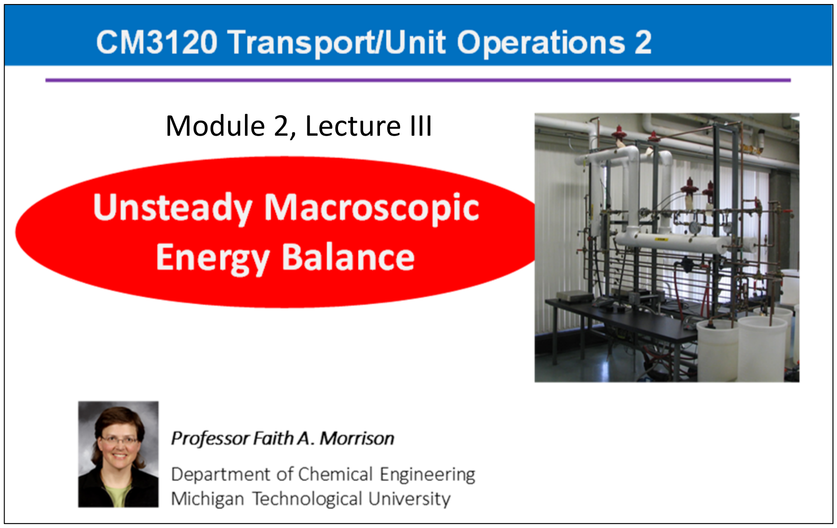
CM3120: Module 2

Unsteady State Heat Transfer

I. Introduction
II. Unsteady Microscopic Energy Balance—(slash and burn)
III. Unsteady Macroscopic Energy Balance
IV. Dimensional Analysis (unsteady)—Biot number, Fourier number
V. Low Biot number solutions—Lumped parameter analysis
VI. Short Cut Solutions—(initial temperature T_0; finite h), Gurney and Lurie charts (as a function of position, $m = \frac{1}{\mathrm{Bi}}$, and Fo); Heissler charts (center point only, as a function of $m = 1/\mathrm{Bi}$, and Fo)
VII. Full Analytical Solutions (stretch)

Another tool for our problem-solving tool belt…



CM3120 Transport/Unit Operations 2

Module 2, Lecture III

Unsteady Macroscopic Energy Balance

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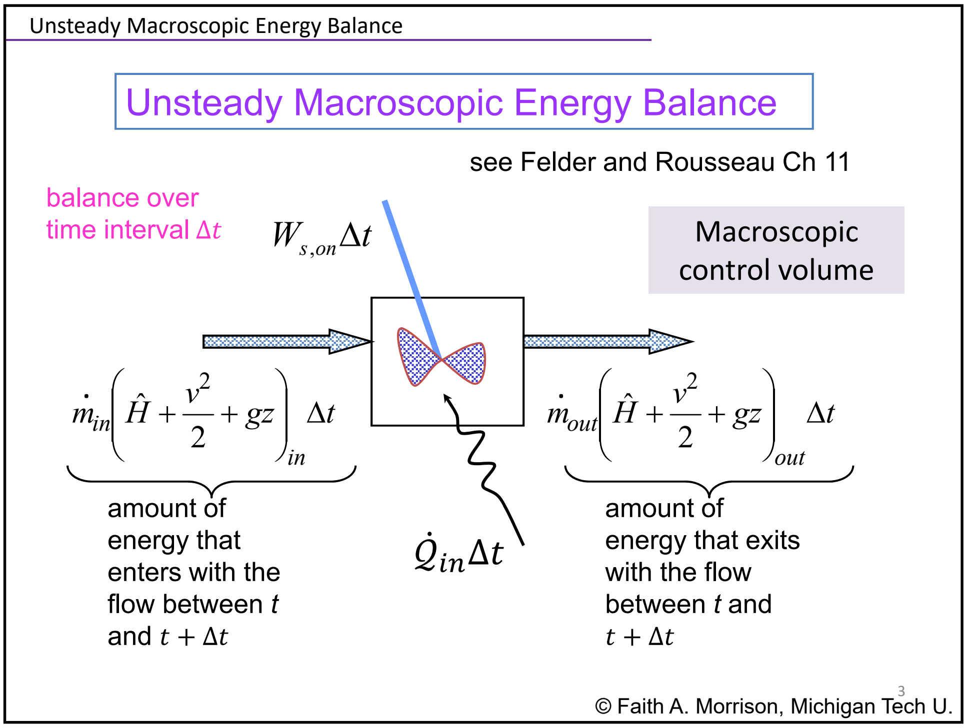


