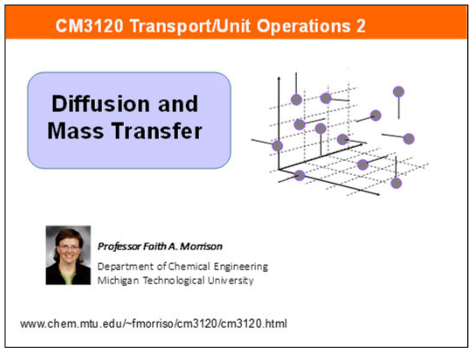
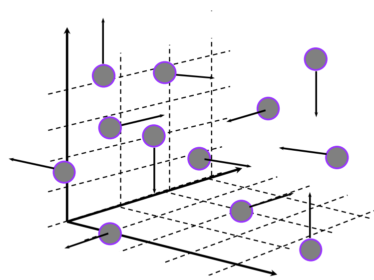
NEXT: Diffusion and Mass Transfer



CM3120 Transport/Unit Operations 2

Diffusion and Mass Transfer



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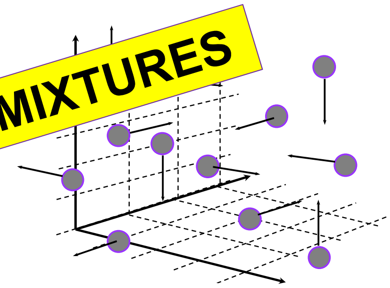
Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

CM3120 Transport/Unit Operations 2

Diffusion and Mass Transfer in MIXTURES



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www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

3

Introduction to Diffusion and Mass Transfer in Mixtures

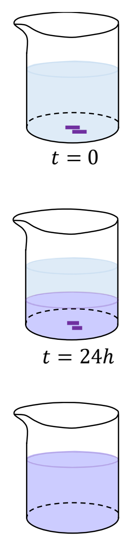
Diffusion

- Is the mixing process caused by random molecular motion.
- Is part of scientific inquiry (explains how nature works)

Diffusion/ mass transfer concerns the physics of **mixtures**.

Mass Transfer

- Encompasses all mass-transfer mechanisms and any issues of mixed physics
- Controls the cost of processes like chemical purification and environmental control
- Is *practical* (is basic to the engineering of chemical processes)



References:
E. L. Cussler, *Diffusion: Mass Transfer in Fluid Systems*, 3rd edition, Cambridge University Press, 2016.
R. B. Bird, W. E. Stewart, E. N. Lightfoot, *Transport Phenomena*, 2nd edition, 2002.
J. R. Welty, G. L. Rorrer, and D. G. Foster, *Fundamentals of Momentum, Heat and Mass Transfer*, 6th edition, 2015.

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Introduction to Diffusion and Mass Transfer in Mixtures

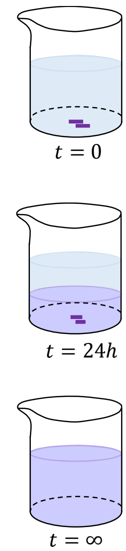
Diffusion

- Is the mixing process caused by random molecular motion (Brownian motion).
- Is part of scientific inquiry (explains how nature works)
- Is ***slow***
- Since it is slow, it acts over short distances

Diffusion progresses at a rate of
- $\sim 5 cm/min$ (gases)
- $\sim 0.05 cm/min$ (liquids)
- $\sim 10^{-5} cm/min$ (solids)

Is the **physics** behind:

- Transport in living cells
- The efficiency of distillation
- The dispersal of pollutants
- Gas absorption
- Fog formed by rain on snow
- The dyeing of wool



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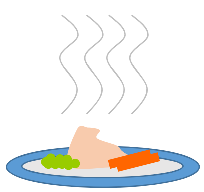
Introduction to Diffusion and Mass Transfer in Mixtures

Example: A friend walks into the far end of the room plates of a delicious-smelling warm lunch including French fries. How did the smell of lunch reach your nostrils?



Diffusion progresses at a rate of
- $\sim 5 cm/min$ (gases)
- $\sim 0.05 cm/min$ (liquids)
- $\sim 10^{-5} cm/min$ (solids)

You try.

