EXAMPLE: Lumped Parameter Analysis

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$$\rho V_{sys} \hat{C}_p \frac{dT}{dt} = hA(T_\infty - T)$$

$$\frac{dT}{dt} = \left(\frac{hA}{\rho V_{sys} \hat{C}_p} \right) (T_\infty - T)$$

$$\frac{dT}{T_\infty - T} = \underbrace{\frac{hA}{\rho V_{sys} \hat{C}_p}}_{\equiv \beta} dt$$

Now, integrate:

note: $\int \frac{du}{u} = \ln u + C_1$

$u \equiv T_\infty - T$

(substitution)

& $du = -dT$

$$\int \frac{(-dT)}{(T_\infty - T)} = -\ln(T_\infty - T)$$

$$\boxed{-\ln(T_\infty - T) = \beta t + C_1}$$

BC: $t = 0 \quad T = T_0$

$$\boxed{-\ln(T_\infty - T_0) = C_1}$$

substitute back

multiply through by (-1):

③

$$+\ln(T_\infty - T) = -\beta t + \ln(T_\infty - T_0)$$

$$\ln(T_\infty - T) - \ln(T_\infty - T_0) = -\beta t$$

$$\ln\left(\frac{T_\infty - T}{T_\infty - T_0}\right) = -\beta t$$

$$\boxed{\left(\frac{T_\infty - T}{T_\infty - T_0}\right) = e^{-\left(\frac{hA}{\rho V_{sp} c_p}\right)t}}$$

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Was it appropriate to use this analysis?

?

$$\underbrace{\frac{hD}{k} = Bi < 0.1}$$

this is the limit for lumped parameter analysis.

We can only estimate h from "usual" magnitudes (see table).

<u>Five</u> Common BC for Heat xfer

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1. Know the Temp

   e.g. $r = R \qquad T = T_0$

2. Know the value of the flux

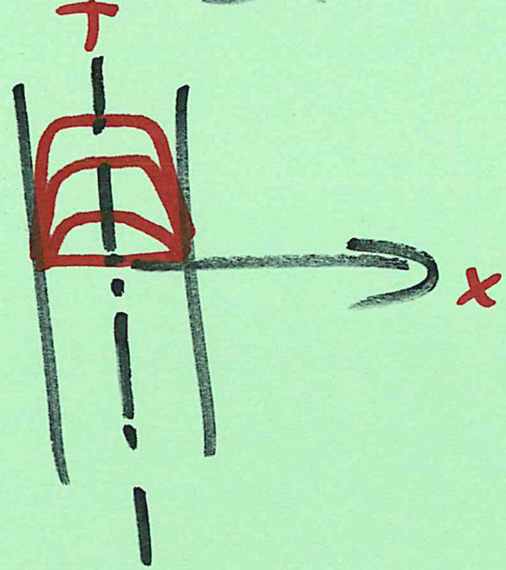
   e.g. $x = L \qquad \frac{q_x}{A} = q_0$ ($q_0 = 0$ insulated wall)

3. Newton's law of cooling

   e.g. $\underbrace{x = B}_{\text{wall location}} \qquad -k\frac{dT}{dx} = \frac{q_x}{A} = h\left|T_{bulk} - \overset{\overset{T(B)}{\downarrow}}{T_{wall}}\right|$