Unsteady Macroscopic Energy Balance

Unsteady Macroscopic Energy Balance

see Felder and Rousseau Ch 11

balance over time interval Δt

Macroscopic control volume

$W_{s,on}\Delta t$

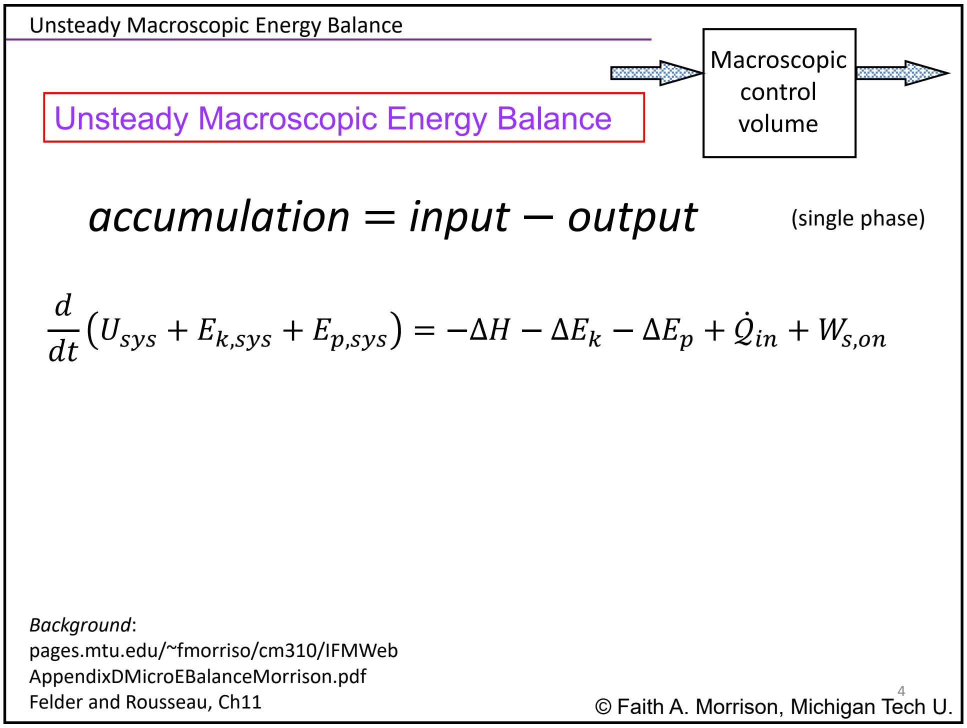
$$\underbrace{\dot{m}_{in}\left(\hat{H}+\frac{v^2}{2}+gz\right)_{in}\Delta t}_{}$$

amount of energy that enters with the flow between t and $t+\Delta t$

$\dot{Q}_{in}\Delta t$

$$\underbrace{\dot{m}_{out}\left(\hat{H}+\frac{v^2}{2}+gz\right)_{out}\Delta t}_{}$$

amount of energy that exits with the flow between t and $t+\Delta t$



Unsteady Macroscopic Energy Balance

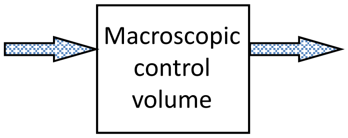
Unsteady Macroscopic Energy Balance

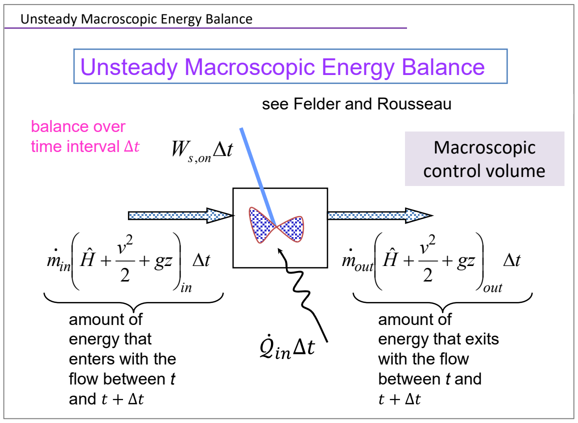
accumulation = input − output (single phase)

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right)=-\Delta H-\Delta E_k-\Delta E_p+\dot{Q}_{in}+W_{s,on}$$

Background:
pages.mtu.edu/~fmorriso/cm310/IFMWeb
AppendixDMicroEBalanceMorrison.pdf
Felder and Rousseau, Ch11

For what type of question would we favor a **macroscopic** control volume?





Unsteady Macroscopic Energy Balance

Unsteady Macroscopic Energy Balance

see Felder and Rousseau

balance over time interval Δt

Macroscopic control volume

$W_{s,on}\Delta t$

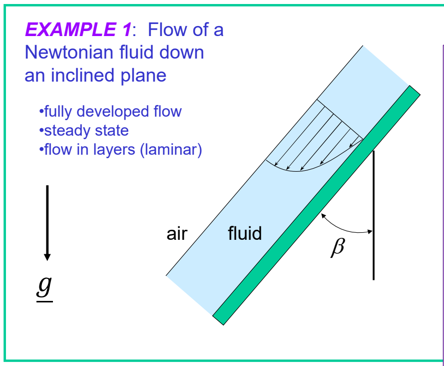
$$\underbrace{\dot{m}_{in}\left(\hat{H}+\frac{v^2}{2}+gz\right)_{in}\Delta t}_{\text{amount of energy that enters with the flow between } t \text{ and } t+\Delta t}$$

$\dot{Q}_{in}\Delta t$

$$\underbrace{\dot{m}_{out}\left(\hat{H}+\frac{v^2}{2}+gz\right)_{out}\Delta t}_{\text{amount of energy that exits with the flow between } t \text{ and } t+\Delta t}$$

Compare choosing a *micro* CV to a *macro* CV in fluids problems **(momentum transfer)**:

