CM3120: Module 4

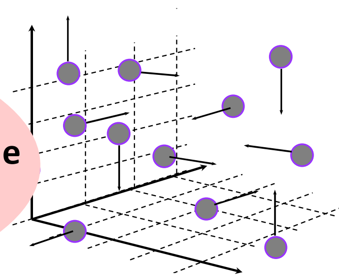
Diffusion and Mass Transfer II

I. Classic diffusion and mass transfer: d) EMCD
II. Classic diffusion and mass transfer: e) Penetration model
III. Unsteady macroscopic species A mass balances (Intro)
IV. Interphase species A mass transfers—To an interface—k_x, k_c, k_p
V. Unsteady macroscopic species A mass balances (Redux)
VI. Interphase species A mass transfers—Across multiple resistances—K_L, K_G
VII. Dimensional analysis
VIII. Data correlations

 1

CM3120: Module 4

Module 4 Lecture VI
Interphase Transfers Across Multiple Resistances
(Overall Mass Transfer Coefficients)



Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

 2

Modeling practical devices involving mass transfer

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Example 6 is presented as a series of **linked examples** that navigate around apparent "dead ends" in modeling mass-transfer units

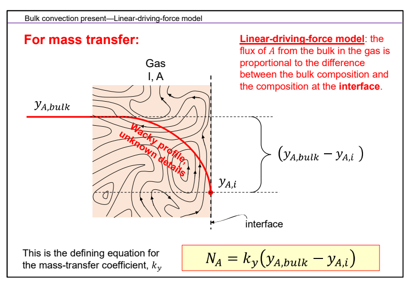
LECTURES

Identify a question	Invent something	Try to use it
1. ✓ How can we model a large, practical device dependent on mass transfer? III	1. ✓ Apply the species A mass balance to a macroscopic C.V. III	1. ✓ Lack a system to account for A going between phases PAUSE III
2. ✓ How an we account for A going between phases? IV	2. ✓ Invent k_x through linear driving force (LDF) model IV	2. ✓ Gets A *to* the boundary, but not *across* PAUSE V
3. How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? VI	3. Write LDF in both phases and combine to create overall effect of multiple resistances VI	3. Working, but can we devise a convenient shorthand? PAUSE VI
4. Can we model a large, practical device, incorporating K_L, K_G to account for mass xfer between phases? VI	4. Yes VI	

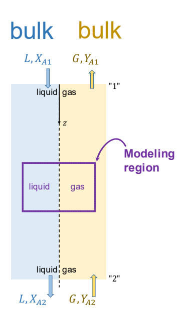
3

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Modeling Mass Transfer Equipment with the Overall Mass-Transfer Coefficient



We have concerned ourselves with mass transfer to and from the ***bulk*** region of a phase and the ***interface*** with another phase



We seek a *combined* model that allows us to describe mass transfer to/from ***bulk*** gas and ***bulk*** liquid.

This will help us to design and optimize chemical engineering mass-transfer units.

Our solution is inspired by how **heat exchangers** are modeled with overall heat transfer coefficient, ***U*** ...

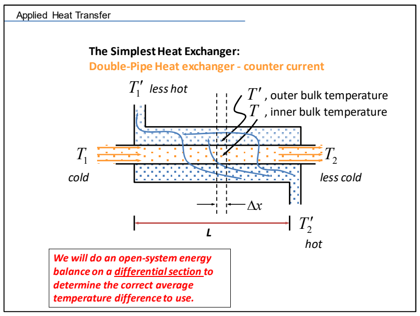
4

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Heat Transfer review

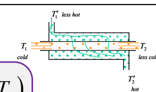
Heat exchangers are modeled with overall heat transfer coefficient, $\boldsymbol{U}$:

$$\dot{Q} = UA\Delta T_{lm}$$



Overall heat transfer coefficient, U

Analysis of double-pipe heat exchanger



FINAL RESULT:

$$Q = U\underbrace{(2\pi RL)}_{A}\underbrace{\frac{(T_1'-T_1)-(T_2'-T_2)}{\ln\frac{(T_1'-T_1)}{(T_2'-T_2)}}}_{\equiv \Delta T_{lm}}$$

$$Q = UA\Delta T_{lm}$$

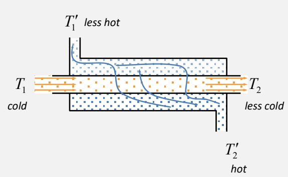
=log-mean temperature difference

Overall **driving force** ΔT_{df} for heat transfer **(bulk to bulk)**

5

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Heat exchangers are modeled with overall heat transfer coefficient, $\boldsymbol{U}$:



$$\dot{Q} = UA\Delta T_{df}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force (df) for heat transfer **(bulk to bulk)**

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, $\boldsymbol{K}$:

$$N_A = K\Delta c_{df}$$

Will this work?

$$\Delta c_{driving\ force} \overset{?}{=} (c_{bulk1} - c_{bulk2})_{av}$$

6

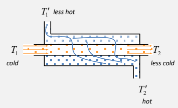
Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Yes, but there are differences between heat and mass transfer that need to be established.

What is Δc_{df}?

There are some subtleties in mass transfer driving force.

Heat exchangers are modeled with overall heat transfer coefficient, U:



$$\dot{Q} = UA\Delta T_{df}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force (df) for heat transfer (bulk to bulk)

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, K:

Will this work?

$$N_A = K\Delta c_{df}$$

$$\Delta c_{driving\ force} \overset{?}{=} (c_{bulk1} - c_{bulk2})_{av}$$

7

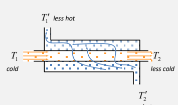
Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Yes, but there are differences between heat and mass transfer that need to be established.

What is Δc_{df}?

There are some subtleties in mass transfer driving force.

Heat exchangers are modeled with overall heat transfer coefficient, U:



$$\dot{Q} = UA\Delta T_{df}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force (df) for heat transfer (bulk to bulk)

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, K:

Will this work?

$$N_A = K\Delta c_{df}$$

$$\Delta c_{driving\ force} \overset{?}{=} (c_{bulk1} - c_{bulk2})_{av}$$

What is K?

We seek to write this in terms of our previous discussions of mass transfer (film coefficients k_x, k_p, k_c, diffusion coefficients $\mathcal{D}_{AB}$).

8

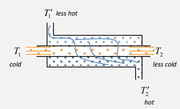
Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Yes, but there are differences between heat and mass transfer that need to be established.

What is Δc_{df}?

There are some subtleties in mass transfer driving force.

Heat exchangers are modeled with overall heat transfer coefficient, U:



$$\dot{Q} = UA\Delta T_{df}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force (df) for heat transfer (bulk to bulk)

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, K:

Will this work?

$$N_A = K\Delta c_{df}$$

?

$$\Delta c_{driving\ force} = (c_{bulk1} - c_{bulk2})_{av}$$

This first

What is K?

We seek to write this in terms of our previous discussions of mass transfer (film coefficients k_x, k_p, k_c, diffusion coefficients $\mathcal{D}_{AB}$).

9

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Driving Force for Mass Transfer

What is Δc_{df}?

(bulk to bulk)

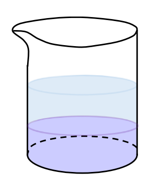
Thought experiment

Bromine has a higher affinity for benzene than for water and benzene is lighter than water.

Two solutions are prepared: bromine in benzene and bromine in water. The concentrations c_A moles bromine/volume solution are the same in both solutions.

The two solutions are put into contact, with the benzene solution floating on top of the water solution.

What happens? Is there mass transfer?



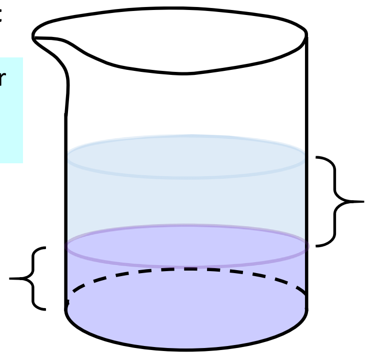
Driving Force for Mass Transfer

What is Δc_{df}?

(bulk to bulk)

Thought experiment

Bromine has a higher affinity for benzene than for water



bromine in **benzene**
$c_A = c_{A,bulk}$

bromine in **water**
$c_A = c_{A,bulk}$

$$N_A \stackrel{?}{=} K\left(c_{A,bulk} - c_{A,bulk}\right)$$

Cussler p262 figure 8.5-1

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

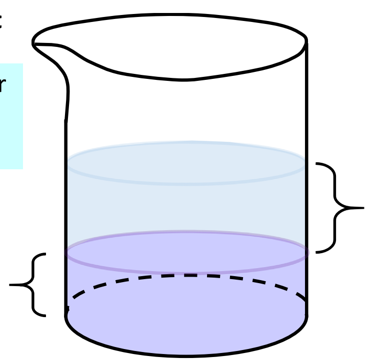
Driving Force for Mass Transfer

What is Δc_{df}?

(bulk to bulk)

Thought experiment

Bromine has a higher affinity for benzene than for water



bromine in **benzene**
$c_A = c_{A,bulk}$

bromine in **water**
$c_A = c_{A,bulk}$

$$N_A \stackrel{?}{=} K\left(c_{A,bulk} - c_{A,bulk}\right) = 0$$

Cussler p262 figure 8.5-1

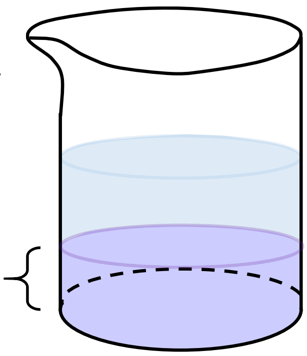
Driving Force for Mass Transfer

What is Δc_{df}?

