CM3120 Transport/Unit Operations 2

Unsteady State Heat Transfer

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www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

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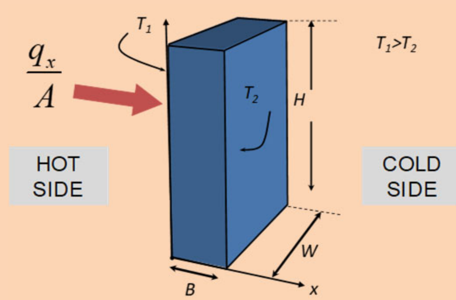
Heat Transfer: Steady vs. Unsteady

To get started, let's contrast the **steady** and **unsteady** cases in a familiar problem:

HEAT TRANSFER

Steady vs. **Unsteady**

1D, heat conduction, in the x-direction, in a slab



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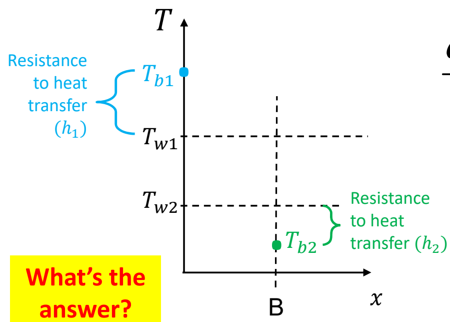
Heat Transfer: Steady vs. Unsteady

Heat Transfer at Steady State

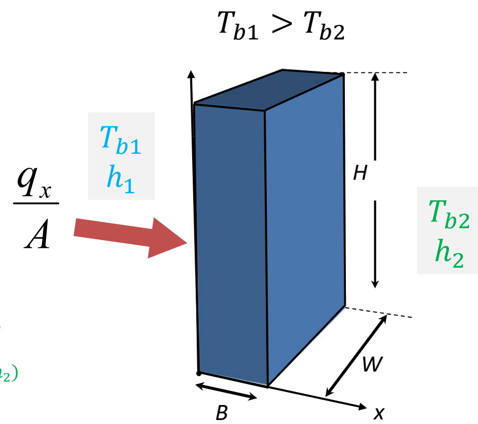
(Newton's law of cooling BCs)

1D, rectangular geometry:
- Independent of time
- Flux $\frac{q_x}{A}=$ **constant**
- **linear** temperature profile
- Steady resistance to heat transfer at both boundaries

$T_{b1} > T_{b2}$



What's the answer?



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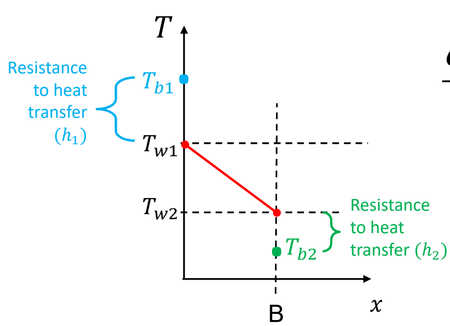

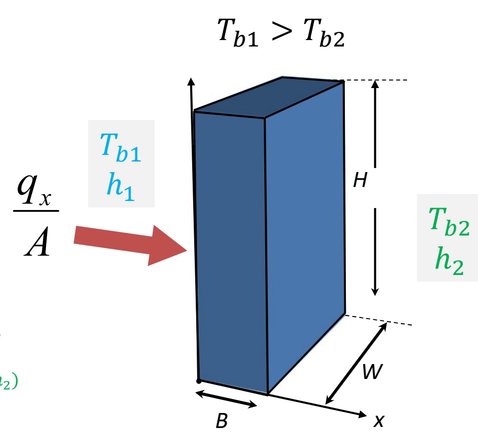

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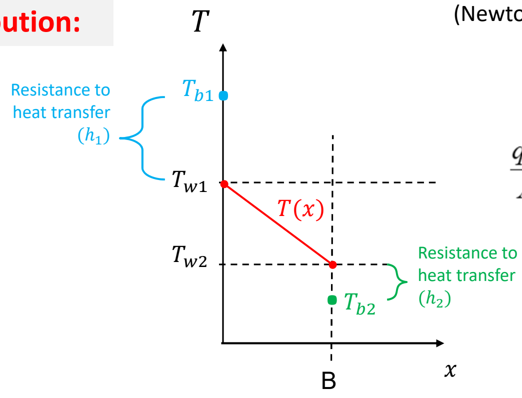
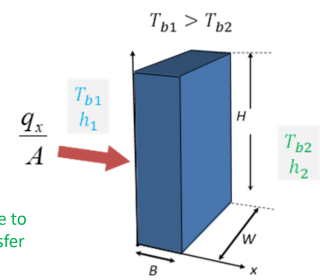
Heat Transfer: Steady vs. Unsteady

