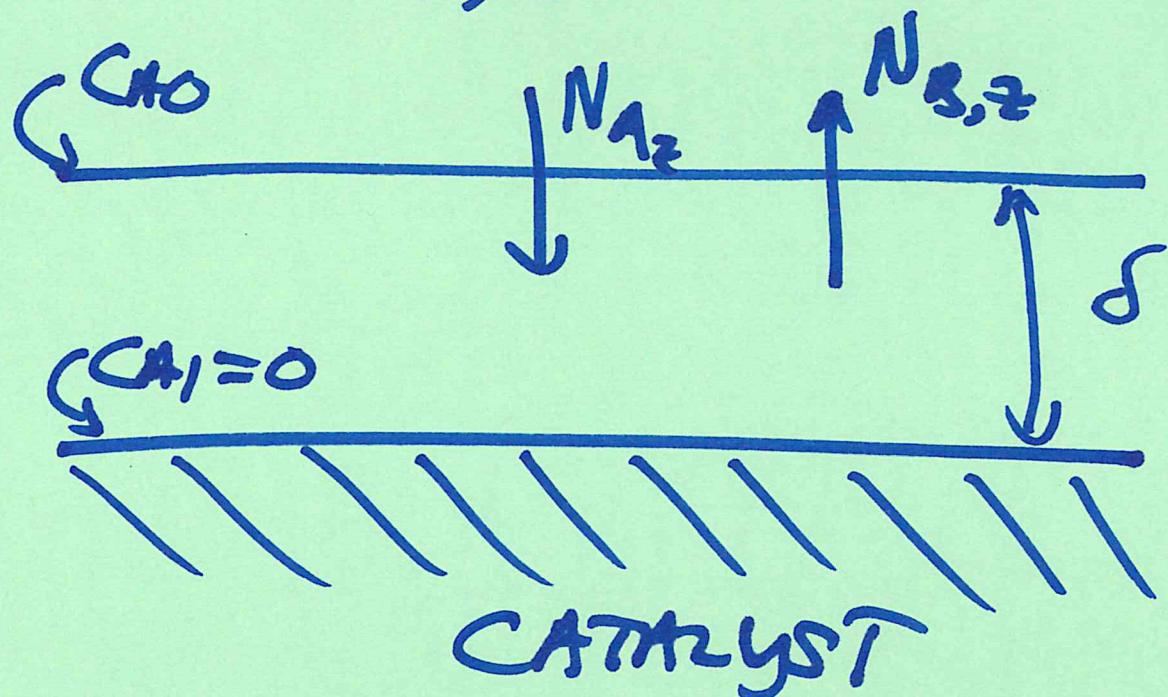


Catalytic converter  
A, B

Fri 20 Mar 19 ①



$$\frac{1}{2} N_{A,z} = -N_{B,z}$$

$$\frac{dN_{A,z}}{dz} = 0$$

$N_{A,z} = C_1$

# The Equation of Species Mass Balance in Terms of Combined Molar quantities

(2)

The general case, where the combined molar flux with respect to molar velocity ( $\underline{N}_A$ ), is given on page 1.

Spring 2019 Faith A. Morrison, Michigan Technological University

Microscopic species mass balance, in terms of molar flux; Gibbs notation

$$\frac{\partial c_A}{\partial t} = -\nabla \cdot \underline{N}_A + R_A$$

Microscopic species mass balance, in terms of combined molar flux; Cartesian coordinates

~~Stays~~

$$\frac{\partial c_A}{\partial t} = -\left(\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

~~No homogeneous term~~

Microscopic species mass balance, in terms of combined molar flux; cylindrical coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r} \frac{\partial(rN_{A,r})}{\partial r} + \frac{1}{r} \frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

In terms of total molar flux,  $\underline{N}_A$

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r^2} \frac{\partial(r^2 N_{A,r})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(N_{A,\theta} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial N_{A,z}}{\partial \phi}\right) + R_A$$

$$\frac{dN_A}{dt} = 0$$

Fick's law of diffusion, Gibbs notation:  $\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB} \nabla x_A$

$$= c_A v^* - cD_{AB} \nabla x_A$$

Fick's law of diffusion, Cartesian coordinates:

$$\begin{pmatrix} N_{A,x} \\ N_{A,y} \\ N_{A,z} \end{pmatrix}_{xyz} = \begin{pmatrix} x_A(N_{A,x} + N_{B,x}) - cD_{AB} \frac{\partial x_A}{\partial x} \\ x_A(N_{A,y} + N_{B,y}) - cD_{AB} \frac{\partial x_A}{\partial y} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB} \frac{\partial x_A}{\partial z} \end{pmatrix}_{xyz}$$