(bulk to bulk)

Thought experiment

Bromine has a higher affinity for benzene than for water



bromine in **benzene**
$c_A = c_{A,bulk}$

bromine in **water**
$c_A = c_{A,bulk}$

$$N_A \stackrel{?}{=} K\left(\cancel{c_{A,bulk} - c_{A,bulk}}\right) = 0$$

Not correct: Species A <u>does</u> move between phases
$\boldsymbol{N_A \neq 0}$

Cussler p262 figure 8.5-1

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

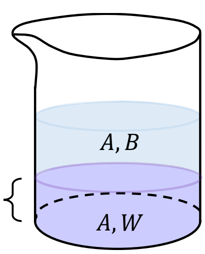
Driving Force for Mass Transfer

What is Δc_{df}?

(bulk to bulk)

What drives the motion?

The two phases are <u>not</u> in equilibrium



bromine in **benzene**
$\tilde{c}_A = \tilde{c}_{A,0}$

bromine in **water**
$c_A = c_{A,0}$

The two phases are brought into contact, and mass transfer occurs, moving bromine (A) from the A,W phase to the A,B phase

When equilibrium is reached, the concentrations of the two phases are on the **equilibrium curve**,
$\tilde{c}_A^* \; (c_A^*)$
in benzene *in water*

Cussler p262 Figure 8.5-1

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

What drives the motion?

What is Δc_{df}?

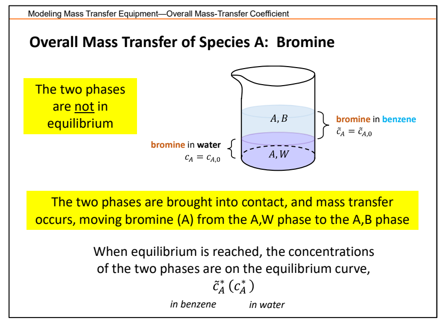
(bulk to bulk)

The **driving force for mass transfer** is not only concentration differences; rather it's differences in chemical nature that an include concentration, but includes other "preferences" that are linked to the species' chemical structures.

Scientists have unified the energetic effects of species moving around in mixtures through the definition of the ***chemical potential***.

Mass transfer occurs in the bromine thought experiment because of the higher affinity bromine has for benzene.

The driving force for mass transfer is distance from chemical equilibrium



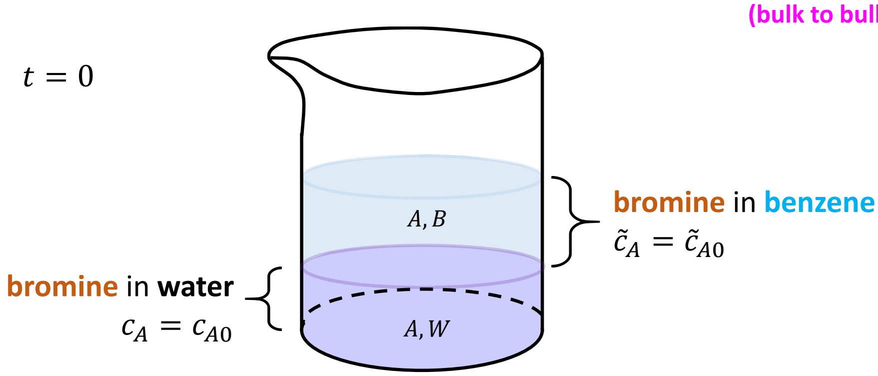
Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Driving Force for Mass Transfer

What is Δc_{df}?

(bulk to bulk)

$t = 0$



In the A,B bromine in benzene phase:

$t \geq 0$ $\qquad N_A = K_{AB}\left(\tilde{c}_A^*(c_A) - \tilde{c}_A\right)$

In the A,W bromine in water phase:

$t \geq 0$ $\qquad N_A = K_{AW}\left(c_A - c_A^*(\tilde{c}_A)\right)$

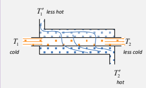
Mass transfer driving force related to distance from chemical equilibrium

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Progress!

Yes, but there are differences that need to be established.

Heat exchangers are modeled with overall heat transfer coefficient, U:



$$\dot{Q} = UA\Delta T_{df}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force (df) for heat transfer (bulk to bulk)

What is Δc_{df}?

There are some subtleties in mass transfer driving force

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, K:

$$N_A = K\Delta c_{df}$$

$$\Delta c_{driving\ force} \overset{?}{=} (c_{bulk1} - c_{bulk2})_{av}$$

Will this work?

Now,

What is K in terms of our previous discussions mass transfer (film coefficients k_x, k_p, k_c, diffusion coefficients $\mathcal{D}_{AB}$)?

What is K?

17

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

We return to modeling a gas absorption column

What is K?

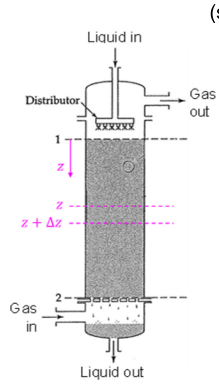
(bulk to bulk)

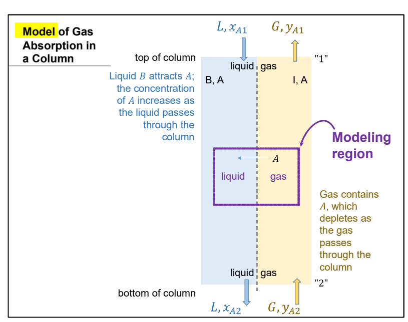
Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Example 6—Revisited, again

(started in Module 4, Lecture III)





18

Modeling practical devices involving mass transfer

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Example 6 is presented as a series of **linked examples** that navigate around apparent "dead ends" in modeling mass-transfer units

LECTURES

Identify a question	Invent something	Try to use it
1. ✓ How can we model a large, practical device dependent on mass transfer? III	1. ✓ Apply the species A mass balance to a macroscopic C.V. III	1. ✓ Lack a system to account for A going between phases PAUSE III
2. ✓ How an we account for A going between phases? IV	2. ✓ Invent k_x through linear driving force (LDF) model IV	2. ✓ Gets A *to* the boundary, but not *across* PAUSE V
3. ✓ How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? VI	3. Write LDF in both phases and combine to create overall effect of multiple resistances VI	3. Working, but can we devise a convenient shorthand? PAUSE VI
4. Can we model a large, practical device, incorporating K_L, K_G to account for mass xfer between phases? VI	4. Yes VI	

19

Applied Heat Transfer

In a previous course, we developed a model so that we can characterize heat exchangers

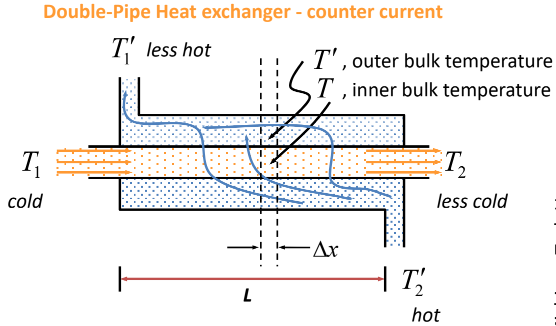
What is K? (bulk to bulk)

We divided the heat exchanger into **slices** with bulk temperatures T and T' at location x and applied Newton's law of cooling (h_1, h_2) and Fourier's law (k)

We integrated over $0 \le x \le L$ to obtain the total heat transferred Q.

This allowed us to define overall heat transfer coefficient U, giving heat transfer at location x:

The Simplest Heat Exchanger:
Double-Pipe Heat exchanger - counter current



$$Q = U_2 A_2 \Delta T$$

$$= \left(\frac{\frac{1}{R_2}}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\frac{R_2}{R_1} + \frac{1}{h_1 R_1}} \right)(2\pi R_2 L)(T' - T)$$

20

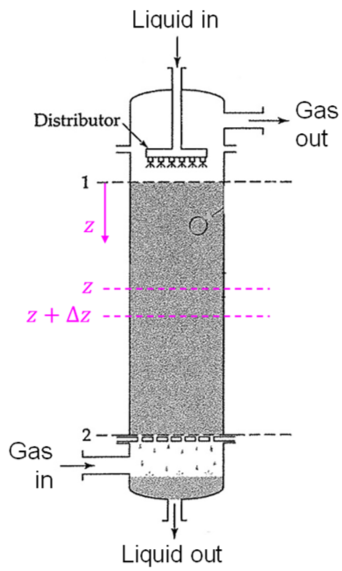
Applied Mass Transfer

Example 6—Revisited, again

Example 6: Height of a packed bed absorber

What is K?

(bulk to bulk)



In Lectures III and V, we began to model a gas absorber, using the same slicing strategy, but we needed a concept of how to model the mass transfer in the slice

We need to apply the film mass transfer coefficients to the slice of the gas absorber column

(*spoiler: This will lead to the overall mass transfer coefficient)

21

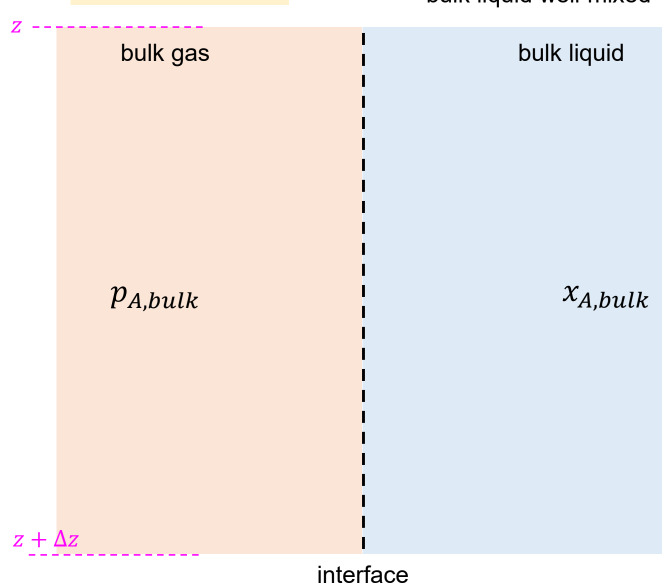
Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

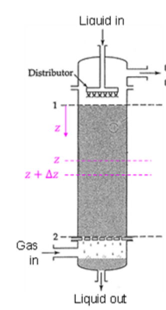
Example 6: Height of a packed bed absorber

Bulk gas well mixed, bulk liquid well mixed

What is K?