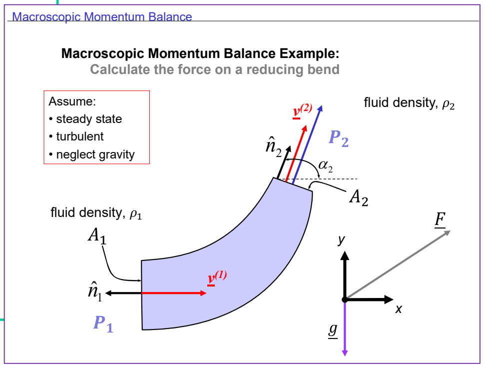
Microscopic control volume



EXAMPLE 1: Flow of a Newtonian fluid down an inclined plane

- •fully developed flow
- •steady state
- •flow in layers (laminar)

Macroscopic control volume

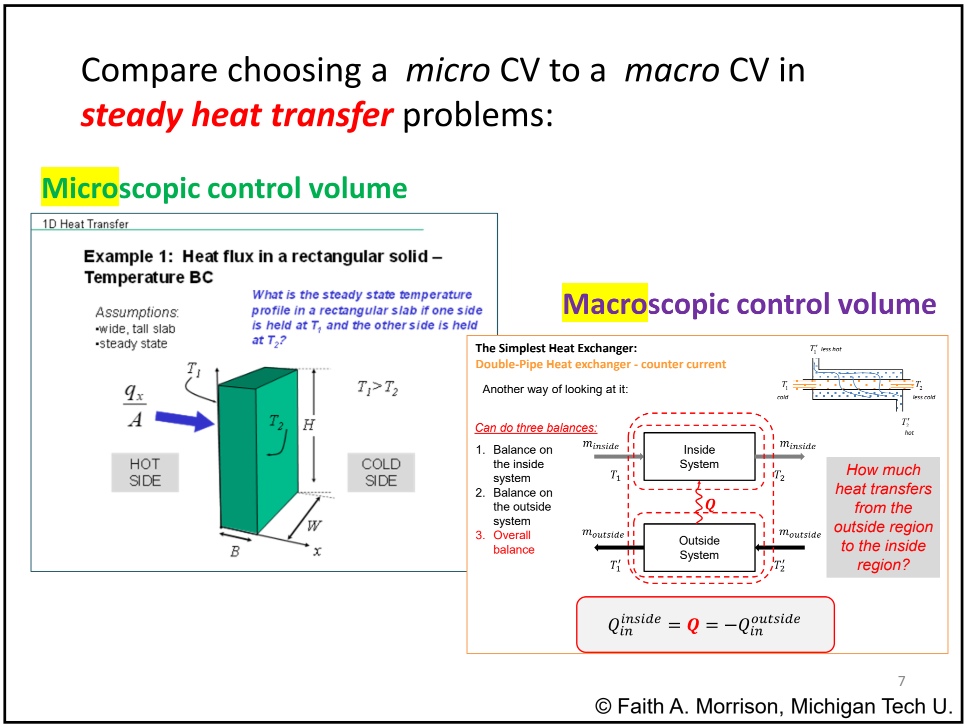


Macroscopic Momentum Balance

Macroscopic Momentum Balance Example:

Calculate the force on a reducing bend

Assume:
- steady state
- turbulent
- neglect gravity



Compare choosing a *micro* CV to a *macro* CV in ***steady heat transfer*** problems:

Microscopic control volume

1D Heat Transfer

Example 1: Heat flux in a rectangular solid – Temperature BC

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Assumptions:
- wide, tall slab
- steady state

Macroscopic control volume

The Simplest Heat Exchanger:

Double-Pipe Heat exchanger - counter current

Another way of looking at it:

Can do three balances:

1. Balance on the inside system
2. Balance on the outside system
3. Overall balance

How much heat transfers from the outside region to the inside region?

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$

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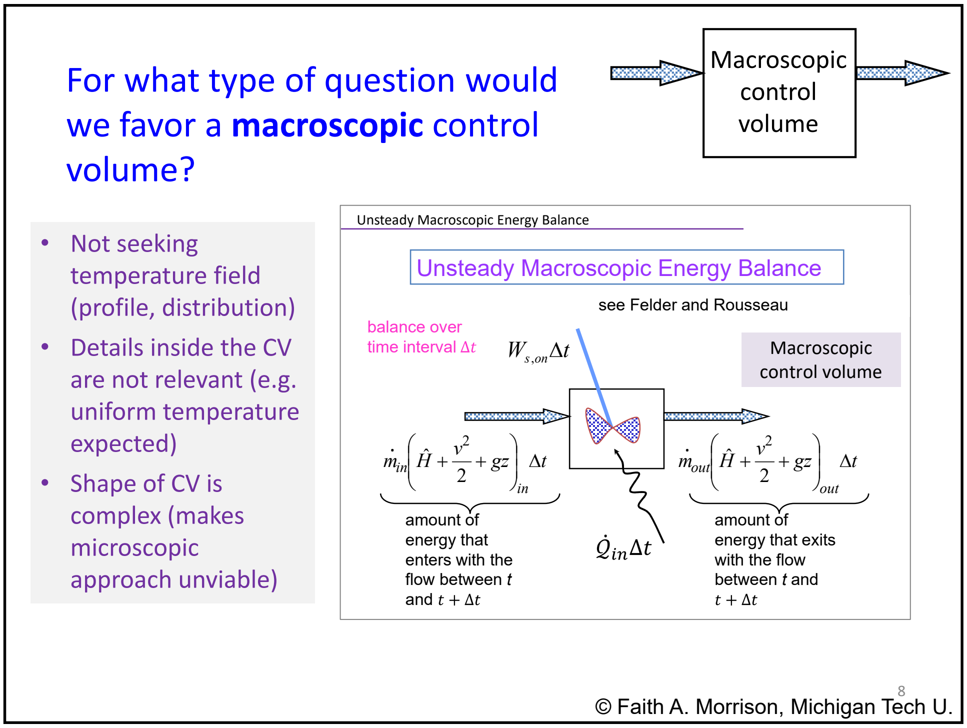

For what type of question would we favor a **macroscopic** control volume?