Heat Transfer at Steady State

(Newton's law of cooling BCs)

Temperature distribution:





$$\frac{q_x}{A} = h_1(T_{b1} - T_{w1}) = -k\frac{dT}{dx} = h_2\,(T_{w2} - T_{b2})$$

Flux $\frac{q_x}{A} =$ **constant**

Independent of time

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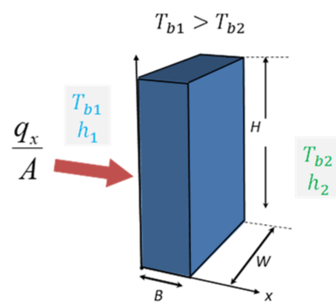
Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



There are many circumstances that cause unsteady heat transfer.

To imagine a case where heat transfer is unsteady:

- We must specify the state of the system at some point in time (***initial conditions***)
- We must specify what then happens to cause heat to start to transfer (**the scenario**).

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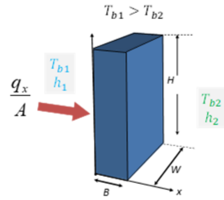
Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



There are many circumstances that cause unsteady heat transfer.

To imagine a case where heat transfer is unsteady:

- We must specify the state of the system at some point in time (***initial conditions***)
- We must specify what then happens to cause heat to start to transfer (**the scenario**).

Can you think of any real situations?
Can you write them in terms of:
- **initial conditions and**
- **a modeling scenario?**

You try.

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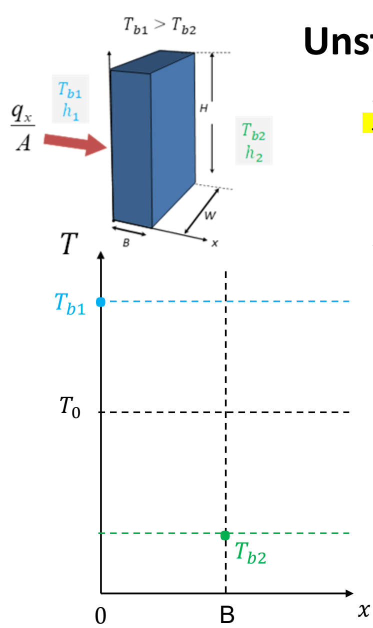
Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



Example: A wide, tall slab initially at T_0 is suddenly subjected to flowing fluid on its two broad faces. The left fluid is at T_{b1} and its heat transfer to the wall is characterized by heat transfer coefficient h_1, while the right side is at T_{b2} and characterized by h_2. What is the temperature distribution across the slab as a function of time?

What do we think will happen?
- Will there be heat transfer resistance at the boundaries?
- Will there be a linear temperature profile in the slab?
- Femtoseconds after the change, what does the profile look like?
- What will the solution trend towards as time goes on ($\rightarrow \infty$)?

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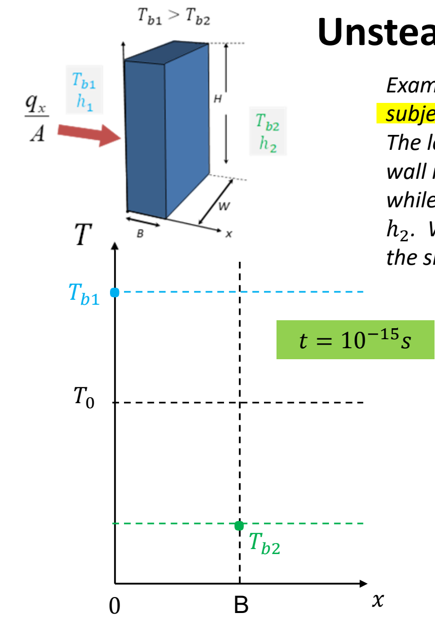
Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



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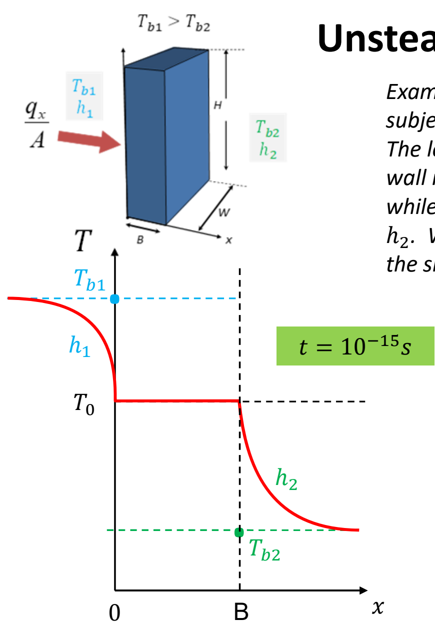
Heat Transfer: Steady vs. Unsteady