   both parts needed

4. Symmetry



$x = 0$

$$\frac{dT}{dx} = 0$$

max or min at plane of symmetry

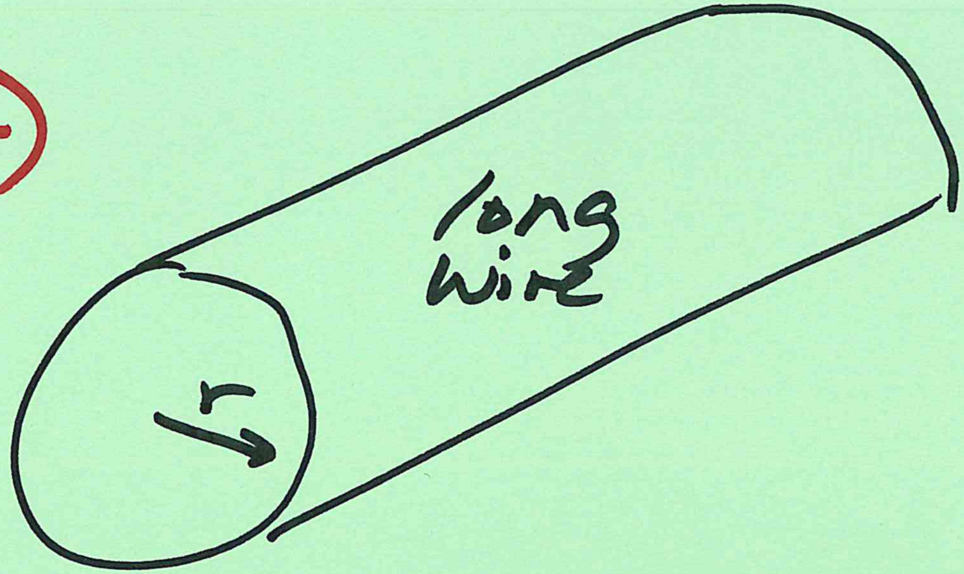
* note: this is only true if the (e.g. r=0) point is in the domain

5. Physically realizable solns *

e.g. $T = (\alpha) r + \frac{c_1}{r}$ at $r = 0$ $\Rightarrow c_1 = 0$

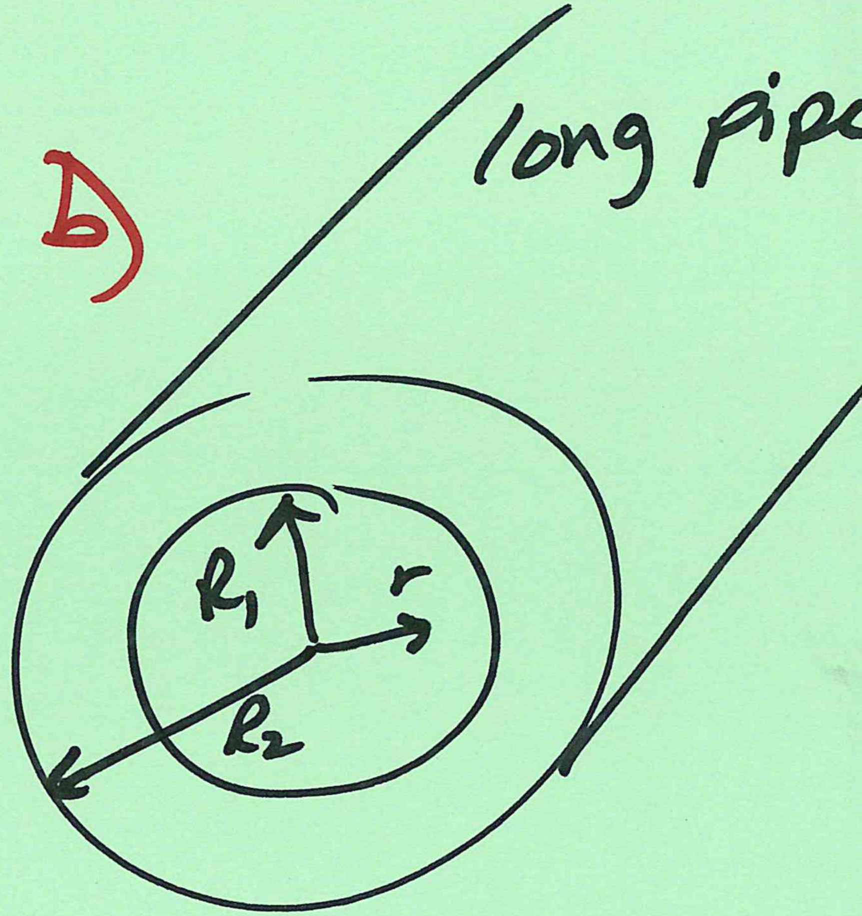
$T = $ finite

a)



$r = 0$ is in the domain

b)



domain: $R_1 \le r \le R_2$

$r = 0$ is NOT in the domain //