- Not seeking temperature field (profile, distribution)
- Details inside the CV are not relevant (e.g. uniform temperature expected)
- Shape of CV is complex (makes microscopic approach unviable)

Unsteady Macroscopic Energy Balance

Unsteady Macroscopic Energy Balance

see Felder and Rousseau

balance over time interval Δt

$$\underbrace{\dot{m}_{in}\left(\hat{H}+\frac{v^2}{2}+gz\right)_{in}\Delta t}_{\text{amount of energy that enters with the flow between } t \text{ and } t+\Delta t} \qquad \dot{Q}_{in}\Delta t \qquad \underbrace{\dot{m}_{out}\left(\hat{H}+\frac{v^2}{2}+gz\right)_{out}\Delta t}_{\text{amount of energy that exits with the flow between } t \text{ and } t+\Delta t}$$

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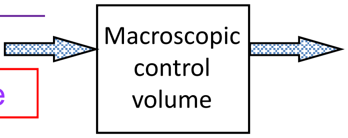
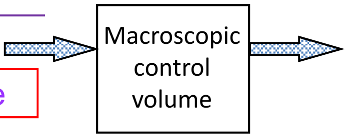


Unsteady Macroscopic Energy Balance

accumulation = input − output (single phase)

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$



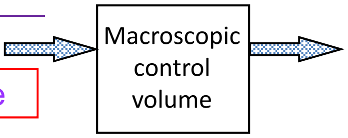
Unsteady Macroscopic Energy Balance

accumulation = input − output (single phase)

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

We can identify the questions that allow us to eliminate (slash) or evaluate each term.

Unsteady Macroscopic Energy Balance



Unsteady Macroscopic Energy Balance

accumulation = input − output (single phase)

$$\frac{d}{dt}\left(\overset{?}{U_{sys}} + \cancel{E_{k,sys}} + \cancel{E_{p,sys}}\right) = \underbrace{-\cancel{\Delta H} - \cancel{\Delta E_k} - \cancel{\Delta E_p}}_{\text{no flow}} + \overset{?}{\dot{Q}_{in}} + \cancel{W_{s,on}}$$

often negligible

no flow

no shafts

(no pump, turbine, mixing shaft)

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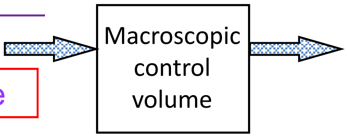
Unsteady Macroscopic Energy Balance



Unsteady Macroscopic Energy Balance

accumulation = input − output (single phase)

$$\frac{d}{dt}\left(\overset{?}{U_{sys}} + \cancel{E_{k,sys}} + \cancel{E_{p,sys}}\right) = \underbrace{-\cancel{\Delta H} - \cancel{\Delta E_k} - \cancel{\Delta E_p}}_{\text{no flow}} + \overset{?}{\dot{Q}_{in}} + \cancel{W_{s,on}}$$

Has there been phase change, chemical rxn, temperature change?

often negligible

no flow

no shafts

(no pump, turbine, mixing shaft)

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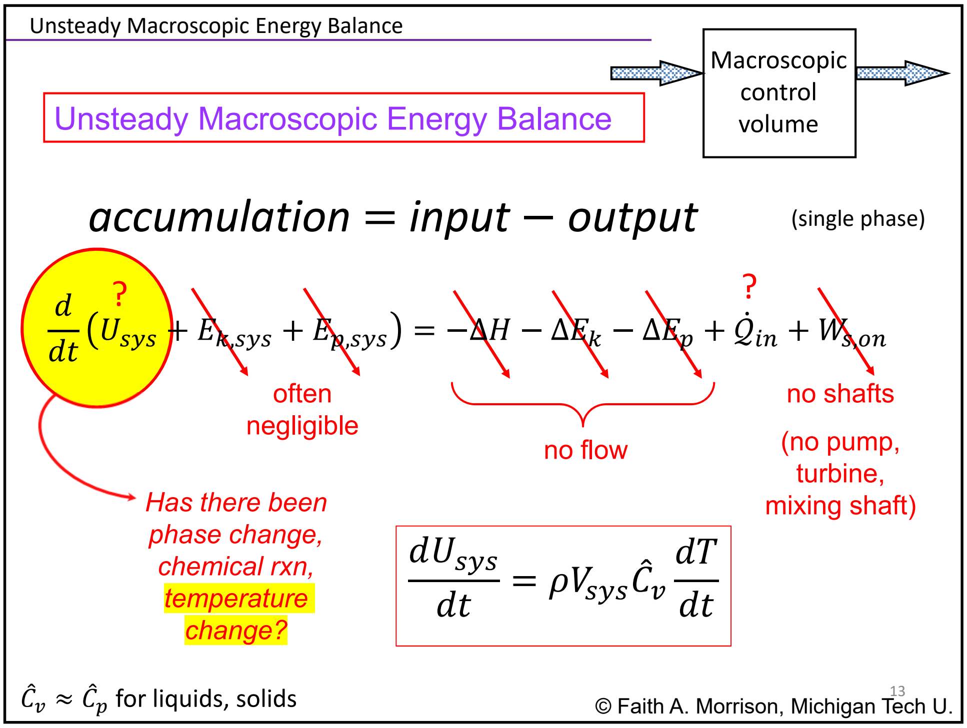