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Introduction to Diffusion and Mass Transfer in Mixtures

Mass Transfer

- Encompasses all mass-transfer mechanisms: random motion, convection, thermodynamics-driven (specific interaction).
- Controls the cost of processes like chemical purification and environmental control
- Is practical (is basic to the engineering of chemical processes)
- Is also slow

- There is an analogy to heat transfer (but care must be taken not to over emphasize)
- Dilute mass transfer is emphasized
- Is the **modeling** behind (for example):
  - ✓ Differential distillation (common) versus staged distillation (less common)
  - ✓ Adsorption
  - ✓ Important applications of mass transfer in biology and medicine
  - ✓ Much more

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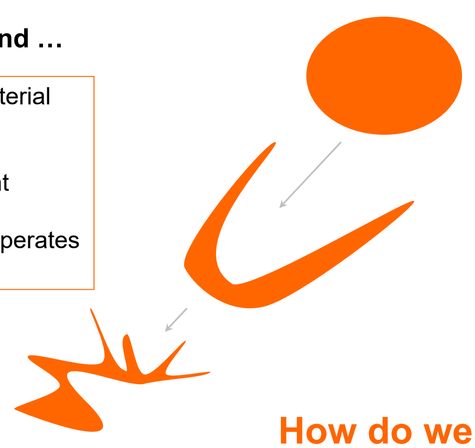
Introduction to Diffusion and Mass Transfer in Mixtures

Mass Transfer

Convection and Diffusion and …

- Agitation or stirring moves material over long distances
- Exposing new fluid elements
- Diffusion mixes newly adjacent material
- Because diffusion is ***slow***, it operates only over short distances



How do we model diffusion?

Reference:
E. L. Cussler, *Diffusion: Mass Transfer in Fluid Systems*, 3rd edition, Cambridge University Press, 2016.

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Summary *(in advance)* Diffusion and Mass Transfer

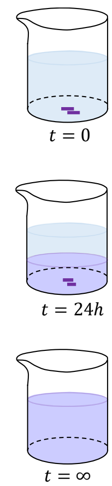
Mixtures

Diffusion:

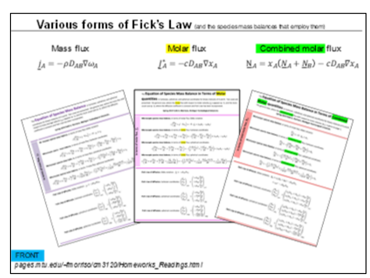
- Brownian motion (random molecular motion),
- Fick's law of diffusion, D_{AB}
- slow,
- operates over short distances

Mass Transfer:

- Includes *all* mechanisms (e.g. diffusion, convection, thermodynamics-driven),
- Linear-driving-force model,
- slow,
- also acts over short distances but convection extends it a bit



Recap: Diffusion/Mass Transfer (so far, and beyond)



Modeling:

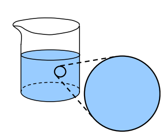
Microscopic species A mass balance (continuum modeling tricky with mixtures; also issues of units, (mass versus moles) and reference coordinates)

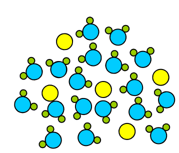
- Four fluxes $\underline{N}_A, \underline{J}_A^*, \underline{n}_A, \underline{j}_A$
- Three summary sheets using $\underline{N}_A, \underline{J}_A^*, \underline{j}_A$

Fick's law (diffusion)

- Diffusion coefficient D_{AB}
- Four forms using $\underline{N}_A, \underline{J}_A^*, \underline{n}_A, \underline{j}_A$

Linear-driving-force model (mass transfer; analogous to Newton's law of cooling)—mass transfer coefficient $k_{various\ designations}$





We start *here*, with diffusion

Diffusion/Mass Transfer Recap and Planning

Recap:
Diffusion/Mass Transfer (so far, and beyond)

Modeling:

Microscopic species A mass balance
(continuum modeling tricky with mixtures; also issues of units, (mass versus moles) and reference coordinates)
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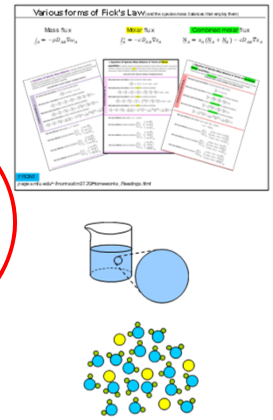

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Diffusion/Mass Transfer Recap and Planning

Recap:
Diffusion/Mass Transfer (so far, and beyond)

Modeling:

Microscopic species A mass balance
(continuum modeling tricky with mixtures; also issues of units, (mass versus moles) and reference coordinates)
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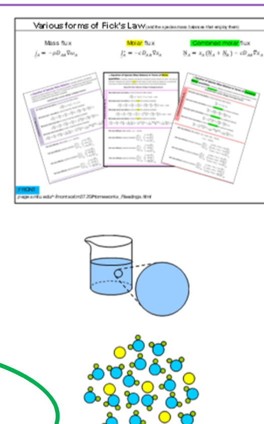