(bulk to bulk)





The mass transfer in a slice of the column can be modeled with film coefficients

WRF Ch29, Fig 29.4

22

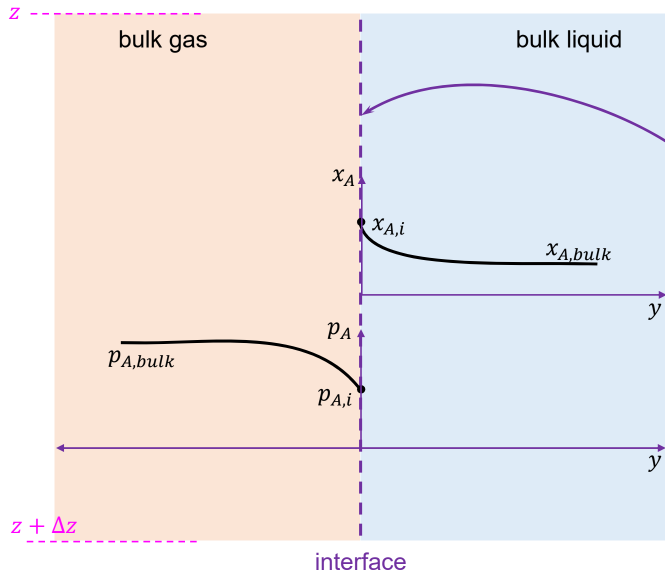
Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Example 6: Height of a packed bed absorber

Bulk gas well mixed, bulk liquid well mixed

What is K?

(bulk to bulk)



Interface: species in the two phases are in equilibrium

Equilibrium is characterized by H, the Henry's law constant:

$$p_A^* = Hx_A^*$$

Cussler, p262

23

Overall Mass-Transfer Coefficient

Example 6: Height of a packed bed absorber

Use the linear driving force model (film coefficients)

What is K?

(bulk to bulk)

Gas: Linear driving force model:

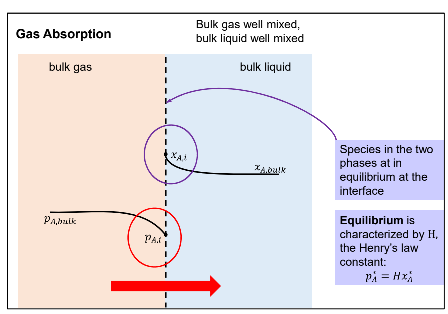
$$(N_A)_G = k_G(p_A - p_{Ai})$$

$$k_G[=]\frac{(moles\ A\ transferred)}{(time \cdot area \cdot pressure)}$$

Liquid: Linear driving force model:

$$(N_A)_L = k_L(x_{A,i} - x_A)$$

$$k_L[=]\frac{(moles\ A\ transferred)}{(time \cdot area \cdot conc)}$$



At steady state:

$$(N_A)_G = (N_A)_L$$

24

Overall Mass-Transfer Coefficient

Example 6: Height of a packed bed absorber

Use the linear driving force model (film coefficients)

What is K? **(bulk to bulk)**

(x_A, p_A) = bulk conditions ("operating point")

(x_{Ai}, p_{Ai}) = interface point

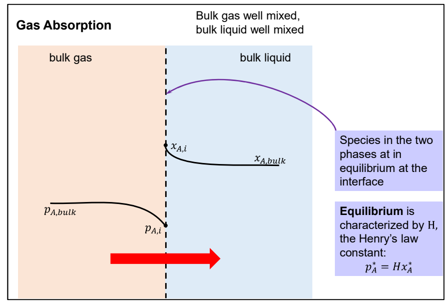
at steady state:

$$(N_A)_G = (N_A)_L$$

In terms of film coefficients:

$$k_p(p_A - p_{Ai}) = k_x(x_{A,i} - x_A)$$

$$(p_A - p_{Ai}) = \frac{-k_x}{k_p}(x_A - x_{A,i})$$



A line of slope $(-k_x/k_p)$ connects the operating point (bulk conditions $x_{A,b}, p_{A,b}$) and the interface point

25

Overall Mass-Transfer Coefficient

Example 6: Height of a packed bed absorber

Use the linear driving force model (film coefficients)

What is K? **(bulk to bulk)**

(x_A, p_A) = bulk conditions ("operating point")

(x_{Ai}, p_{Ai}) = interface point

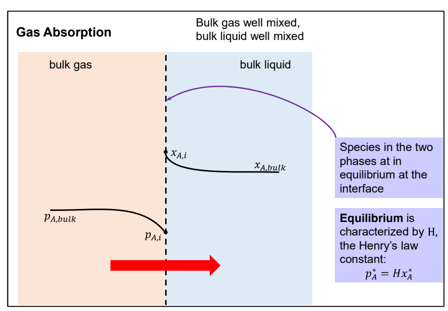
at steady state:

$$(N_A)_G = (N_A)_L$$

In terms of film coefficients:

$$k_p(p_A - p_{Ai}) = k_x(x_{A,i} - x_A)$$

$$(p_A - p_{Ai}) = \frac{-k_x}{k_p}(x_A - x_{A,i})$$

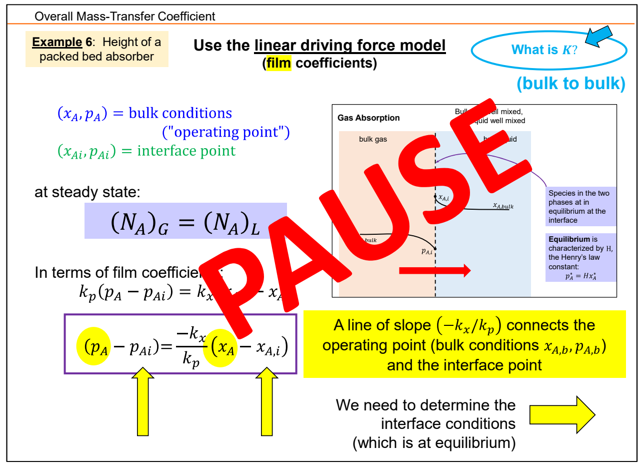


A line of slope $(-k_x/k_p)$ connects the operating point (bulk conditions $x_{A,b}, p_{A,b}$) and the interface point

We need to determine the interface conditions (which is at equilibrium)

26

We **PAUSE** **Example 6** for the third time to pursue the relationship between overall mass transfer coefficient K and the interface *film* mass transfer coefficients k_x and k_p



27

Modeling practical devices involving mass transfer

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Example 6 is a part of a series of **linked examples** that navigate around apparent "dead ends" in modeling mass-transfer units

LECTURES

Identify a question

1. ✓ How can we model a large, practical device dependent on mass transfer? **III**
2. ✓ How an we account for A going between phases? **IV**
3. ✓ How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? **VI**
4. Can we model a large, practical device, incorporating K_L, K_G to account for mass xfer between phases? **VI**

Invent something

1. ✓ Apply the species A mass balance to a macroscopic C.V. **III**
2. ✓ Invent k_x through linear driving force (LDF) model **IV**
3. ✓ Write LDF in both phases and combine to create overall effect of multiple resistances **VI**
4. Yes **VI**

Try to use it

1. ✓ Lack a system to account for A going between phases **PAUSE III**
2. ✓ Gets A *to* the boundary, but not *across* **PAUSE V**
3. Working, but can we devise a convenient shorthand? **PAUSE VI**

28

Overall Mass-Transfer Coefficient

What is K?

(bulk to bulk)

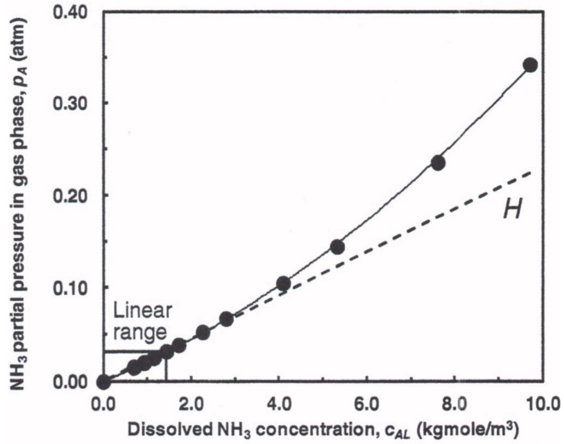
Phase Equilibrium Data (*material function*)

Phase equilibrium data are published in the literature

In the **dilute** regime, the equilibrium curve is linear **(Henry's law)**:

$$p_A^* = Hx_A^*$$

In the non-dilute (concentrated) regime, the equilibrium curve is not linear



WRF Ch29, Fig 29.2

(there is no resistance at the interface, equilibrium is established)

29

Overall Mass-Transfer Coefficient

What is K?

(bulk to bulk)

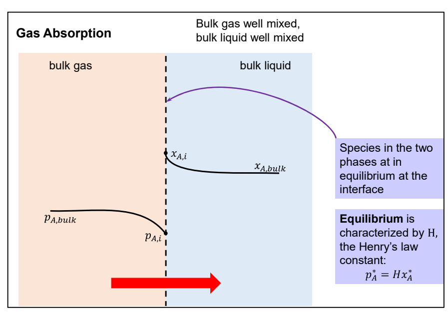
Example 11: What are the liquid and gas concentrations at the interface?