Unsteady Macroscopic Energy Balance

Unsteady Macroscopic Energy Balance

accumulation = input − output

(single phase)

$$\frac{d}{dt}\left(\overset{?}{U_{sys}} + \cancel{E_{k,sys}} + \cancel{E_{p,sys}}\right) = \underbrace{-\cancel{\Delta H} - \cancel{\Delta E_k} - \cancel{\Delta E_p}}_{\text{no flow}} + \overset{?}{\dot{Q}_{in}} + \cancel{W_{s,on}}$$

often negligible

no shafts

(no pump, turbine, mixing shaft)

Has there been phase change, chemical rxn, temperature change?

$$\boxed{\frac{dU_{sys}}{dt} = \rho V_{sys} \hat{C}_v \frac{dT}{dt}}$$

$\hat{C}_v \approx \hat{C}_p$ for liquids, solids

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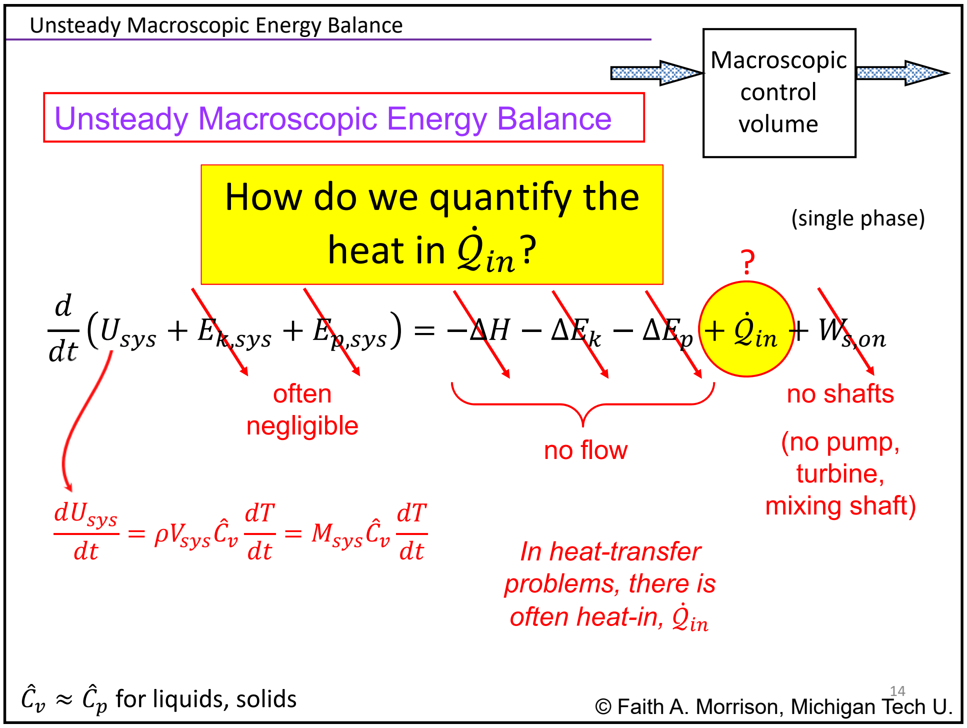

Unsteady Macroscopic Energy Balance

Unsteady Macroscopic Energy Balance

How do we quantify the heat in $\dot{Q}_{in}$?

(single phase)

$$\frac{d}{dt}\left(U_{sys} + \cancel{E_{k,sys}} + \cancel{E_{p,sys}}\right) = \underbrace{-\cancel{\Delta H} - \cancel{\Delta E_k} - \cancel{\Delta E_p}}_{\text{no flow}} + \overset{?}{\dot{Q}_{in}} + \cancel{W_{s,on}}$$

often negligible

no shafts

(no pump, turbine, mixing shaft)

$$\frac{dU_{sys}}{dt} = \rho V_{sys} \hat{C}_v \frac{dT}{dt} = M_{sys} \hat{C}_v \frac{dT}{dt}$$

In heat-transfer problems, there is often heat-in, $\dot{Q}_{in}$

$\hat{C}_v \approx \hat{C}_p$ for liquids, solids

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Unsteady Macroscopic Energy Balance

accumulation = input − output

$\dot{Q}_{in}$ = Heat ***in***to the chosen macroscopic control volume

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

$\dot{Q}_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- Thermal conduction: $q_{in} = -kA\frac{dT}{dx}$
- Convection heat xfer: $|q_{in}| = |hA(T_b - T)|$
- Radiation: $q_{in} = \varepsilon\sigma A\left(T^4_{surroundings} - T^4_{surface}\right)$
- Electric current: $q_{in} = I^2 R_{elec} L$
- Chemical Reaction: $q_{in} = S_{rxn} V_{sys}$