Later, we turn to the linear-driving-force model (mass transfer coefficient)

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Linear Driving Force Model for Mass Transfer

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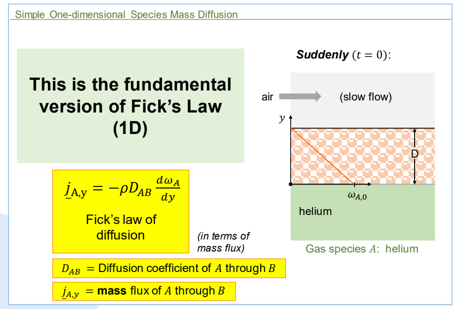
CM3120
Transport II
Part II: Diffusion and Mass Transfer



Diffusion — Fick's Law of Mass Transfer

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Introduction to Diffusion and Mass Transfer in Mixtures

Transport Analogy (flux proportional to driving gradient)

Momentum

$$-\tilde{\tau}_{yz} = -\mu \frac{\partial v_y}{\partial z}$$ Newton's Law

Heat

$$\frac{q_z}{A} = -k \frac{\partial T}{\partial z}$$ Fourier's Law

Species A Mass *in a mixture with B*

$$j_{A,z} = -\rho D_{AB} \frac{\partial \omega_A}{\partial z}$$ Fick's Law

- ✓ **Momentum** goes down a velocity gradient
- ✓ **Heat** goes down a temperature gradient
- ✓ **Mass of species A** goes down a gradient in concentration of A *in a mixture*

Transport Analogy Reference:
R. B. Bird, W. E. Stewart, E. N. Lightfoot, *Transport Phenomena*, 2nd edition, Wiley, 2002.
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Introduction to Diffusion and Mass Transfer in Mixtures

Transport Analogy (flux proportional to driving gradient)

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Species A Mass *in a mixture with B*

$$j_{A,z} = -\rho D_{AB} \frac{\partial \omega_A}{\partial z}$$ Fick's Law

Mass of species A diffusing in the z-direction, per area per time

✓ **Mass of species A** goes down a gradient in concentration of A *in a mixture*

Transport Analogy Reference:
R. B. Bird, W. E. Stewart, E. N. Lightfoot,
Transport Phenomena, 2nd edition, Wiley, 2002.
p. xxi

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Introduction to Diffusion and Mass Transfer in Mixtures

Transport Analogy (flux proportional to driving gradient)

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Species A Mass *in a mixture with B*

$$j_{A,z} = -\rho D_{AB} \frac{\partial \omega_A}{\partial z}$$ Fick's Law

There is a transport analogy but

- topics important to diffusion but not to fluid flow tend to be omitted or deemphasized when the transport analogy is emphasized (e.g. simultaneous diffusion and chemical reaction)
- Numerous topics unrelated to the transport law are deemphasized (in fluid mechanics non-Newtonian flow and heat transfer & some aspects of macroscopic modeling)

Transport Analogy Reference:
R. B. Bird, W. E. Stewart, E. N. Lightfoot,
Transport Phenomena, 2nd edition, Wiley, 2002.

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Introduction to Diffusion and Mass Transfer in Mixtures

Transport Analogy (flux proportional to driving gradient)

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$$-\tilde{\tau}_{yz} = -\mu \frac{\partial v_y}{\partial z}$$ Newton's Law

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- Numerous topics unrelated to the trans fluid mechanics non-Newtonian flow and he macroscopic modeling)

How do we model diffusion?

Transport Analogy Reference:
R. B. Bird, W. E. Stewart, E. N. Lightfoot, *Transport Phenomena*, 2nd edition, Wiley, 2002.

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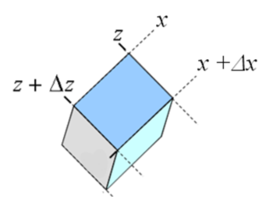
Introduction to Diffusion and Mass Transfer in Mixtures

Modeling Diffusion/Mass Transfer:

Mass is Conserved

Both:
- *overall mass*
- *individual species' masses in a mixture*

As was true in momentum transfer and heat transfer, solving problems with shell balances on individual control volumes is tricky, and it is easy to make errors.