Equilibrium in mixtures dilute in A is characterized by H, the Henry's law constant:

$$p_A^* = Hx_A^*$$

dilute regime

Let's try



$$N_A = k_p(p_A - p_{Ai}) = k_x(x_{A,i} - x_A)$$

30

Overall Mass-Transfer Coefficient

Example 11: What are the liquid and gas concentrations at the interface?

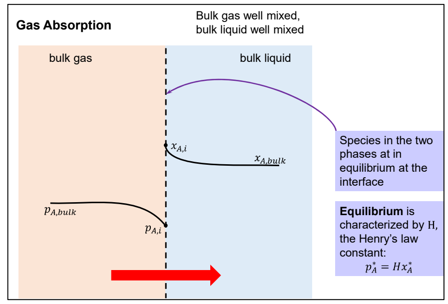
What is K?

(bulk to bulk)

Equilibrium in mixtures dilute in A is characterized by H, the Henry's law constant:

$$p_A^* = Hx_A^*$$

dilute regime



Answer:

$$\frac{p_{Ai}}{H} = x_{Ai} = \frac{k_p p_A + k_x x_A}{k_p H + k_x}$$

(Henry's law regime)

31

Overall Mass-Transfer Coefficient

Example 12: What is the flux from gas to liquid?

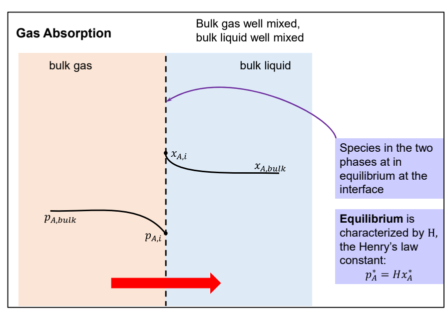
What is K?

(bulk to bulk)

Equilibrium in mixtures dilute in A is characterized by H, the Henry's law constant:

$$p_A^* = Hx_A^*$$

dilute regime



Let's try

$$N_A = k_p(p_A - p_{Ai}) = k_x(x_{A,i} - x_A)$$

32

Overall Mass-Transfer Coefficient

Example 12: What is the flux from gas to liquid?

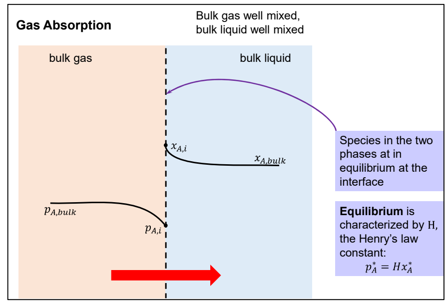
What is K?

(bulk to bulk)

Equilibrium in mixtures dilute in A is characterized by H, the Henry's law constant:

$$p_A^* = Hx_A^*$$

dilute regime



Answer:

$$N_A = \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_p}}\right)[p_A - Hx_A]$$

Includes both resistances

(Henry's law regime)

33

Overall Mass-Transfer Coefficient

What was our goal?

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Heat exchangers are modeled with overall heat transfer coefficient, U:

$$\dot{Q} = UA\Delta T_{df}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force (df) for heat transfer (bulk to bulk)

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, K:

$$N_A = K\Delta c_{df}$$

What is K?

What is Δc_{df}?

34

Overall Mass-Transfer Coefficient

What was our goal?

Develop a mass-transfer device analog to U in heat exchangers

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Heat exchangers are modeled with overall heat transfer coefficient, $\boldsymbol{U}$:

$$\dot{Q} = \boldsymbol{UA\Delta T_{df}}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force ($\boldsymbol{df}$) for heat transfer (bulk to bulk)

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, $\boldsymbol{K}$:

$$\boldsymbol{N_A = K\Delta c_{df}}$$

What is K?

What is Δc_{df}?

Overall Mass-Transfer Coefficient

What was our goal?

Develop a mass-transfer device analog to U in heat exchangers

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Heat exchangers are modeled with overall heat transfer coefficient, $\boldsymbol{U}$:

$$\dot{Q} = \boldsymbol{UA\Delta T_{df}}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force ($\boldsymbol{df}$) for heat transfer (bulk to bulk)

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, $\boldsymbol{K}$:

$$\boldsymbol{N_A = K\Delta c_{df}}$$

Done.

p_A = bulk concentration (partial pressure) of the gas

x_A = bulk concentration mole fraction) of the liquid

Gas side units

$$N_A = \underbrace{\left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_p}}\right)}_{K_G} \underbrace{[p_A - Hx_A]}_{\Delta c_{df}}$$

dilute regime

Overall Mass-Transfer Coefficient

Overall Mass-Transfer Coefficient

What was our goal?

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Heat exchangers are modeled with overall heat transfer coefficient, U:

$$\dot{Q} = UA\Delta T_{df}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force (df) for heat transfer (bulk to bulk)

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, K:

$$N_A = K\Delta c_{df}$$

p_A = bulk concentration (partial pressure) of the gas

x_A = bulk concentration mole fraction) of the liquid

$$N_A = \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_p}}\right)[p_A - Hx_A]$$

Henry's law:

$$p_A^* = Hx_A^*$$

dilute regime

Includes both L & G resistances

$$N_A = K_G[p_A - Hx_A]$$

$$K_G \equiv \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_p}}\right)$$

K_G is the **gas-side-units** overall mass transfer coefficient

37

Overall Mass-Transfer Coefficient

Alternatively, focusing on the liquid side:

$$N_A = \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_p}}\right)[p_A - Hx_A]\left(\frac{\frac{1}{H}}{\frac{1}{H}}\right)$$

$$N_A = \underbrace{\left(\frac{1}{\frac{1}{k_x} + \frac{1}{Hk_p}}\right)}_{K_L}\underbrace{\left[\frac{p_A}{H} - x_A\right]}_{\Delta c_{df}}$$

Overall Mass-Transfer Coefficient

What was our goal?

Modeling Mass Transfer Equipment—Overall Mass-Transfer Coefficient

Heat exchangers are modeled with overall heat transfer coefficient, U:

$$\dot{Q} = UA\Delta T_{df}$$

$$\Delta T_{driving\ force} = (T_{bulk1} - T_{bulk2})_{lm\ av}$$

Overall driving force (df) for heat transfer (bulk to bulk)

Gas absorbers and distillation columns are modeled with overall mass transfer coefficient, K:

$$N_A = K\Delta c_{df}$$

p_A = bulk concentration (partial pressure) of the gas

x_A = bulk concentration mole fraction) of the liquid

$$N_A = \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_p}}\right)[p_A - Hx_A]$$

Henry's law:

$$p_A^* = Hx_A^*$$

Includes both L & G resistances

Liquid side

$$N_A = K_L\left[\frac{p_A}{H} - x_A\right]$$

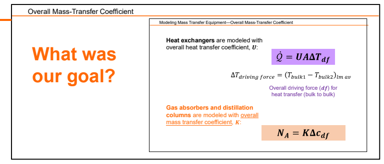
$$K_L \equiv \left(\frac{1}{\frac{1}{k_x} + \frac{1}{Hk_p}}\right)$$

K_L is the **liquid-side-units** overall mass transfer coefficient

38

Overall Mass-Transfer Coefficient

The **Overall Mass-Transfer Coefficients** *equivalently* describe the steady state relationship between the bulk concentrations (gas and liquid) of a composite scenario (slice of gas absorber)



Henry's law:
$$p_A^* = Hx_A^*$$
dilute regime

$$N_A = K_G[p_A - Hx_A]$$
$$K_G \equiv \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_p}}\right)$$

K_G is the gas-side-units overall mass transfer coefficient

Both include **both** resistances

$$N_A = K_L\left[\frac{p_A}{H} - x_A\right]$$
$$K_L \equiv \left(\frac{1}{\frac{1}{k_x} + \frac{1}{Hk_p}}\right)$$

K_L is the liquid-side-units overall mass transfer coefficient

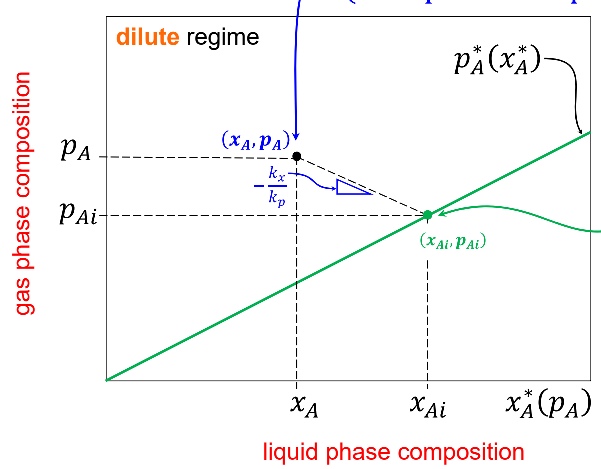
Driving forces written differently, but are equivalent

39

Overall Mass-Transfer Coefficient **dilute** regime

We can visualize the various driving forces graphically:

(x_A, p_A) = operating point (bulk phase compositions)



(x_{Ai}, p_{Ai}) = interface point (interface phase compositions; in equilibrium)

If film coefficients k_G, k_L known from data correlations (dimensional analysis, literature), the operating point and the interface point may be linked.

