\cancel{S}$$

steady

no rxn
no elec. current

$\underline{v}=0$

θ, ϕ symmetry

$$0=\cancel{k}\,\cancel{\frac{1}{r^2}}\left(\frac{d}{dr}\left(\underbrace{r^2\frac{dT}{dr}}_{\Phi=}\right)\right)$$

Now — solve

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

$$\frac{d\Phi}{dr} = 0$$

integrate: $\Phi = c_1 = r^2 \frac{dT}{dr}$

$$\frac{dT}{dr} = \frac{c_1}{r^2}$$

integrate:

$$\boxed{T = -\frac{c_1}{r} + c_2}$$

B.C.: $r = R \quad T = T_R$

$r = \infty \quad T = T_\infty$

or could use Newton's law of cooling – both work in this problem substituting the B.C.

$$\begin{cases} T_R = -\dfrac{C_1}{R} + C_2 \\ T_\infty = C_2 \end{cases}$$

$$\boxed{C_1 = -(T_R - T_\infty)R}$$

$$\boxed{T = \frac{(T_R - T_\infty)R}{r} + T_\infty}$$

Final temperature distribution

match flux at $r = R$: ⑤

$$h(T_R - T_\infty) = \left(-k\frac{dT}{dr}\right)\Big|_{r=R}$$

$$\frac{c_1}{r^2}\Big|_{r=R}$$

$$h\cancel{(T_R - T_\infty)} = -\frac{k}{R^2}(-)\cancel{(T_R - T_\infty)}R$$

diameter

$$Nu = \frac{hD}{k}$$

$D = 2R$

multiply by 2R:

$$(2R)\,h = \frac{k}{\cancel{R}}(2R) \qquad Nu = \frac{h(2R)}{k}$$

$$\boxed{Nu = \frac{h(2R)}{k} = 2}$$

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The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = -\nabla \cdot \underline{\tilde{q}} + S$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{1}{r} \frac{\partial (r \tilde{q}_r)}{\partial r} + \frac{1}{r} \frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = -\left(\frac{1}{r^2} \frac{\partial (r^2 \tilde{q}_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\tilde{q}_\theta \sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \tilde{q}_\phi}{\partial \phi} \right) + S$$

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = -k \nabla T$

The flux in liquid is here

Fourier's law of heat conduction, Cartesian coordinates:

$$\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial x} \\ -k \frac{\partial T}{\partial y} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

Fourier's law of heat conduction, cylindrical coordinates:

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$$

Fourier's law of heat conduction, spherical coordinates:

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_\phi \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -\frac{k}{r \sin\theta} \frac{\partial T}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$