Instead, we use the general equation, derived for all circumstances:

Equation of Species *A* Mass Balance
(microscopic species mass balance)

Recall the other microscopic balances, all written in terms of **Continuum Modeling**

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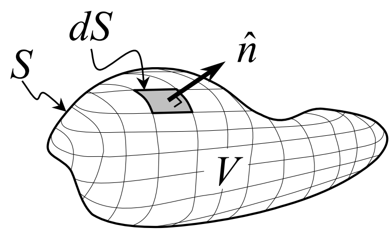
Microscopic Momentum Balance:

Equation of Motion



Microscopic **momentum** balance written on an arbitrarily shaped control volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \nabla\cdot\underline{\underline{\tilde{\tau}}} + \rho\underline{g}$$ **general fluid**

Gibbs notation: $$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}$$ **Newtonian fluid**

Navier-Stokes Equation; constant viscosity

Microscopic momentum balance is a vector equation.

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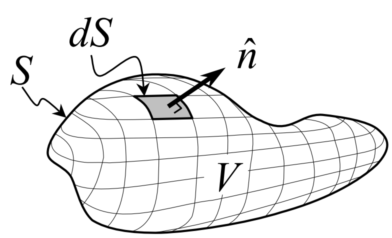
Microscopic Energy Balance:

Equation of Thermal Energy



Microscopic **energy** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\rho\left(\frac{\partial \hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = -\nabla\cdot\underline{\tilde{q}} + S_e$$ **general conduction**

Gibbs notation: $$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$$ **Fourier conduction**

(incompressible fluid, constant pressure, neglect $\hat{E}_k, \hat{E}_p$, viscous dissipation, constant k)

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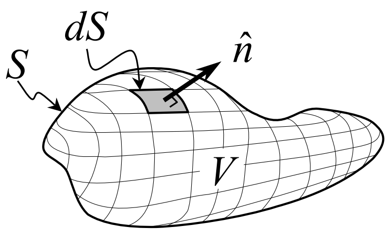
Microscopic Species A Mass Balance:

Equation of Species Mass Balance



Microscopic **species A mass** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\rho\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla\omega_A\right) = -\nabla\cdot\underline{j}_A + r_A$$ **general mass transfer**

Gibbs notation: $$\rho\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla\omega_A\right) = \rho D_{AB}\nabla^2\omega_A + r_A$$ **Fickean diffusion**

(written in terms of mass quantities; constant ρD_{AB})

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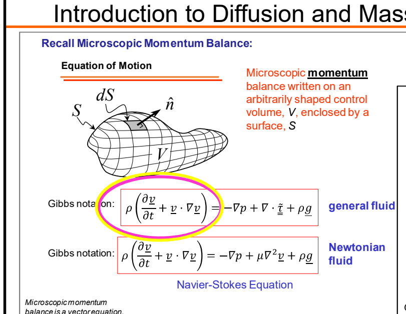
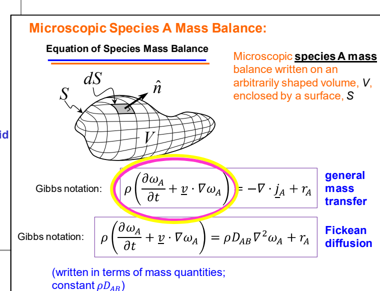
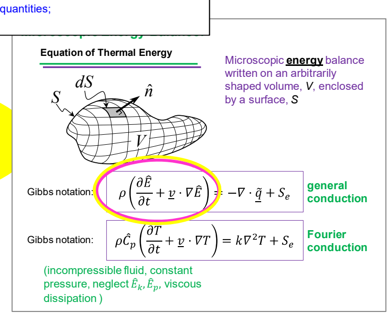
Introduction to Diffusion and Mass Transfer in Mixtures







Microscopic Balances:

- **All three have a convective term on the left-hand side (due to use of <u>control volume</u> as the system and mass or per mass basis)**
- All three have two forms, one including the flux and one with the transport law embedded

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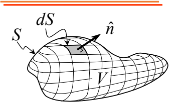
Introduction to Diffusion and Mass Transfer in Mixtures

Recall Microscopic Momentum Balance:

Equation of Motion



Microscopic **momentum** balance written on an arbitrarily shaped control volume, V, enclosed by a surface, S

Gibbs notation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$ — general fluid

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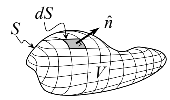
Navier-Stokes Equation

Microscopic momentum balance is a vector equation.