$$(p_A - p_{Ai}) = \frac{-k_x}{k_p}(x_A - x_{A,i})$$

WRF Ch29, Fig 29.6

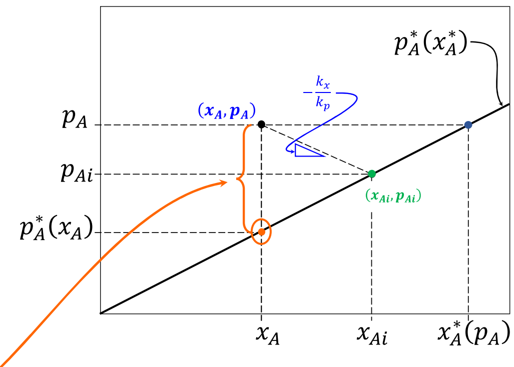
40

We can visualize the gas-phase-units **driving force** as a vertical segment:

Gas-phase-units:
Overall Linear driving force model:

$$N_A = K_G(p_A - Hx_A)$$

Δc_{df}

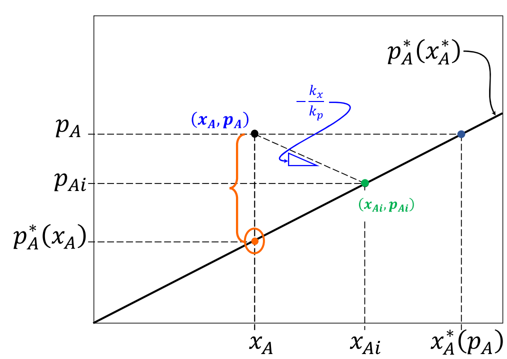


We can visualize the gas-phase-units **driving force** as a vertical segment:

Gas-phase-units:
Overall Linear driving force model:

$$N_A = K_G(p_A - Hx_A)$$

Δc_{df}



It turns out: $Hx_A = p_A^*(x_A) =$ The gas partial pressure that a liquid of concentration x_A (the liquid bulk mole fraction) would be in equilibrium with

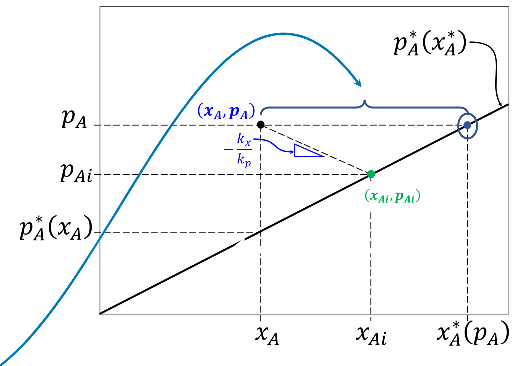
dilute regime

Overall Mass-Transfer Coefficient **dilute** regime

We can visualize the liquid-phase-units **driving force** as a horizontal segment:

Liquid-phase-units
Overall Linear driving force model:

$$N_A = K_L \underbrace{\left(\frac{p_A}{H} - x_A\right)}_{\Delta c_{df}}$$



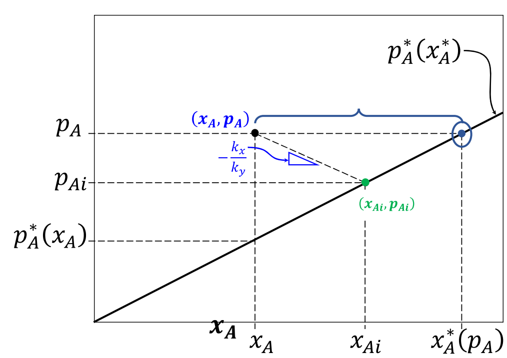
 43

Overall Mass-Transfer Coefficient **dilute** regime

We can visualize the liquid-phase-units **driving force** as a horizontal segment:

Liquid-phase-units
Overall Linear driving force model:

$$N_A = K_L \underbrace{\left(\frac{p_A}{H} - x_A\right)}_{\Delta c_{df}}$$



It turns out: $\frac{p_A}{H} = x_A^*(p_A) =$ The liquid mole fraction that a gas of partial pressure p_A (the gas bulk partial pressure) would be in equilibrium with

dilute regime

 44

Overall Mass-Transfer Coefficient dilute regime

Overall Mass Transfer Coefficients

dilute regime

Two equivalent versions of $\boldsymbol{K}$ and $\boldsymbol{\Delta c_{df}}$; one based on gas-phase customary units, one based on liquid-phase customary units

Liquid-phase-units
Overall Linear driving force model:

$$N_A = K_L\left(\frac{p_{A,b}}{H} - x_{A,b}\right)$$

Gas-phase-units:
Overall Linear driving force model:

$$N_A = K_G\left(p_{A,b} - H x_{A,b}\right)$$

The expressions relate steady state ***bulk*** concentrations in the liquid and gas phases, reflecting the net effect of multiple (two) resistances

both versions include **both** resistances

45

Overall Mass-Transfer Coefficient concentrated and dilute regimes

Overall Mass Transfer Coefficients

concentrated regime *(and dilute)*

Two equivalent versions of $\boldsymbol{K}$ and $\boldsymbol{\Delta c_{df}}$; one based on gas-phase customary units, one based on liquid-phase customary units

Liquid-phase-units
Overall Linear driving force model:

$$N_A \equiv K_L\left(x_A^*\left(p_{A,b}\right) - x_{A,b}\right)$$

Gas-phase-units:
Overall Linear driving force model:

$$N_A \equiv K_G\left(p_{A,b} - p_A^*\left(x_{A,b}\right)\right)$$

We can choose to use the driving-force relationships that resulted from the dilute regime to ***define*** overall mass-transfer coefficients for the concentrated regime

both versions include **both** resistances

46

Overall Mass-Transfer Coefficient — **concentrated and dilute** regimes

Many **concentration units may be used**

$\boldsymbol{K_L}$

Liquid-phase-units
Overall Linear driving force model, defined:

$$N_A \equiv K_x\left(x_A^*(\quad) - x_{A,b}\right)$$
$$N_A \equiv K_{cL}\left(c_{AL}^*(\quad) - c_{AL,b}\right)$$

$$(\quad) = p_{A,b} \; or \; c_{A,b} \; or \; y_{A,b}$$

$\boldsymbol{K_G}$

Gas-phase-units:
Overall Linear driving force model, defined:

$$N_A \equiv K_p\left(p_{A,b} - p_A^*(\quad)\right)$$
$$N_A \equiv K_{cG}\left(c_{AG,b} - c_{AG}^*(\quad)\right)$$
$$N_A \equiv K_y\left(y_{A,b} - y_A^*(\quad)\right)$$

$$(\quad) = x_{A,b} \; or \; c_{AL,b}$$

47

Overall Mass-Transfer Coefficient — **concentrated and dilute** regimes

But, how do we relate these overall mass-transfer coefficients back to the film coefficients?

Liquid-phase-units
Overall Linear driving force model:

$$N_A \equiv K_x\left(x_A^*\left(y_{A,b}\right) - x_{A,b}\right)$$

Gas-phase-units:
Overall Linear driving force model:

$$N_A \equiv K_y\left(y_{A,b} - y_A^*\left(x_{A,b}\right)\right)$$

Let's return to the **graphical representation**

48

Overall Mass-Transfer Coefficient

Gas-phase-units

concentrated regime

$$\frac{N_A}{K_y} = \left(y_{Ab} - y_A^*(x_{Ab})\right) = a + b = (y_{Ab} - y_{Ai}) + b$$

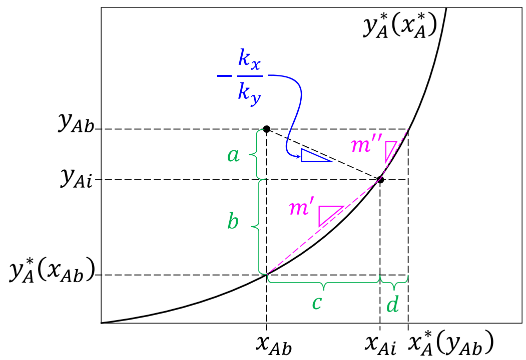
$$m' = \frac{\text{rise}}{\text{run}} = \frac{b}{(x_{Ai} - x_{Ab})}$$

$$\frac{N_A}{K_y} = (y_{Ab} - y_{Ai}) + m'(x_{Ai} - x_{Ab})$$

$$\frac{N_A}{K_y} = \frac{N_A}{k_y} + m'\left(\frac{N_A}{k_x}\right)$$

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m'}{k_x}$$

$$K_y = \frac{1}{\frac{1}{k_y} + \frac{m'}{k_x}}$$



WRF pp600-603

49

Overall Mass-Transfer Coefficient

Liquid-phase-units

concentrated regime

$$\frac{N_A}{K_x} = \left(x_A^*(y_{Ab}) - x_{Ab}\right) = c + d = (x_{Ai} - x_{Ab}) + d$$

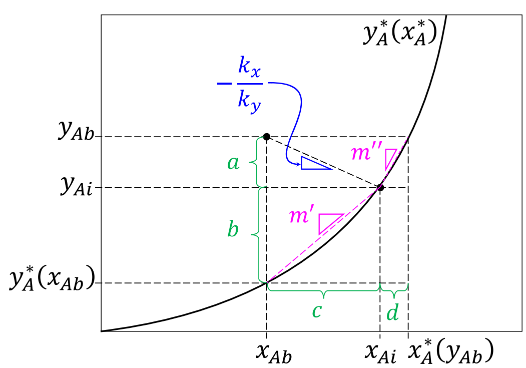
$$m'' = \frac{\text{rise}}{\text{run}} = \frac{y_{Ab} - y_{Ai}}{d}$$

$$\frac{N_A}{K_x} = (x_{Ai} - x_{Ab}) + \frac{(y_{Ab} - y_{Ai})}{m''}$$

$$\frac{N_A}{K_x} = \frac{N_A}{k_x} + \frac{1}{m''}\left(\frac{N_A}{k_y}\right)$$