Fick's law of diffusion, cylindrical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,z} \end{pmatrix}_{r\theta z} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB} \frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r} \frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB} \frac{\partial x_A}{\partial z} \end{pmatrix}_{r\theta z}$$

Fick's law of diffusion, spherical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,\phi} \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB} \frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r} \frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,\phi} + N_{B,\phi}) - \frac{cD_{AB}}{r \sin \theta} \frac{\partial x_A}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$

(3)

## Fick's Law

$$N_{A_z} = x_A \left( N_{A_z} + N_{B_z} \right) - c D_{AB} \frac{dx_A}{dz}$$

↑

$$N_{A_z} = -2 N_{B_z}$$

at steady state

:

Solve

$$\text{BC: } z=0 \quad x_A = x_{A0}$$

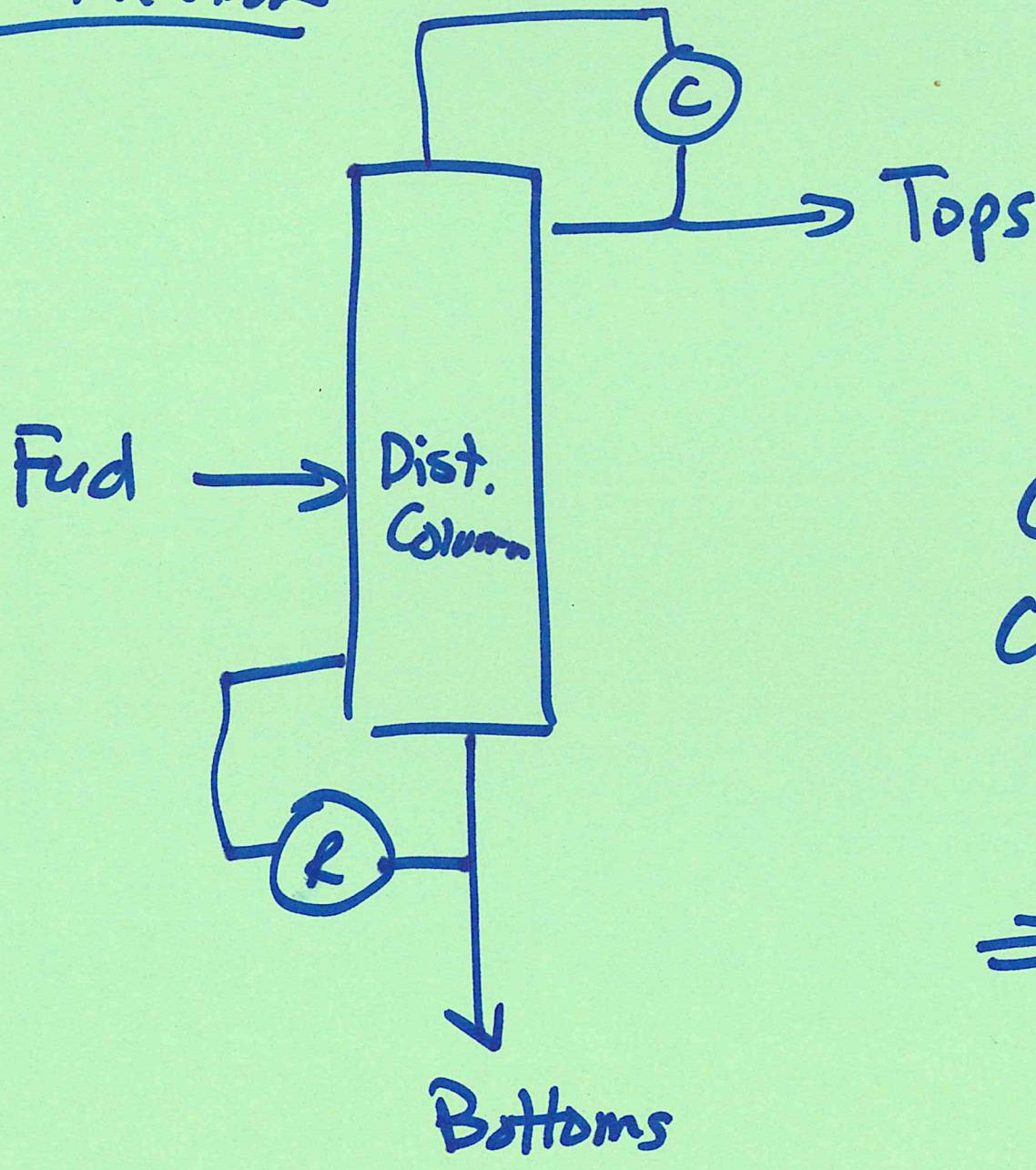
$$z=\delta \quad x_A = 0$$

(fast heterogeneous rxn) //

# Distillation

FAM  
20 Mar 19

①

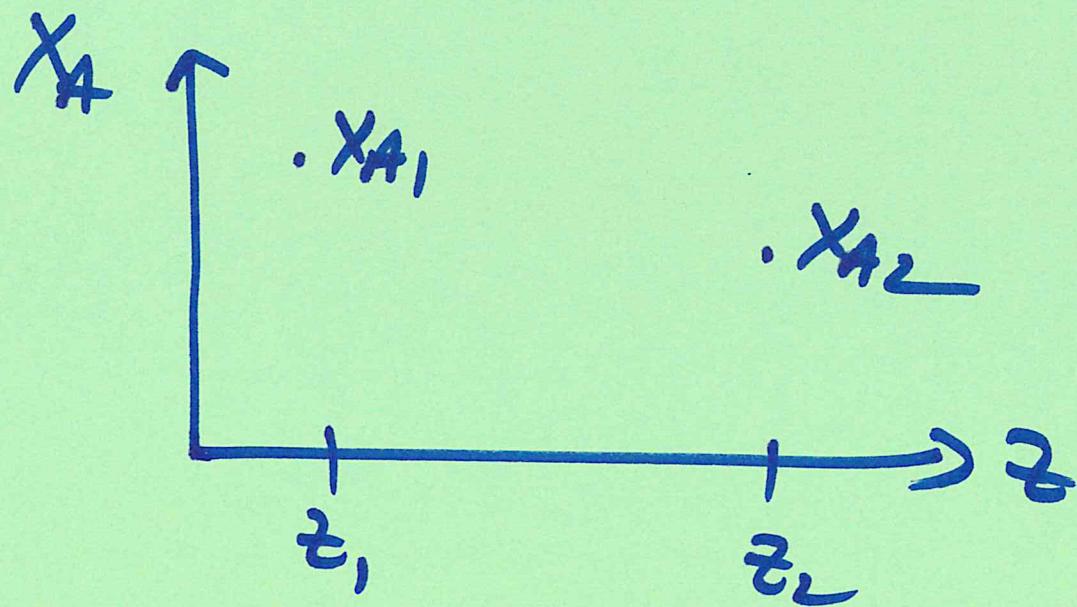


CMO:  
constant  
molar  
overflow  
 $\Rightarrow$  equimolar  
counter diffn  
in gas  $\phi$

Equimolar Counter diffusion ②

$$N_{A2} = -N_{AB}$$

$$\underline{V}^* = 0$$



# Equation of Species Mass Balance in Terms of Combined Molar quantities



The Equation of Species Mass Balance in Terms of **Combined Molar quantities** in Cartesian, cylindrical, and spherical coordinates for binary mixtures of A and B.

The general case, where the **combined molar flux** with respect to molar velocity ( $\underline{N}_A$ ), is given on page 1.

Spring 2019 Faith A. Morrison, Michigan Technological University

Microscopic species mass balance, in terms of molar flux; Gibbs notation

$$\frac{\partial c_A}{\partial t} = -\nabla \cdot \underline{N}_A + R_A$$

Microscopic species mass balance, in terms of **combined molar flux**; Cartesian coordinates