Microscopic Species A Mass Balance:

Equation of Species Mass Balance



Microscopic **species A mass** balance written on an arbitrarily shaped volume, V, enclosed by a surface, S

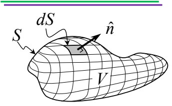
Gibbs notation: $\rho\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla\omega_A\right) = -\nabla\cdot\underline{j}_A + r_A$ — general mass transfer

Gibbs notation: $\rho\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla\omega_A\right) = \rho D_{AB}\nabla^2\omega_A + r_A$ — Fickean diffusion

(written in terms of mass quantities; constant ρD_{AB})

Microscopic Energy Balance:

Equation of Thermal Energy



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Gibbs notation: $\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$ — Fourier conduction

(incompressible fluid, constant pressure, neglect $\hat{E}_k$, $\hat{E}_p$, viscous dissipation)

Microscopic Balances:

- All three have a convective term on the left-hand side (due to use of control volume as the system and mass or per mass basis)
- **All three have two forms, one including the flux and one with the transport law embedded**

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Introduction to Diffusion and Mass Transfer in Mixtures

Microscopic species A mass balance

in a mixture

$$\rho\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla\omega_A\right) = \rho D_{AB}\nabla^2\omega_A + r_A$$

- $\frac{\partial \omega_A}{\partial t}$: **rate of change**
- $\underline{v}\cdot\nabla\omega_A$: **convection** — *Appears due to use of stationary coordinates (control volume)*
- $\rho D_{AB}\nabla^2\omega_A$: **diffusion** (all directions) — *Appears due to diffusive transport through a surface (control surface)*
- r_A: **source** *(mass of species A generated by homogeneous reaction per time)*

velocity must satisfy equation of motion, equation of continuity

The types of terms that appear are very much like similar mechanisms that we have seen in the other transport fields.

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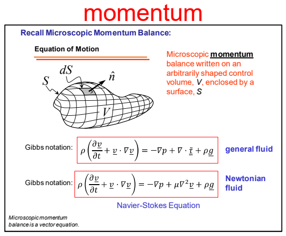
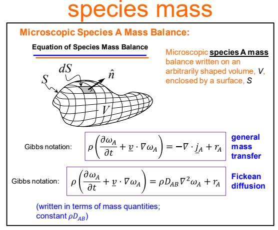
Introduction to Diffusion and Mass Transfer in Mixtures

An underlying feature of these balances is the assumption that matter forms a **continuum**.

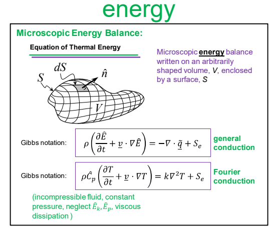
momentum



species mass



energy



To model diffusion and mass transfer within this familiar structure, we must adapt our notion of the ***continuum***.

to accommodate aspects that are important in a mixture

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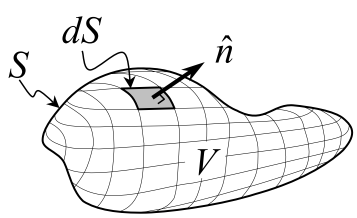
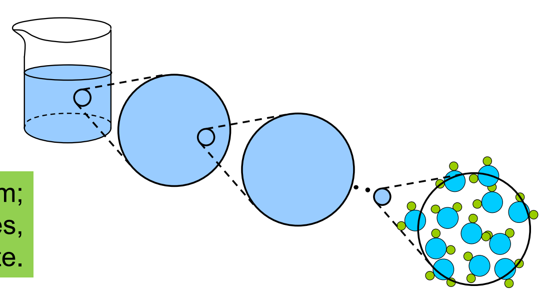
Continuum Modeling



Microscopic balances are written on an arbitrarily shaped microscopic volume, V, enclosed by a surface, S

- A continuum is infinitely divisible
- Material properties (μ, k, ρ) are shared by all volume elements



BUT:

Real matter is *not* a continuum; at small enough length scales, molecules are discrete.

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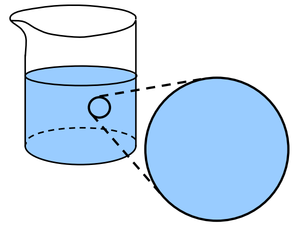
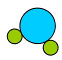
Continuum Modeling

- A **continuum** is infinitely divisible
- Material properties (μ, k, ρ) are shared by all volume elements



- In a **binary mixture**, different pieces of matter have different material identities and different material properties

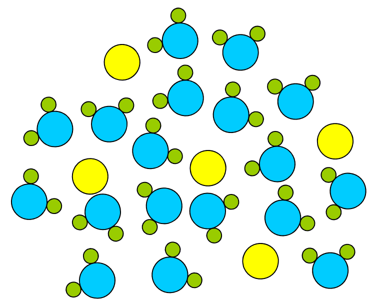
Species A: 