Unsteady Heat Transfer



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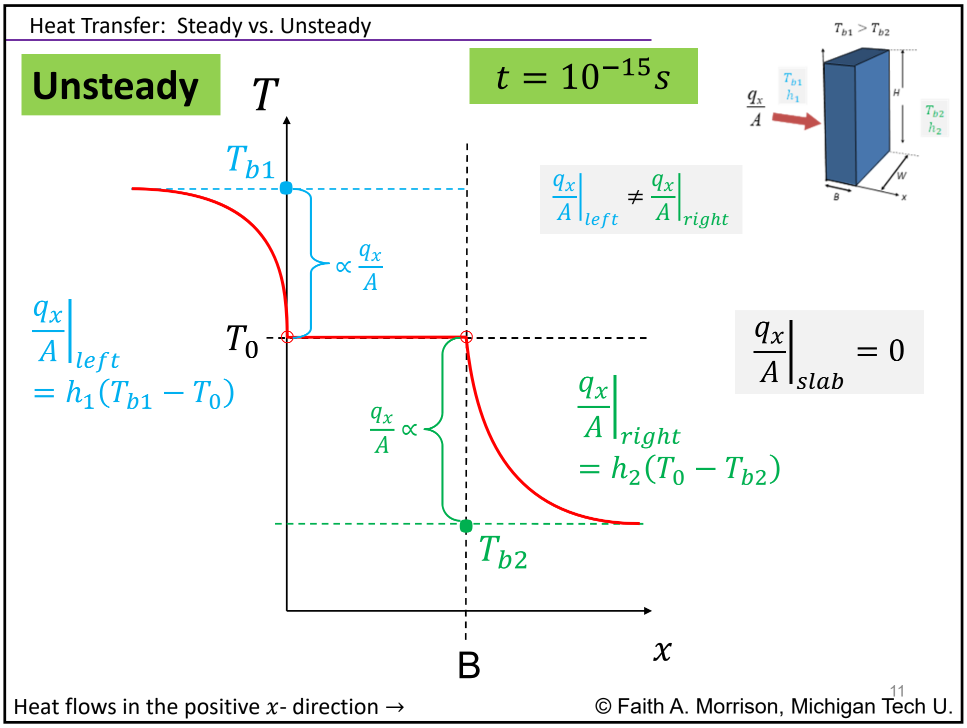
Heat Transfer: Steady vs. Unsteady
Unsteady
$t = 10^{-15} s$
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
T
T_{b1}
$\propto \frac{q_x}{A}$
$\left.\frac{q_x}{A}\right|_{left} \neq \left.\frac{q_x}{A}\right|_{right}$
$\left.\frac{q_x}{A}\right|_{left} = h_1(T_{b1} - T_0)$
T_0
$\left.\frac{q_x}{A}\right|_{slab} = 0$
$\frac{q_x}{A} \propto$
$\left.\frac{q_x}{A}\right|_{right} = h_2(T_0 - T_{b2})$
T_{b2}
B
x
Heat flows in the positive x- direction →
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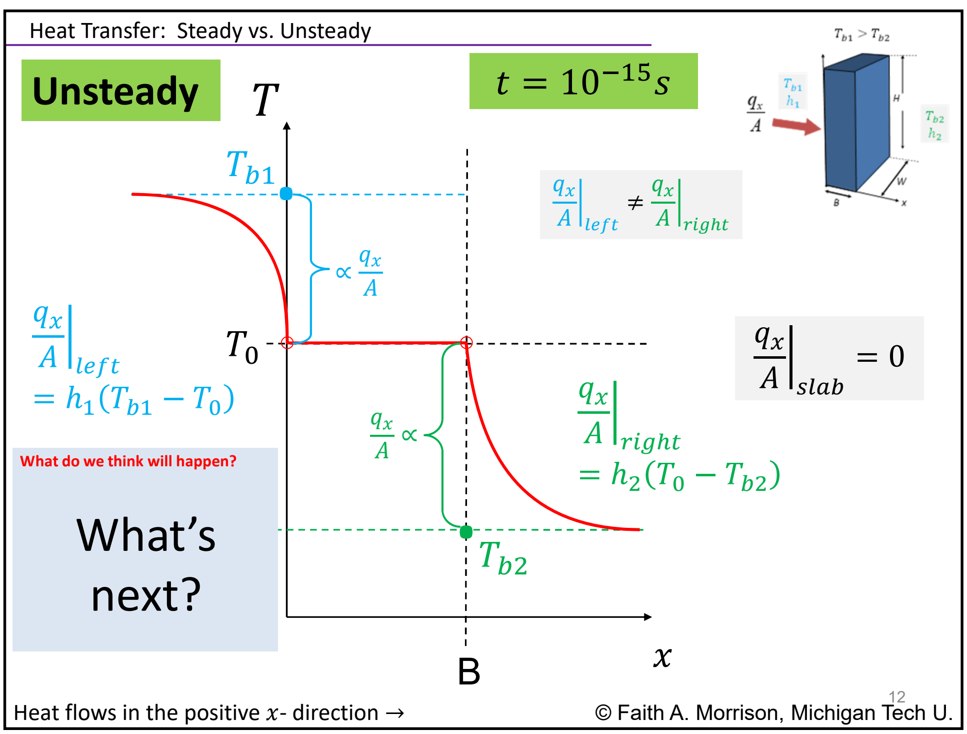
Heat Transfer: Steady vs. Unsteady
Unsteady
$t = 10^{-15} s$
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
T
T_{b1}
$\propto \frac{q_x}{A}$
$\left.\frac{q_x}{A}\right|_{left} \neq \left.\frac{q_x}{A}\right|_{right}$
$\left.\frac{q_x}{A}\right|_{left} = h_1(T_{b1} - T_0)$
T_0
$\left.\frac{q_x}{A}\right|_{slab} = 0$
What do we think will happen?
What's next?
$\frac{q_x}{A} \propto$
$\left.\frac{q_x}{A}\right|_{right} = h_2(T_0 - T_{b2})$
T_{b2}
B
x
Heat flows in the positive x- direction →
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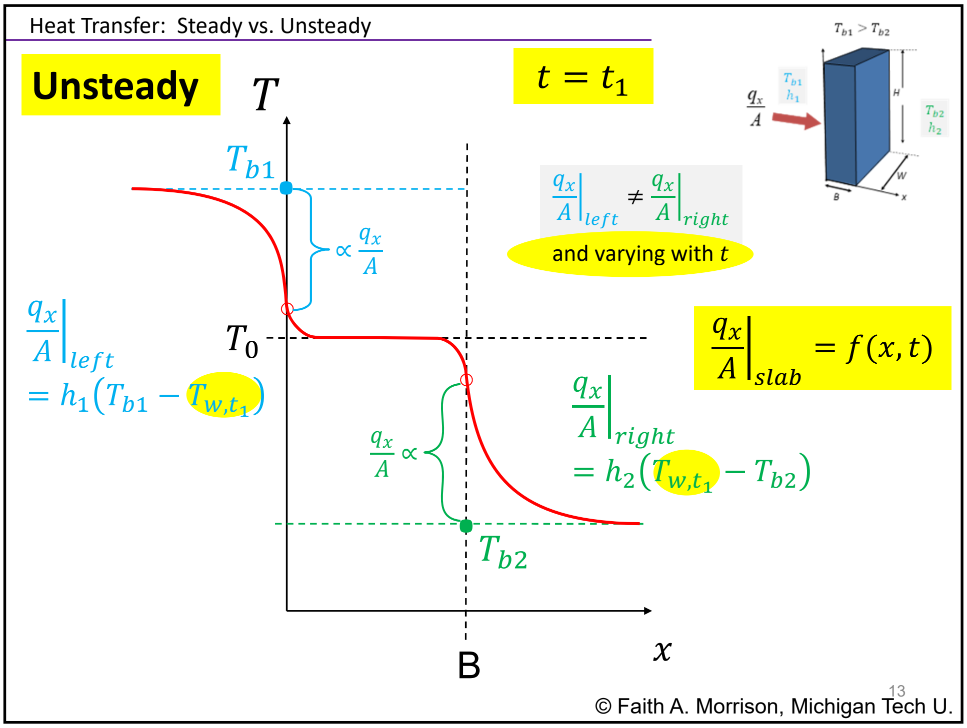