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{m'' k_y}$$

$$K_x = \frac{1}{\frac{1}{k_x} + \frac{1}{m'' k_y}}$$



50

Overall Mass-Transfer Coefficient — **concentrated and dilute** regimes

Liquid-phase-units
Film Linear driving force model:

$$N_A \equiv k_x(x_{A,i} - x_{A,b})$$
$$N_A \equiv k_{cL}(c_{AL,i} - c_{AL,b})$$

Gas-phase-units:
Film Linear driving force model:

$$N_A \equiv k_p(p_{A,b} - p_{A,i})$$
$$N_A \equiv k_{cG}(c_{AG,b} - c_{A,i})$$
$$N_A \equiv k_y(y_{A,b} - y_{A,i})$$

Liquid-phase-units
Overall Linear driving force model:

$$N_A \equiv K_x(x_A^*(\ \ ) - x_{A,b})$$
$$N_A \equiv K_{cL}(c_{AL}^*(\ \ ) - c_{AL,b})$$

$(\ \ ) = p_{A,b}\ or\ c_{A,b}\ or\ y_{A,b}$

Gas-phase-units:
Overall Linear driving force model:

$$N_A \equiv K_p\left(p_{A,b} - p_A^*(\ \ )\right)$$
$$N_A \equiv K_{cG}\left(c_{AG,b} - c_{AG}^*(\ \ )\right)$$
$$N_A \equiv K_y\left(y_{A,b} - y_A^*(\ \ )\right)$$

$(\ \ ) = x_{A,b}\ or\ c_{AL,b}$

Let's take these tools out for a spin!

$$K_x = \frac{1}{\frac{1}{k_x} + \frac{1}{m'' k_y}}, \quad K_y = \frac{1}{\frac{1}{k_y} + \frac{m'}{k_x}}, \text{ etc.}$$

51

Overall Mass-Transfer Coefficient — **Example 13**: Liquid stripping process (remove H_2S from water)

Overall Mass Transfer Coefficients

Example 13:

A liquid stripping process (20^oC, $1.5\ atm$) is used to transfer hydrogen sulfide (H_2S = species A) dissolved in water into an air stream. At a particular elevation in the column, the mole fraction of H_2S in the gas phase is 0.010 and the mole fraction of H_2S in the liquid phase is 6.0×10^{-5}. Calculate the flux, the overall mass transfer coefficients, and the interface composition.

1. Film coefficients: $k_x = 0.30 \frac{kmol}{m^2 s}$, $k_y = 4.5 \times 10^{-3} \frac{kmol}{m^2\ s}$ (obtained from data correlations).
2. Equilibrium relationship: $p_A^*(atm) = \bar{H}\ c_A^* \left(\frac{mol\ H_2S}{m^3}\right)$, where $\bar{H} = 8.8 \times 10^{-3}\ m^3 atm/mol$.

WRF, Ch29, Ex 1

52

Overall Mass-Transfer Coefficient

Example 13: Liquid stripping process (remove H_2S from water)

Overall Mass Transfer Coefficients

Example 13:

A liquid stripping process $(20^oC,\ 1.5\ atm)$ is used to transfer hydrogen sulfide (H_2S = species A) dissolved in water into an air stream. At a particular elevation in the column, the mole fraction of H_2S in the gas phase is 0.010 and the mole fraction of H_2S in the liquid phase is 6.0×10^{-5}. Calculate the flux, the overall mass transfer coefficients, and the interface composition.

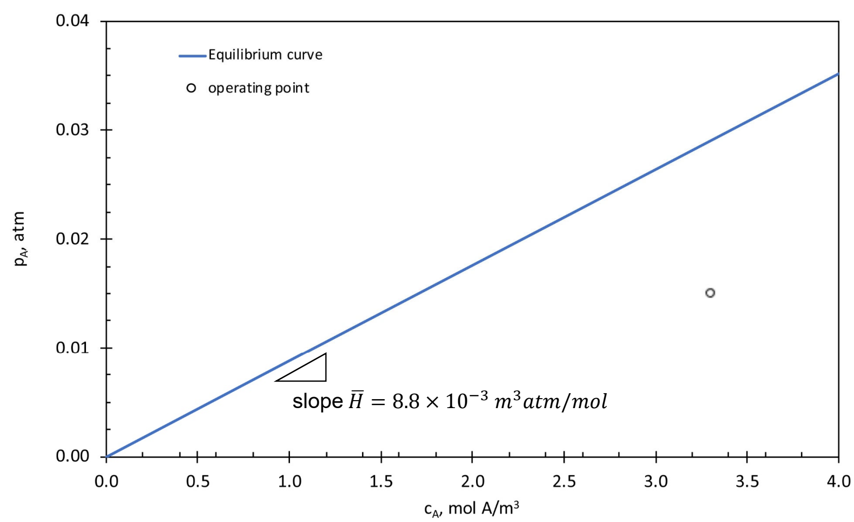
1. Film coefficients: $k_x = 0.30 \frac{kmol}{m^2} s$, $k_y = 4.5 \times 10^{-3} \frac{kmol}{m^2\ s}$ (obtained from data correlations).
2. Equilibrium relationship: $p_A^*(atm) = \bar{H}\ c_A^* \left(\frac{mol\ H_2S}{m^3}\right)$, where $\bar{H} = 8.8 \times 10^{-3}\ m^3 atm/mol$.

Let's try

WRF, Ch29, Ex 1

Overall Mass-Transfer Coefficient

Example 13: Liquid stripping process (remove H_2S from water)

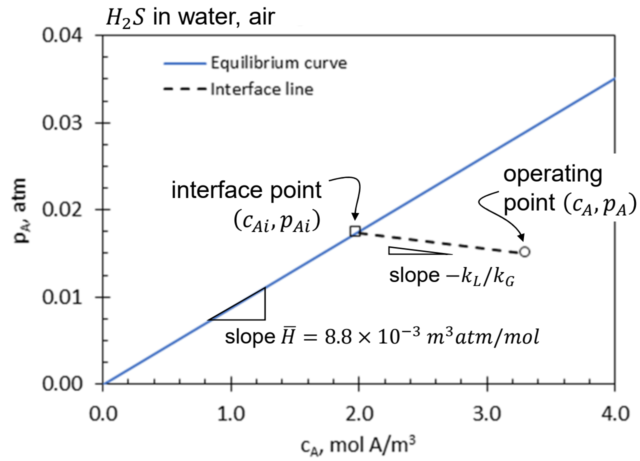


Overall Mass-Transfer Coefficient

Example 13: Liquid stripping process (remove H_2S from water)

- Choose units: liquid c_A, gas p_A
- Convert units
- Plot equilibrium curve, operating point
-
-
-

For mass transfer from liquid to gas (stripping), the operating point is below the equilibrium line



WRF p604, Fig 29.8
Excel 3/22/21 solution hw4 prob 15 overal xfer coef

55

Overall Mass-Transfer Coefficient

Example 13: Liquid stripping process (remove H_2S from water)

Many concentration units may be used

K_L	K_G
Liquid-phase-units **<u>Overall</u> Linear driving force model:**	**Gas-phase-units:** **<u>Overall</u> Linear driving force model:**
$N_A \equiv K_x(x_A^*(\ \ ) - x_{A,b})$ $N_A \equiv K_{cL}(c_{AL}^*(\ \ ) - c_{AL,b})$	$N_A \equiv K_p\left(p_{A,b} - p_A^*(\ \ )\right)$ $N_A \equiv K_{cG}\left(c_{AG,b} - c_{AG}^*(\ \ )\right)$ $N_A \equiv K_y\left(y_{A,b} - y_A^*(\ \ )\right)$
$(\ \ ) = p_{A,b}\ or\ c_{A,b}\ or\ y_{A,b}$	$(\ \ ) = x_{A,b}\ or\ c_{AL,b}$

In the chosen units:

slope $\bar{H} = 8.8 \times 10^{-3}\ m^3 atm/mol$

$$K_G = K_p = \frac{1}{\frac{\bar{H}}{k_c} + \frac{1}{k_p}} = 5.1 \times 10^{-4} \frac{kmol}{m^2 atm\ s}$$

$$K_L = K_c = \frac{1}{\frac{1}{k_c} + \frac{1}{\bar{H} k_p}} = 4.5 \times 10^{-3} \frac{m}{s}$$

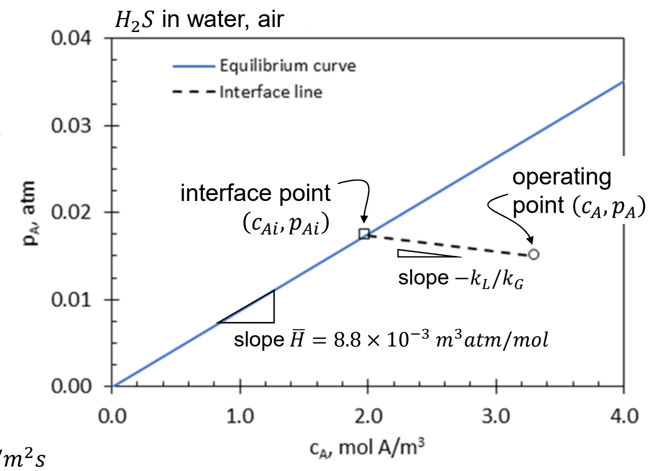
56

Overall Mass-Transfer Coefficient

Example 13: Liquid stripping process (remove H_2S from water)

- Choose units: liquid c_A, gas p_A
- Convert units
- Plot equilibrium curve, operating point
- Calculate interface point (convert units of k_y, k_x
- Calculate K_G, K_L
- Calculate N_A

See hand notes for start



$N_A = -7.2 \times 10^{-6} kmol/m^2 s$

Homework 4, problem 4.15

We must use care with units!