Species B: 

x_A, mole fraction A

x_B, mole fraction B

$c, \ \dfrac{\text{(moles mixture)}}{\text{(volume mixture)}}$



MOLAR basis

Moles are easier when reactions occur…

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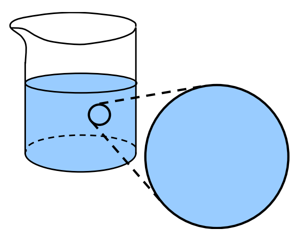
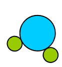
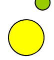
Continuum Modeling

- A **continuum** is infinitely divisible
- Material properties (μ, k, ρ) are shared by all volume elements



- In a **binary mixture**, different pieces of matter have different material identities and different material properties

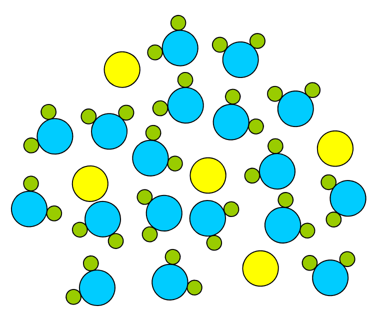
Species A: 

Species B: 

ω_A, mass fraction A

ω_B, mass fraction B

$\rho, \ \dfrac{\text{(mass mixture)}}{\text{(volume mixture)}}$



MASS basis

Only mass is conserved…

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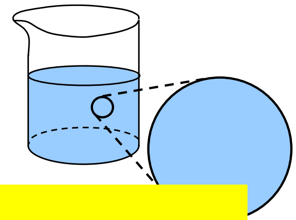
Continuum Modeling

- A **continuum** is infinitely divisible
- Material properties (μ, k, ρ) are shared by all volume elements



- In a **binary m**
matter have di
and different m

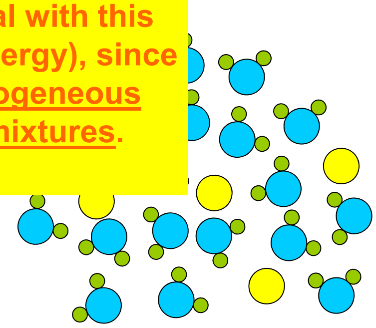
We didn't have to deal with this before (momentum, energy), since we considered homogeneous materials and not mixtures.

Species A:

Species B:

ω_B, mass fraction B

$\rho, \dfrac{(\text{mass mixture})}{(\text{volume mixture})}$



MASS basis

Only mass is conserved…

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Continuum Modeling

Mass versus Moles

- A complication with the microscopic species mass balance is that we are accustomed to modeling systems as a **continuum**.
- In a continuum, material properties (μ, k, ρ) are shared by all volume elements.
- But now, we're interested in species A and B as separate entities.
- Chemical identity manifests as a distribution of atoms/molecules (or **moles** of either) and also as a distribution of **mass.**
- Molar and mass distributions **are not the same distribution.**

x_A, mole fraction A

x_B, mole fraction B

$c, \dfrac{(\text{moles mixture})}{(\text{volume mixture})}$

Species A:

Species B:

ω_A, mass fraction A

ω_B, mass fraction B

$\rho, \dfrac{(\text{mass mixture})}{(\text{volume mixture})}$

MOLAR basis

Moles are easier when reactions occur…

Only mass is conserved…

MASS basis

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Continuum Modeling

Mass versus Moles

ω_A, mass fraction A
ω_B, mass fraction B
ρ, $\frac{\text{(mass mixture)}}{\text{(volume mixture)}}$

Species A:
Species B:

x_A, mole fraction A
x_B, mole fraction B
c, $\frac{\text{(moles mixture)}}{\text{(volume mixture)}}$

Should we express the diffusion of molecules in terms of moles or in terms of mass?

Does it matter?

Answer? It depends.

MASS! Fits well with previous microscopic balances (in a mixture, $\underline{v}$ is the **mass average velocity**)

MOLES! When reactions take place, reactions are naturally analyzed in terms of **moles**

This tradeoff has led to an increase of nomenclature.

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Species Fluxes

The community has found use for four (actually more) different fluxes. The differences in the various fluxes are related to several questions:

Flux of what? And due to what mechanism?