Heat Transfer: Steady vs. Unsteady
Unsteady
$t = t_1$
T
T_{b1}
T_0
T_{b2}
x
B
$\propto \frac{q_x}{A}$
$\frac{q_x}{A} \propto$
$\left.\frac{q_x}{A}\right|_{left} \neq \left.\frac{q_x}{A}\right|_{right}$
and varying with t
$\left.\frac{q_x}{A}\right|_{left} = h_1(T_{b1} - T_{w,t_1})$
$\left.\frac{q_x}{A}\right|_{slab} = f(x,t)$
$\left.\frac{q_x}{A}\right|_{right} = h_2(T_{w,t_1} - T_{b2})$
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
13
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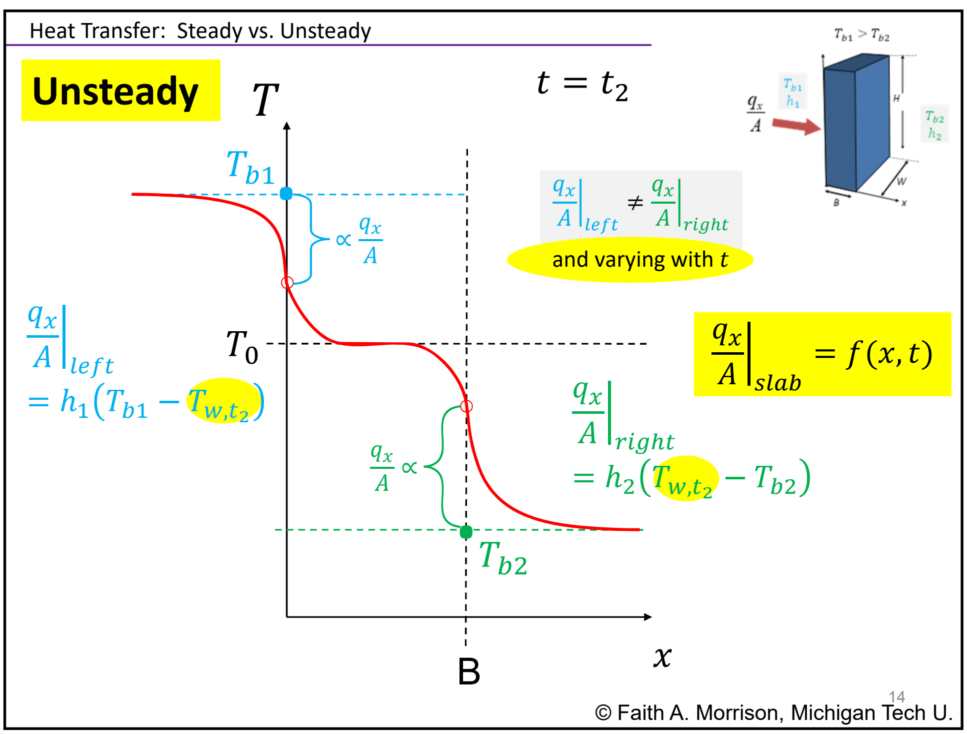