WRF p604, Fig 29.8
Excel 3/22/21 solution hw4 prob 15 overal xfer coef

57

Overall Mass-Transfer Coefficient

Now, what can we do with this new tool, **overall mass transfer coefficient K**, in a device?

Unsteady Macroscopic Species A Mass Balance

Previous lectures

Let's try a practical problem

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Let's pick up where we paused last …

BSL2 p742

58

Modeling practical devices involving mass transfer

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Example 6 is presented as a series of **linked examples** that navigate around apparent "dead ends" in modeling mass-transfer units

LECTURES

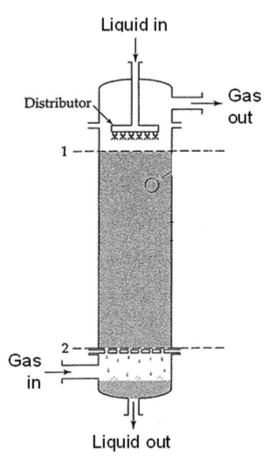
Identify a question	Invent something	Try to use it
✓ 1. How can we model a large, practical device dependent on mass transfer? **III**	✓ 1. Apply the species A mass balance to a macroscopic C.V. **III**	✓ 1. Lack a system to account for A going between phases **PAUSE III**
✓ 2. How an we account for A going between phases? **IV**	✓ 2. Invent k_x through linear driving force (LDF) model **IV**	✓ 2. Gets A *to* the boundary, but not *across* **PAUSE V**
✓ 3. How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? **VI**	✓ 3. Write LDF in both phases and combine to create overall effect of multiple resistances **VI**	✓ 3. Working, but can we devise a convenient shorthand? **PAUSE VI**
4. Can we model a large, practical device, incorporating K_L, K_G to account for mass xfer between phases? **VI**	4. Yes **VI**	

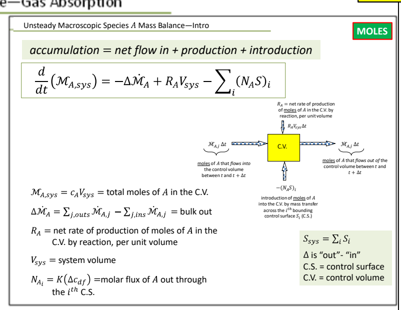
59

Unsteady Macroscopic Species A Mass Balance—Gas Absorption

Example 6—Continued

Unsteady Macroscopic Species A Mass Balance—Gas Absorption





?

You try.

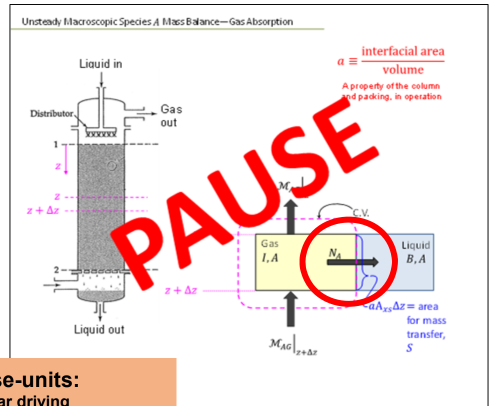
60

Unsteady Macroscopic Species A Mass Balance—Gas Absorption

Example 6—Continued

We ***paused***, most recently, because we did not have a tool to account for the **bulk-to-bulk mass transfer** between the phases in the apparatus

Now, we do.



Liquid-phase-units
<u>Overall</u> Linear driving force model:

$$N_A \equiv K_x\left(x_A^*(y_{A,b}) - x_{A,b}\right)$$

Gas-phase-units:
<u>Overall</u> Linear driving force model:

$$N_A \equiv K_y\left(y_{A,b} - y_A^*(x_{A,b})\right)$$

61

Unsteady Macroscopic Species A Mass Balance

Example 6—concluded

Now, let's <u>*finish*</u> our practical problem

<u>Example</u> 6: Height of a packed bed absorber

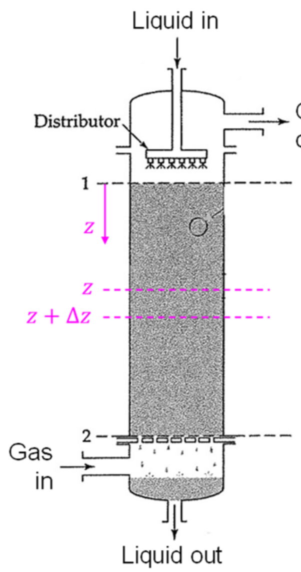
How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation? We assume dilute concentrations in both gas and liquid.

Cussler p314

62

Unsteady Macroscopic Species A Mass Balance—Gas Absorption

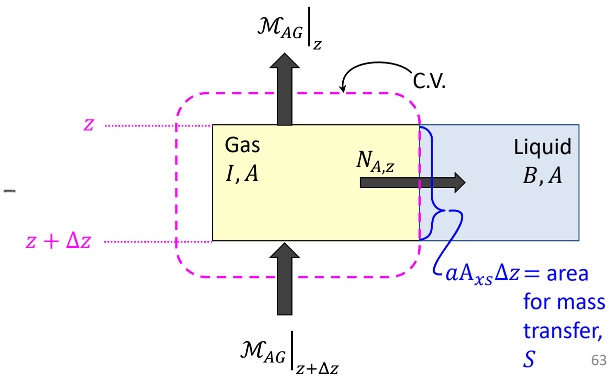
Example 6—concluded



$a \equiv \frac{\text{interfacial area}}{\text{volume}}$ A property of the column and packing, in operation

Combined molar flux of species A **from gas to liquid** $\equiv N_{A,z}$

A_{xs} = column cross section



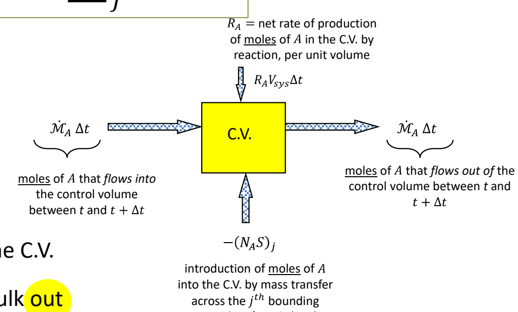
63

Unsteady Macroscopic Species A Mass Balance—Intro

MOLES

accumulation = net flow in + production + introduction

$$\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$



$\mathcal{M}_{A,sys} = c_A V_{sys}$ = total moles of A in the C.V.

$\Delta\dot{\mathcal{M}}_A = \sum_{j,outs}\dot{\mathcal{M}}_{A,j} - \sum_{j,ins}\dot{\mathcal{M}}_{A,j}$ = bulk out

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

V_{sys} = system volume

$N_{A_j} = K\left(\Delta c_{df}\right)$ =molar flux of A out through the j^{th} C.S.

We now have expressions for K and Δc_{df}

$S_{sys} = \sum_j S_j$

Δ is "out"- "in"

C.S. = control surface

C.V. = control volume

64

Overall Mass-Transfer Coefficient — concentrated and dilute regimes

Liquid-phase-units
Film Linear driving force model:

$$N_A \equiv k_x(x_{A,i} - x_{A,b})$$
$$N_A \equiv k_{cL}(c_{AL,i} - c_{AL,b})$$

Gas-phase-units:
Film Linear driving force model:

$$N_A \equiv k_p(p_{A,b} - p_{A,i})$$
$$N_A \equiv k_{cG}(c_{AG,b} - c_{A,i})$$
$$N_A \equiv k_y(y_{A,b} - y_{A,i})$$

Liquid-phase-units
Overall Linear driving force model:

$$N_A \equiv K_x(x_A^*(\ \ ) - x_{A,b})$$
$$N_A \equiv K_{cL}(c_{AL}^*(\ \ ) - c_{AL,b})$$

$(\ \ ) = p_{A,b}$ or $c_{A,b}$ or $y_{A,b}$

Gas-phase-units:
Overall Linear driving force model:

$$N_A \equiv K_p\left(p_{A,b} - p_A^*(\ \ )\right)$$
$$N_A \equiv K_{cG}\left(c_{AG,b} - c_{AG}^*(\ \ )\right)$$
$$N_A \equiv K_y\left(y_{A,b} - y_A^*(\ \ )\right)$$

$(\ \ ) = x_{A,b}$ or $c_{AL,b}$

Let's ~~take these tools out for a spin!~~
choose our mass transfer units (our tools)

$$K_x = \frac{1}{\frac{1}{k_x} + \frac{1}{m''k_y}}, \quad K_y = \frac{1}{\frac{1}{k_y} + \frac{m'}{k_x}}, \text{ etc.}$$

65

Unsteady Macroscopic Species A Mass Balance—Gas Absorption

Example 6—concluded

$$\frac{d}{dt}(\mathcal{M}_{A,sys}) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$

steady

No homogenous reaction

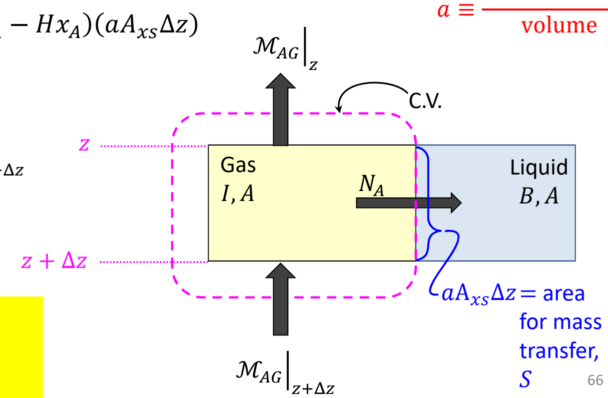
$$L, G = \frac{moles}{column\ area \cdot time}$$

A_{xs} = column cross section

$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

$$N_A S = (K_y \Delta c_{df})(aA_{xs}\Delta z)$$
$$= \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_y}}\right)(y_A - Hx_A)(aA_{xs}\Delta z)$$