~~$$\frac{\partial c_A}{\partial t} = -\left(\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$~~

Microscopic species mass balance, in terms of **combined molar flux**; cylindrical coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r} \frac{\partial(rN_{A,r})}{\partial r} + \frac{1}{r} \frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

Microscopic species mass balance, in terms of **combined molar flux**; spherical coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r^2} \frac{\partial(r^2 N_{A,r})}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(N_{A,\theta} \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial N_{A,z}}{\partial \phi}\right) + R_A$$

In terms of total molar flux,  $\underline{N}_A$

Fick's law of diffusion, Gibbs notation:  $\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB} \nabla x_A$

$$= c_A \underline{v}^* - cD_{AB} \nabla x_A$$

Fick's law of diffusion, Cartesian coordinates:

$$\begin{pmatrix} N_{A,x} \\ N_{A,y} \\ N_{A,z} \end{pmatrix}_{xyz} = \begin{pmatrix} x_A(N_{A,x} + N_{B,x}) - cD_{AB} \frac{\partial x_A}{\partial x} \\ x_A(N_{A,y} + N_{B,y}) - cD_{AB} \frac{\partial x_A}{\partial y} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB} \frac{\partial x_A}{\partial z} \end{pmatrix}_{xyz}$$

Fick's law of diffusion, cylindrical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,z} \end{pmatrix}_{r\theta z} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB} \frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r} \frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB} \frac{\partial x_A}{\partial z} \end{pmatrix}_{r\theta z}$$

**SOLVE**

Fick's law of diffusion, spherical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,\phi} \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB} \frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r} \frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,\phi} + N_{B,\phi}) - \frac{cD_{AB}}{r \sin \theta} \frac{\partial x_A}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$