Heat Transfer: Steady vs. Unsteady
Unsteady
$t = t_2$
T
T_{b1}
T_0
T_{b2}
x
B
$\propto \frac{q_x}{A}$
$\frac{q_x}{A} \propto$
$\left.\frac{q_x}{A}\right|_{left} \neq \left.\frac{q_x}{A}\right|_{right}$
and varying with t
$\left.\frac{q_x}{A}\right|_{left} = h_1(T_{b1} - T_{w,t_2})$
$\left.\frac{q_x}{A}\right|_{slab} = f(x,t)$
$\left.\frac{q_x}{A}\right|_{right} = h_2(T_{w,t_2} - T_{b2})$
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
14
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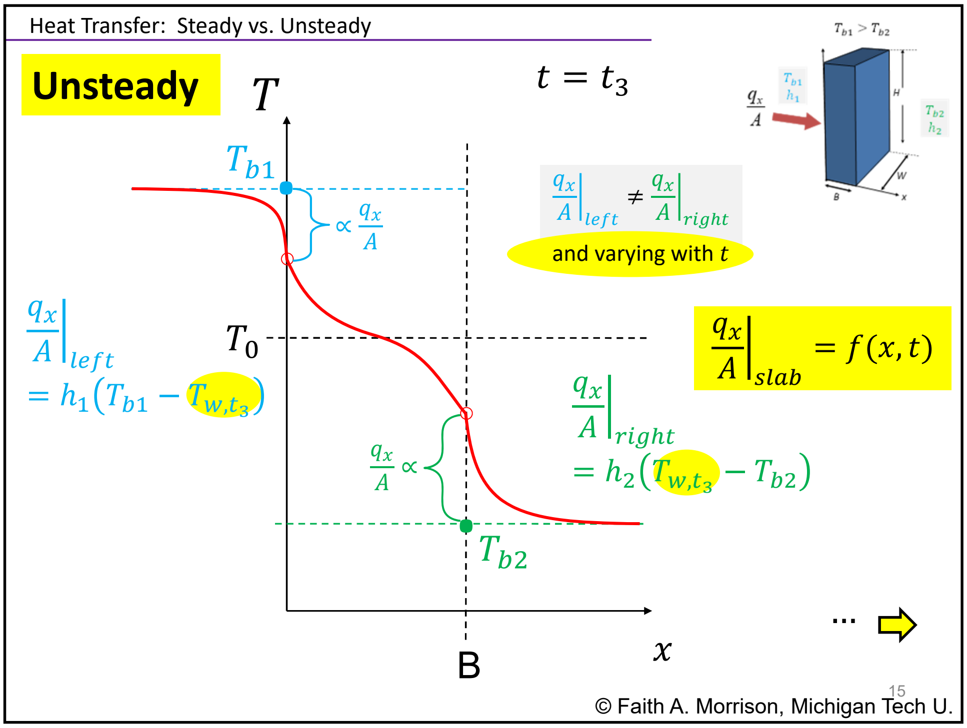
Heat Transfer: Steady vs. Unsteady
Unsteady
t = t3
T
Tb1
∝ qx/A
qx/A|left = h1(Tb1 − Tw,t3)