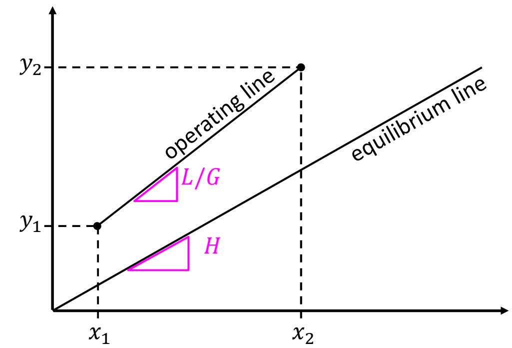
$$\mathcal{M}_{AG}\big|_{z+\Delta z} = GA_{xs}\, y_A\big|_{z+\Delta z}$$
$$\mathcal{M}_{AG}\big|_{z} = GA_{xs}\, y_A\big|_{z}$$

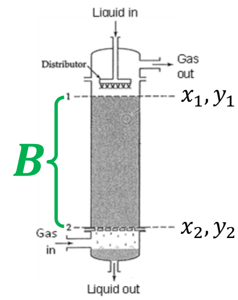


See Handnotes

66

Unsteady Macroscopic Species A Mass Balance—Absorption





$G[=]\frac{mol}{m^2 s}$

$L[=]\frac{mol}{m^2 s}$

$A_{xs}[=]\,m^2$

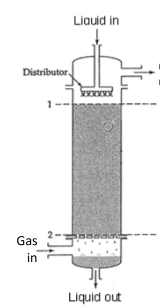
67

Unsteady Macroscopic Species A Mass Balance—Gas Absorption

Example 6—concluded

Example 6 Solution:

"rate" equation (gas-side, species A mass balance)	$\frac{dy}{dz} = \frac{K_y a}{G}(y - Hx)$
"operating line" equation (overall, species A mass balance)	$y = \frac{L}{G}(x - x_1) + y_1$
"equilibrium line" equation (thermodynamic equilibrium, dilute mixtures)	$y^* = Hx^*$
Column height, $\boldsymbol{B}$ (result)	$\boldsymbol{B} = \frac{G}{K_y a}\left(\frac{1}{1 - \frac{G}{L}H}\right)\ln\left(\frac{y_2 - y_2^*}{y_1 - y_1^*}\right)$



$A_{xs}[=]\,m^2$

$y_2^* = y^*(x_2)$
$y_1^* = y^*(x_1)$

$G[=]\frac{mol}{m^2 s}$

$L[=]\frac{mol}{m^2 s}$

See Handnotes

HW 4.16

68

Modeling practical devices involving mass transfer

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Example 6 is presented as a series of **linked examples** that navigate around apparent "dead ends" in modeling mass-transfer units

Identify a question	Invent something	Try to use it
1. How can we model a large, practical device dependent on mass transfer?	1. Apply the species A mass balance to a macroscopic C.V.	1. Lack a system to account for A going between phases
2. How an we account for A going between phases?	2. Invent k_x through linear driving force (LDF) model	2. Gets A *to* the boundary, but not *across*
3. How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)?	3. Write LDF in both phases and combine to create overall effect of multiple resistances	3. Working, but can we devise a convenient shorthand?
4. **Can we model a large, practical device, incorporating K_L, K_G to account for mass xfer between phases?**	4. **Yes**	

69

Unsteady Macroscopic Species A Mass Balance—Absorption

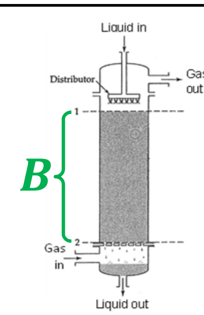
Mass Transfer in an Absorption Column:

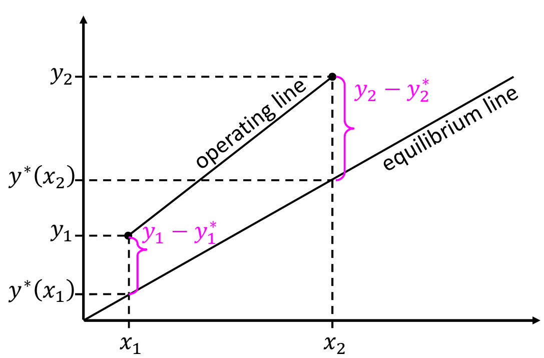
Species A from gas to liquid = Gas scrubbing

- operating line is above the equilibrium line

Species A from liquid to gas = Liquid stripping

- operating line is below the equilibrium line





$G[=]\frac{mol}{m^2 s}$

$L[=]\frac{mol}{m^2 s}$

$A_{xs}[=]m^2$

Column height, B (result)

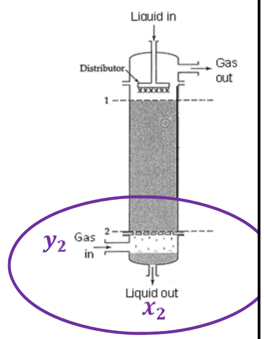
$$B = \frac{G}{K_y a}\left(\frac{1}{1-\frac{G}{L}H}\right)\ln\left(\frac{y_2 - y_2^*}{y_1 - y_1^*}\right)$$

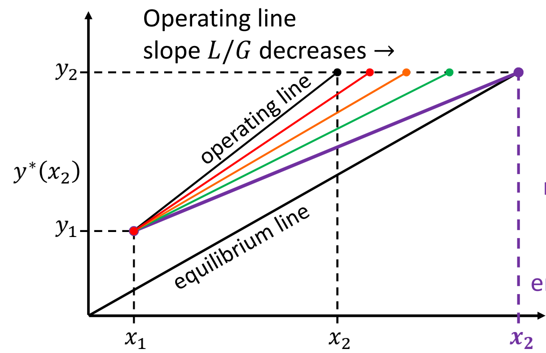
70

Unsteady Macroscopic Species A Mass Balance—Absorption

Mass Transfer in an Absorption Column:

- operating line slope $= \frac{L}{G}$
- equilibrium line slope $= H = m$





Minimum slope, minimum L/G ratio, results when exit liquid condition x_2 is in equilibrium with the entering gas condition y_2

(cannot go lower)

71

Unsteady Macroscopic Species A Mass Balance—Absorption

Column Performance: HTU/NTU

Column height, B

$$B = \frac{G}{K_y a}\left(\frac{1}{1-\frac{G}{L}H}\right)\ln\left(\frac{y_2 - y_2^*}{y_1 - y_1^*}\right)$$

"rate" equation (gas-side, species A mass balance)

$$\frac{dy}{dz} = \frac{K_y a}{G}(y - Hx) \qquad y^* = y^*(x) = Hx$$

$$\frac{dy}{dz} = \frac{K_y a}{G}(y - y^*)$$

$$\int_{y_1}^{y^2} \frac{dy}{(y-y^*)} = \int_0^B \frac{K_y a}{G} dz$$

$$B = \frac{G}{K_y a}\int_{y_1}^{y^2} \frac{dy}{(y-y^*)}$$

$$B = \left(\frac{G}{K_y a}\right)\left(\int_{y_1}^{y^2} \frac{dy}{(y-y^*)}\right)$$

$$B = \mathbf{HTU} \cdot \mathbf{NTU}$$

HTU = height of a transfer unit

A measure of the efficiency of the equipment

NTU = number of transfer units

A measure of difficulty of separation

$G[=]\frac{mol}{m^2 s}$

$L[=]\frac{mol}{m^2 s}$

$A_{xs}[=]m^2$

Cussler p318

72

Column Performance: HTU/NTU

- As the gap between the operating line and the equilibrium line narrows, NTU increases ($B \propto$ integral of 1/gap)

$$B = \left(\frac{G}{K_y a}\right)\left(\int_{y_1}^{y^2} \frac{dy}{(y - y^*)}\right)$$

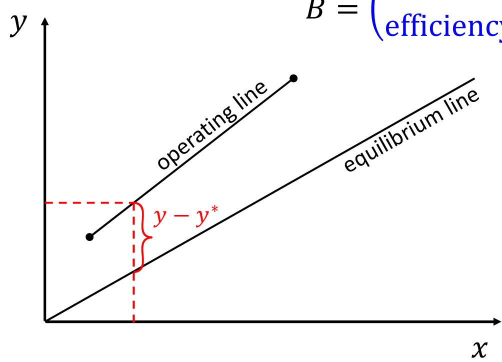
$$B = \boldsymbol{HTU} \cdot \boldsymbol{NTU}$$

$$B = \binom{\text{column}}{\text{efficiency}}\binom{\text{difficulty}}{\text{of separation}}$$

$G[=]\frac{mol}{m^2 s}$

$L[=]\frac{mol}{m^2 s}$

$A_{xs}[=]\, m^2$



HW 4.17

- Both operating and equilibrium lines are straight for dilute systems; When not dilute, both lines may be curved; integration then is done numerically (Excel)

73

Overall Mass-Transfer Coefficient

Overall Mass Transfer Coefficients Summary

- The driving force for species A mass transfer is distance from equilibrium (not bulk concentration difference)
- Both overall mass transfer coefficients K_G, K_L are the aggregation of all the resistances (gas side and liquid side)
- *Film* and *overall* mass transfer coefficients may be expressed in many equivalent units
- K_L, K_G are specific to a scenario or device (not a material property, not a detailed model of interphase mass transfer)
- Individual film mass transfer coefficients are needed to predict the overall transfer coefficients (obtain from literature); K_L, K_G may also be measured
- The correct average mass transfer driving force is determined by integrating over the entire column
- The efficiency of the column is reflected in the HTU
- The difficulty of the separation is reflected by the NTU

74