$S_{rxn}[=]\frac{\text{energy}}{\text{time volume}}$

pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf
Incropera and DeWitt, 6th edition p18

Unsteady Macroscopic Energy Balance

accumulation = input − output

$\dot{Q}_{in}$ = Heat ***in***to the chosen macroscopic control volume

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

Signs must match transfer from outside CV (e.g. bulk fluid) to inside CV (e.g. metal)

$\dot{Q}_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- ➡ Thermal conduction: $q_{in} = -kA\frac{dT}{dx}$
- ➡ Convection heat xfer: $|q_{in}| = |hA(T_b - T)|$
- ➡ Radiation: $q_{in} = \varepsilon\sigma A\left(T^4_{surroundings} - T^4_{surface}\right)$
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Unsteady Macroscopic Energy Balance

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right) = -\Delta H-\Delta E_k-\Delta E_p+\boxed{\dot{Q}_{in}}+W_{s,on}$$

$\dot{Q}_{in}=\sum_i q_{in,i}$ comes from a variety of sources:

- **Thermal conduction:** $q_{in}=-kA\frac{dT}{dx}$

  e.g. device held by bracket; a solid phase that extends through boundaries of control volume

- **Convection heat xfer:** $|q_{in}|=|hA(T_b-T)|$

  e.g. device dropped in stirred liquid; forced air stream flows past, natural convection occurs outside system; phase change at boundary

- **Radiation:** $q_{in}=\varepsilon\sigma A\left(T^4_{surroundings}-T^4_{surface}\right)$

  e.g. device at high temp. exposed to a gas/vacuum; hot enough to produce nat. conv.=possibly hot enough for radiation

S-B constant: $\sigma=5.676\times10^{-8}\frac{W}{m^2K^4}$

- **Electric current:** $q_{in}=I^2R_{elec}L$

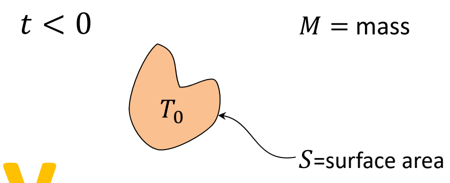
  e.g. if electric current is flowing within the device/control volume/ system

- **Chemical Reaction:** $q_{in}=S_{rxn}V_{sys}$

  e.g. if a homogeneous reaction is taking place throughout the device/ control volume/system

CM3120 Module 2—Cooling of a recently manufactured part

Example: Brass parts (oddly shaped, mass M with surface area S) are ejected at regular intervals from a machine that fabricates them. When ejected, the very hot parts at temperature T_0 enter a moving air stream where the air temperature is T_{bulk}. Create a model that will allow us to calculate the temperature of the part as a function of time. Using Excel, calculate $T(t)$ for the parts.



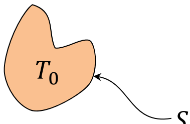
You try.

Exam 2 2019, problem 5

CM3120 Module 2—Cooling of a recently manufactured part

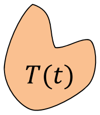
$t < 0$

M = mass



S=surface area

$t \geq 0$



Cooling in air

Forced convection, h, T_{bulk}

CM3120 Module 2—Cooling of a recently manufactured part

S=surface area

$$M\hat{C}_V\frac{dT}{dt} = hS(T_{bulk} - T) + \varepsilon\sigma S\left(T_{bulk}^4 - T^4\right)$$

Solve for $T(t)$

(Excel)

CM3120 Module 2—Cooling of a recently manufactured part

S=surface area

$$M\hat{C}_V\frac{dT}{dt} = hS(T_{bulk} - T) + \varepsilon\sigma S(T_{bulk}^4 - T^4)$$

$$\frac{dT}{dt} = \frac{hS}{M\hat{C}_V}(T_{bulk} - T) + \frac{\varepsilon\sigma S}{M\hat{C}_V}(T_{bulk}^4 - T^4)$$

$$\frac{dT}{dt} = \underbrace{\left[\frac{hS}{M\hat{C}_V}(T_{bulk}) + \frac{\varepsilon\sigma S}{M\hat{C}_V}(T_{bulk}^4)\right]}_{\Phi_0 \equiv} - \left[T\left(\frac{hS}{M\hat{C}_V}\right) + \mathrm{T}^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]$$

$$\frac{dT}{dt} = \Phi_0 - \left[T\left(\frac{hS}{M\hat{C}_V}\right) + \mathrm{T}^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]$$

$$\frac{dT}{\Phi_0 - \left[T\left(\frac{hS}{M\hat{C}_V}\right) + \mathrm{T}^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]} = dt$$

CM3120 Module 2—Cooling of a recently manufactured part

S=surface area

$$\int_{T_0}^{T}\frac{dT'}{\Phi_0 - \left[T'\left(\frac{hS}{M\hat{C}_V}\right) + \mathrm{T}'^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]} = \int_0^t dt'$$

We can use trapezoidal rule to integrate $f(T)$ in Excel

$$\text{area} = \frac{1}{2}h(B_1 + B_2)$$

$$\int_{T_0}^{T} f(T')dT' = t$$

$$f(T') = \frac{1}{\Phi_0 - \left[T'\left(\frac{hS}{M\hat{C}_V}\right) + \mathrm{T}'^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]}$$