T0
qx/A|left ≠ qx/A|right
and varying with t
qx/A|slab = f(x,t)
qx/A|right = h2(Tw,t3 − Tb2)
qx/A ∝
Tb2
B
x
Tb1 > Tb2
qx/A
Tb1 h1
Tb2 h2
…
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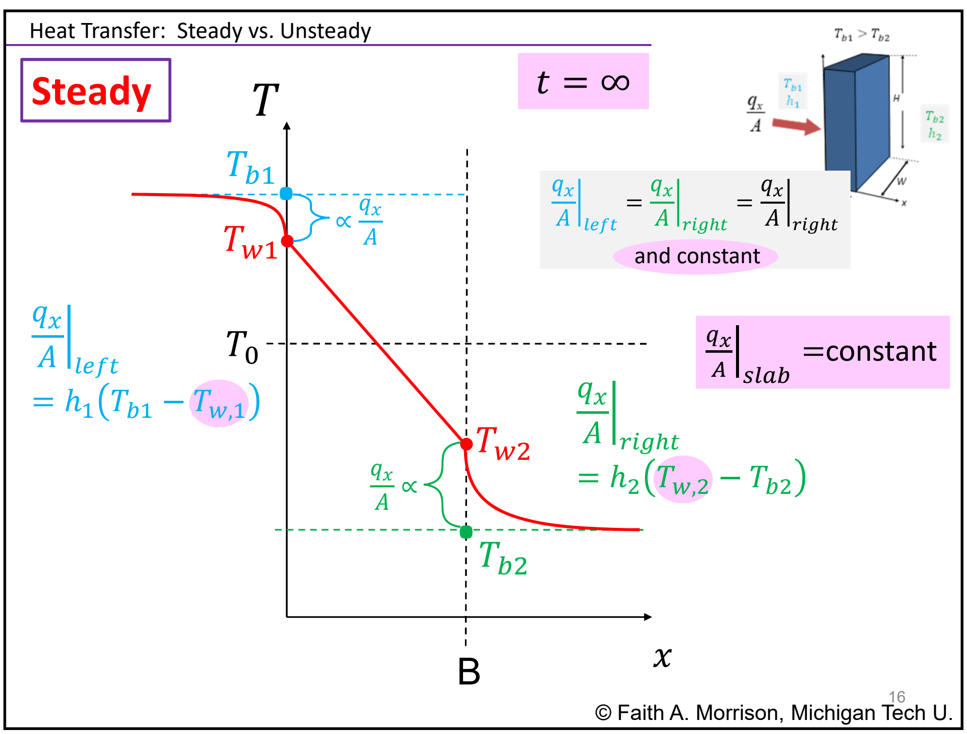
Heat Transfer: Steady vs. Unsteady
Steady
t = ∞
T
Tb1
Tw1
∝ qx/A
qx/A|left = h1(Tb1 − Tw,1)
T0
qx/A|left = qx/A|right = qx/A|right
and constant
qx/A|slab =constant
Tw2
qx/A|right = h2(Tw,2 − Tb2)
qx/A ∝
Tb2
B
x
Tb1 > Tb2
qx/A
Tb1 h1
Tb2 h2
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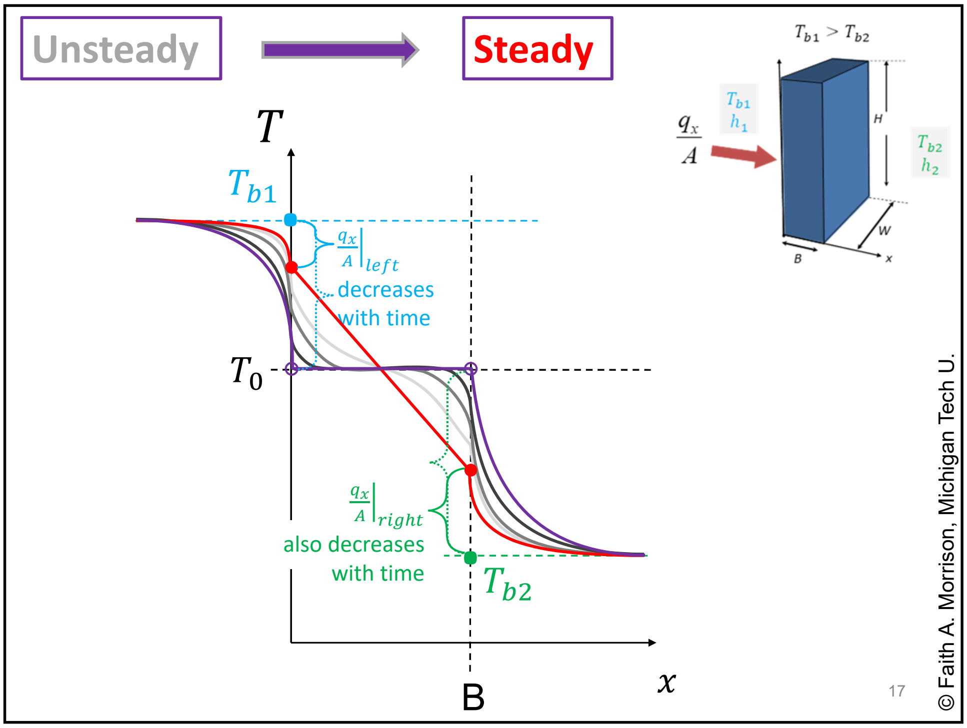
Unsteady
Steady
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