$\underline{N}_A$ – combined molar flux (includes both convection and diffusion)
$\underline{n}_A$ – combined mass flux (includes both convection and diffusion)
$\underline{j}_A$ – mass flux (diffusion only)
$\underline{J}_A^*$ – molar flux (diffusion only)

Microscopic species A mass balance

$$\rho\left(\frac{\partial\omega_A}{\partial t}+\underline{v}\cdot\nabla\omega_A\right)=\rho D_{AB}\nabla^2\omega_A+r_A$$

rate of change; convection; diffusion (all directions); source (mass of species A generated by homogeneous reaction per time)

Written relative to what velocity?

$\underline{N}_A$ – relative to stationary coordinates
$\underline{n}_A$ –relative to stationary coordinates
$\underline{j}_A$ – relative to the mass average velocity $\underline{v}$
$\underline{J}_A^*$ – relative to the molar average velocity $\underline{v}^*$

These different definitions lead to different forms for the **microscopic species mass** balance and for **the transport law**.

BSL2, p552

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These different definitions lead to different forms for the **microscopic species mass** balance and for **the transport law**.

These different fluxes are a significant complication.

It will take some time and practice to get used to all this

Microscopic species A mass balance—Five forms

In terms of mass flux and mass concentrations	$\rho\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla\omega_A\right) = -\nabla\cdot\underline{j}_A + r_A$ $= \rho D_{AB}\nabla^2\omega_A + r_A$
In terms of molar flux and molar concentrations	$c\left(\frac{\partial x_A}{\partial t} + \underline{v}^*\cdot\nabla x_A\right) = -\nabla\cdot\underline{J}_A^* + (x_B R_A - x_A R_B)$ $= cD_{AB}\nabla^2 x_A + (x_B R_A - x_A R_B)$
In terms of combined molar flux and molar concentrations	$\frac{\partial c_A}{\partial t} = -\nabla\cdot\underline{N}_A + R_A$

These different definitions lead to **different forms** for the **microscopic species mass balance** and for the **species transport law**, Fick's law.

It will take some time and practice to get used to all this

Various quantities in diffusion and mass transfer

How much is present: $cx_A = c_A = \frac{1}{M_A}(\rho_A) = \frac{1}{M_A}(\rho\omega_A)$

$\underline{j}_A \equiv$ **mass** flux of species A relative to a mixture's **mass average velocity**, $\underline{v}$
$= \rho_A(\underline{v}_A - \underline{v})$
$\underline{j}_A + \underline{j}_B = 0$, i.e. these fluxes are measured relative to the mixture's center of mass

$\underline{n}_A \equiv \rho_A \underline{v}_A = \underline{j}_A + \rho_A \underline{v} =$ **combined mass** flux relative to **stationary coordinates**
$\underline{n}_A + \underline{n}_B = \rho \underline{v}$

$\underline{J}_A^* \equiv$ **molar** flux relative to a mixture's **molar average velocity**, $\underline{v}^*$
$= c_A(\underline{v}_A - \underline{v}^*)$
$\underline{J}_A^* + \underline{J}_B^* = 0$

$\underline{N}_A \equiv c_A \underline{v}_A = \underline{J}_A^* + c_A \underline{v}^* =$ **combined molar** flux relative to **stationary coordinates**
$\underline{N}_A + \underline{N}_B = c\underline{v}^*$

$\underline{v}_A \equiv$ velocity of species A in a mixture, i.e. average velocity of all molecules of species A within a small volume
$\underline{v} = \omega_A \underline{v}_A + \omega_B \underline{v}_B \equiv$ mass average velocity; same velocity as in the microscopic momentum and energy balances
$\underline{v}^* = x_A \underline{v}_A + x_B \underline{v}_B \equiv$ molar average velocity

Part of the problem is that we have grown comfortable with the continuum, but now we are peering into the details of the continuum

It will take some time and practice to get used to all this

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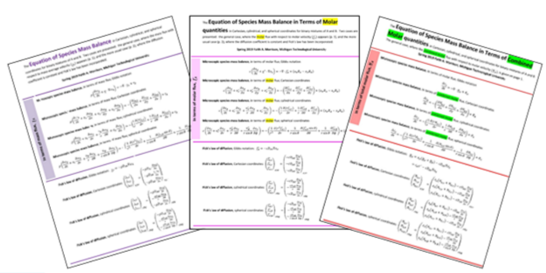
Various forms of Fick's Law (and the species mass balances that employ them)

Mass flux	Molar flux	Combined molar flux
$\underline{j}_A = -\rho D_{AB} \nabla \omega_A$	$\underline{J}_A^* = -c D_{AB} \nabla x_A$	$\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - c D_{AB} \nabla x_A$



FRONT
pages.mtu.edu/~fmorriso/cm3120/Homeworks_Readings.html

We will be introduced to handy worksheets and to the common assumptions and boundary conditions (just like in momentum and energy balances)

It will take some time and practice to get used to all this

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It turns out that there are many interesting and applicable problems we can address readily with **this** form of the species mass balance.