CM3120 Module 2—Cooling of a recently manufactured part

S=surface area

$$\int_{T_0}^{T} \frac{dT'}{\Phi_0 - \left[T'\left(\frac{hS}{M\hat{C}_V}\right) + T'^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]} = \int_0^t dt'$$

We can use **trapezoidal rule** to integrate $f(T)$ in Excel

$$\int_{T_0}^{T} f(T')dT' = t$$

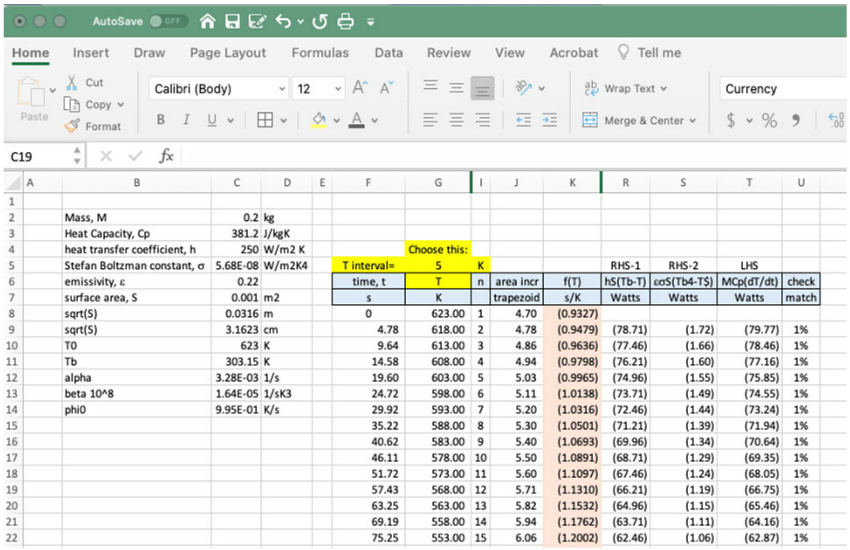
$$f(T') = \frac{1}{\Phi_0 - \left[T'\left(\frac{hS}{M\hat{C}_V}\right) + T'^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]}$$

$$\text{area} = \frac{1}{2}h(B_1 + B_2)$$

$\alpha \equiv$ $\beta \equiv$

CM3120 Module 2—Cooling of a recently manufactured part

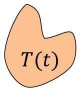
We can use trapezoidal rule to integrate $f(T)$ in Excel



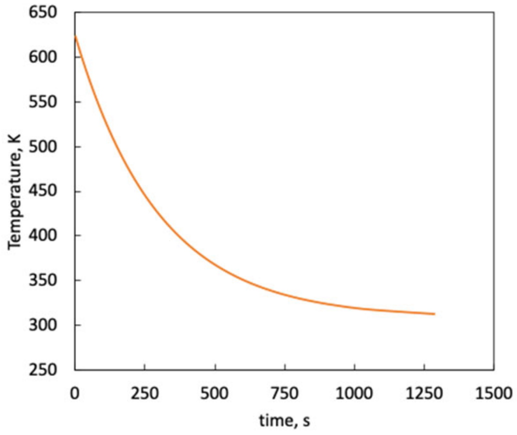
	A	B	C	D	E	F	G	I	J	K	R	S	T	U
1														
2		Mass, M	0.2	kg										
3		Heat Capacity, Cp	381.2	J/kgK										
4		heat transfer coefficient, h	250	W/m2 K			Choose this:							
5		Stefan Boltzman constant, σ	5.68E-08	W/m2K4		T interval=	5	K			RHS-1	RHS-2	LHS	
6		emissivity, ε	0.22			time, t	T	n	area incr	f(T)	hS(Tb-T)	εσS(Tb4-T$)	MCp(dT/dt)	check
7		surface area, S	0.001	m2		s	K		trapezoid	s/K	Watts	Watts	Watts	match
8		sqrt(S)	0.0316	m		0	623.00	1	4.70	(0.9327)				
9		sqrt(S)	3.1623	cm		4.78	618.00	2	4.78	(0.9479)	(78.71)	(1.72)	(79.77)	1%
10		T0	623	K		9.64	613.00	3	4.86	(0.9636)	(77.46)	(1.66)	(78.46)	1%
11		Tb	303.15	K		14.58	608.00	4	4.94	(0.9798)	(76.21)	(1.60)	(77.16)	1%
12		alpha	3.28E-03	1/s		19.60	603.00	5	5.03	(0.9965)	(74.96)	(1.55)	(75.85)	1%
13		beta 10^8	1.64E-05	1/sK3		24.72	598.00	6	5.11	(1.0138)	(73.71)	(1.49)	(74.55)	1%
14		phi0	9.95E-01	K/s		29.92	593.00	7	5.20	(1.0316)	(72.46)	(1.44)	(73.24)	1%
15						35.22	588.00	8	5.30	(1.0501)	(71.21)	(1.39)	(71.94)	1%
16						40.62	583.00	9	5.40	(1.0693)	(69.96)	(1.34)	(70.64)	1%
17						46.11	578.00	10	5.50	(1.0891)	(68.71)	(1.29)	(69.35)	1%
18						51.72	573.00	11	5.60	(1.1097)	(67.46)	(1.24)	(68.05)	1%
19						57.43	568.00	12	5.71	(1.1310)	(66.21)	(1.19)	(66.75)	1%
20						63.25	563.00	13	5.82	(1.1532)	(64.96)	(1.15)	(65.46)	1%
21						69.19	558.00	14	5.94	(1.1762)	(63.71)	(1.11)	(64.16)	1%
22						75.25	553.00	15	6.06	(1.2002)	(62.46)	(1.06)	(62.87)	1%