T
T_{b1}
$\frac{q_x}{A}\big|_{left}$ decreases with time
T_0
$\frac{q_x}{A}\big|_{right}$ also decreases with time
T_{b2}
B
x
17
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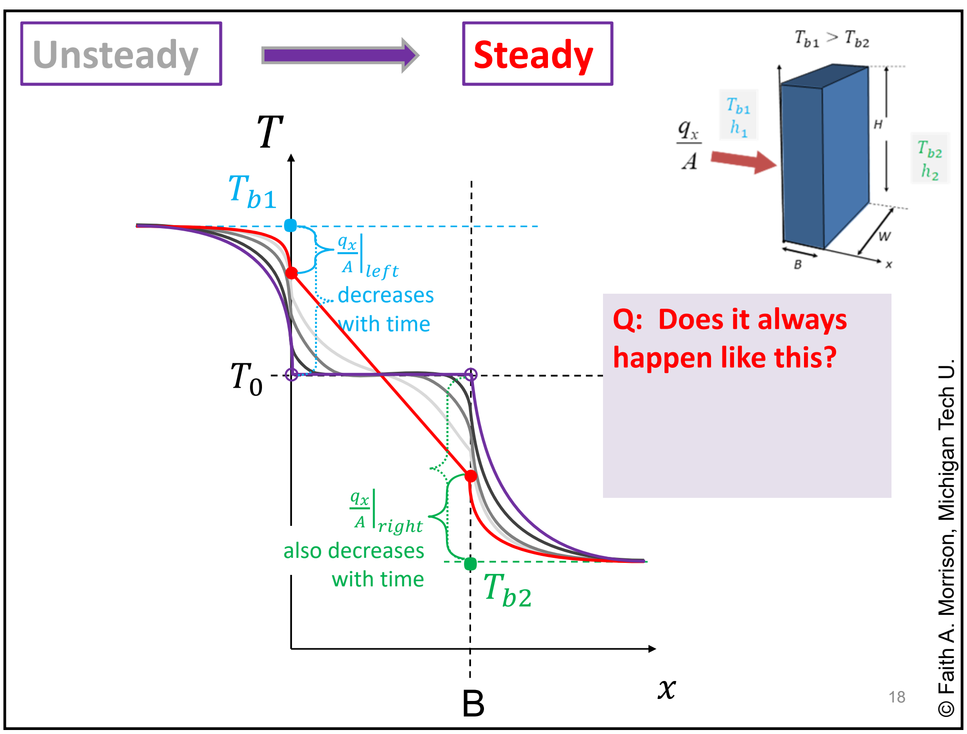
Unsteady
Steady
$T_{b1} > T_{b2}$
$\frac{q_x}{A}$
T_{b1} h_1
T_{b2} h_2
H
W
B
x
T
T_{b1}
$\frac{q_x}{A}\big|_{left}$ decreases with time
T_0
Q: Does it always happen like this?
$\frac{q_x}{A}\big|_{right}$ also decreases with time
T_{b2}
B
x
18
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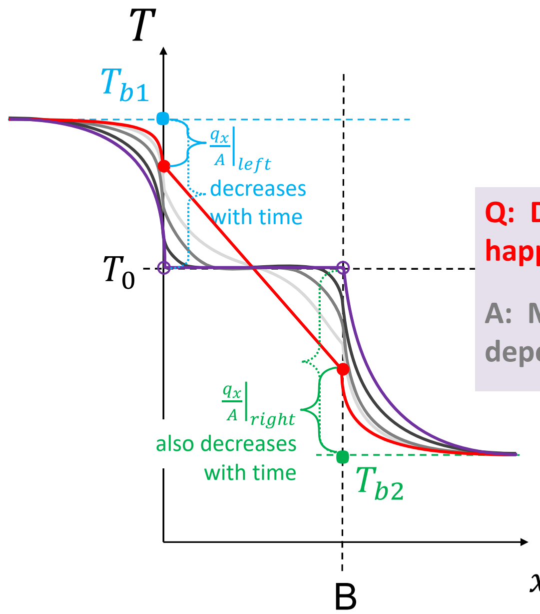
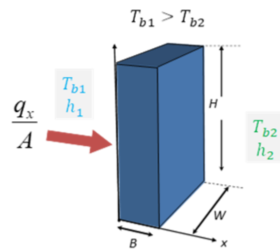
Unsteady → **Steady**