Microscopic species A mass balance—Five forms

In terms of mass flux and mass concentrations
$$\rho\left(\frac{\partial \omega_A}{\partial t} + \underline{v}\cdot\nabla\omega_A\right) = -\nabla\cdot\underline{j}_A + r_A$$
$$= \rho \mathcal{D}_{AB}\nabla^2\omega_A + r_A$$

In terms of molar flux and molar concentrations
$$c\left(\frac{\partial x_A}{\partial t} + \underline{v}^*\cdot\nabla x_A\right) = -\nabla\cdot\underline{J}_A^* + ($$
$$= c\mathcal{D}_{AB}\nabla^2 x_A$$

In terms of combined molar flux and molar concentrations
$$\frac{\partial c_A}{\partial t} = -\nabla\cdot\underline{N}_A + R_A$$

Microscopic species mass balance in terms of combined molar flux $\underline{N}_A$

Let's jump in!

We'll do a "**Quick Start**" and get into some examples and return to the "why" of it all a bit later.

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Diffusion and Mass Transfer QUICK START

Using the **microscopic species mass balance** in terms of combined molar flux and molar concentrations

$$\frac{\partial c_A}{\partial t} = -\nabla\cdot\underline{N}_A + R_A$$

QUICK START

$c_A [=] \frac{moles\ A}{volume\ mix} = x_A c =$ the concentration of A in the mixture

$\underline{N}_A [=] \frac{moles\ A}{area\cdot time} =$ combined molar flux of A (both diffusion and convection) relative to stationary coordinates

$R_A [=] \frac{moles\ A}{volume\ mix\cdot time} =$ rate of production of A by reaction per unit volume mixture

$c [=] \frac{moles\ mix}{volume\ mix} =$ molar density of the mixture (for ideal gases $c = \frac{n}{V} = \frac{P}{RT}$)

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Diffusion and Mass Transfer QUICK START

Using **Fick's law of diffusion** in terms of the same combined molar flux:

$$\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB}\nabla x_A$$

QUICK START

$\underline{N}_A [=] \frac{moles\ A}{area \cdot time}$ = combined molar flux of A (both diffusion and convection) relative to stationary coordinates

$x_A [=] \frac{moles\ A}{moles\ mix}$ = mole fraction of A

$D_{AB} [=] \frac{cm^2}{s}$ = diffusion coefficient (diffusivity) of A in B

$c [=] \frac{moles\ mix}{volume\ mix}$ = molar density of the mixture (for ideal gases $c = \frac{n}{V} = \frac{P}{RT}$)

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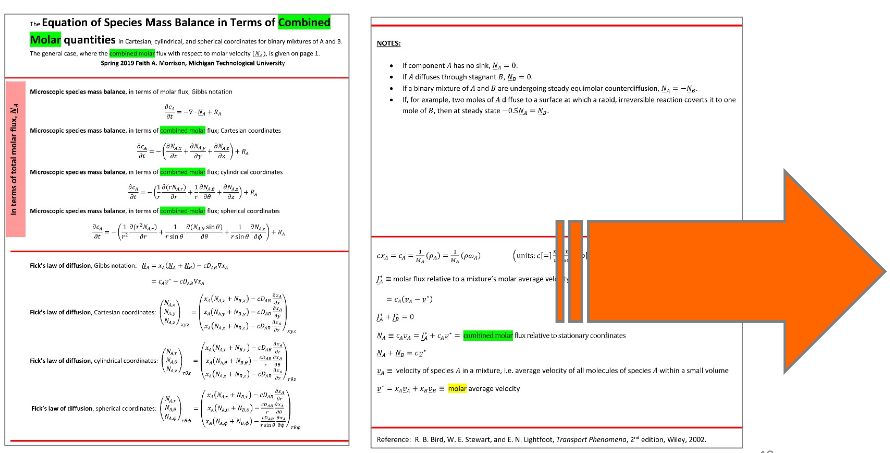
Diffusion and Mass Transfer QUICK START

Using **handy worksheets** to learn the common modeling assumptions

QUICK START



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https://pages.mtu.edu/~fmorriso/cm3120/species_mass_bal_3_combinedmolarflux.pdf