CM3120 Module 2—Cooling of a recently manufactured part

$t \geq 0$

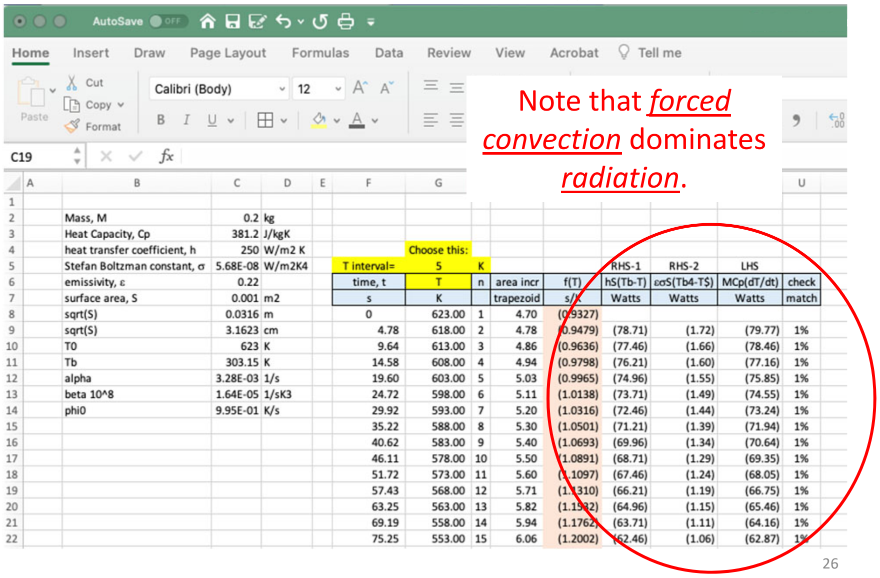


Cooling in air
Forced convection, h, T_{bulk}



CM3120 Module 2—Cooling of a recently manufactured part

We can use trapezoidal rule to integrate $f(T)$ in Excel



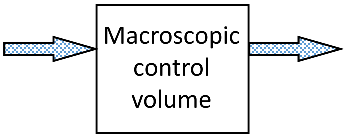
Note that *forced convection* dominates *radiation*.

	B	C	D
1			
2	Mass, M	0.2	kg
3	Heat Capacity, Cp	381.2	J/kgK
4	heat transfer coefficient, h	250	W/m2 K
5	Stefan Boltzman constant, σ	5.68E-08	W/m2K4
6	emissivity, ε	0.22	
7	surface area, S	0.001	m2
8	sqrt(S)	0.0316	m
9	sqrt(S)	3.1623	cm
10	T0	623	K
11	Tb	303.15	K
12	alpha	3.28E-03	1/s
13	beta 10^8	1.64E-05	1/sK3
14	phi0	9.95E-01	K/s

F	G								
	Choose this:								
T interval=	5	K			RHS-1	RHS-2	LHS		
time, t	T	n	area incr	f(T)	hS(Tb-T)	εσS(Tb4-T$)	MCp(dT/dt)	check
s	K		trapezoid	s/K	Watts	Watts	Watts	match
0	623.00	1	4.70	(0.9327)				
4.78	618.00	2	4.78	(0.9479)	(78.71)	(1.72)	(79.77)	1%
9.64	613.00	3	4.86	(0.9636)	(77.46)	(1.66)	(78.46)	1%
14.58	608.00	4	4.94	(0.9798)	(76.21)	(1.60)	(77.16)	1%
19.60	603.00	5	5.03	(0.9965)	(74.96)	(1.55)	(75.85)	1%
24.72	598.00	6	5.11	(1.0138)	(73.71)	(1.49)	(74.55)	1%
29.92	593.00	7	5.20	(1.0316)	(72.46)	(1.44)	(73.24)	1%
35.22	588.00	8	5.30	(1.0501)	(71.21)	(1.39)	(71.94)	1%
40.62	583.00	9	5.40	(1.0693)	(69.96)	(1.34)	(70.64)	1%
46.11	578.00	10	5.50	(1.0891)	(68.71)	(1.29)	(69.35)	1%
51.72	573.00	11	5.60	(1.1097)	(67.46)	(1.24)	(68.05)	1%
57.43	568.00	12	5.71	(1.1310)	(66.21)	(1.19)	(66.75)	1%
63.25	563.00	13	5.82	(1.1532)	(64.96)	(1.15)	(65.46)	1%
69.19	558.00	14	5.94	(1.1762)	(63.71)	(1.11)	(64.16)	1%
75.25	553.00	15	6.06	(1.2002)	(62.46)	(1.06)	(62.87)	1%

Unsteady Macroscopic Energy Balance

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$



Summary

- We have another tool for our problem-solving tool belt
- Similar to other macroscopic problem-solving protocols
- Useful for systems with unusual shapes or with multiple types of physics contributing
- Computer solutions