Q: Does it always happen like this?

A: Mmmm. No. It depends. . . .

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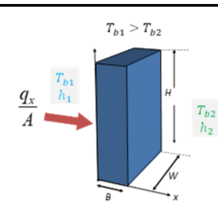
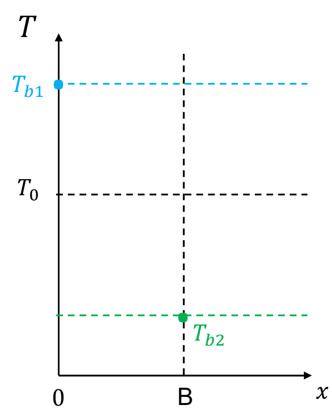
Heat Transfer: Steady vs. Unsteady

What are the various cases that are seen?

- h_i are large

Let's try.





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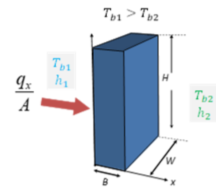
Heat Transfer: Steady vs. Unsteady

What are the various cases that are seen?

- k is large



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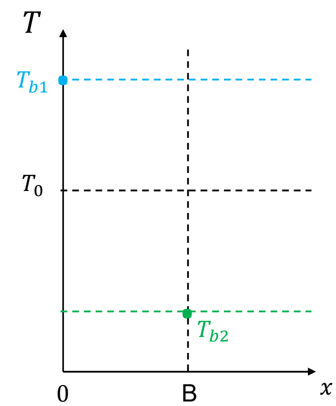

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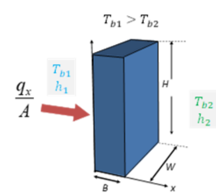
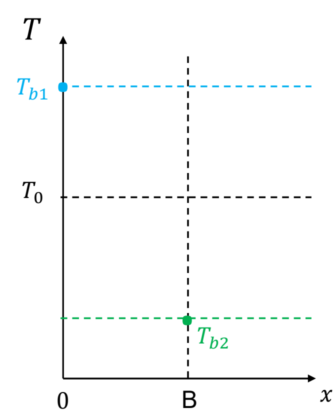
Heat Transfer: Steady vs. Unsteady

What are the various cases that are seen?

- Neither slab conduction nor fluid convection dominates



Let's try.



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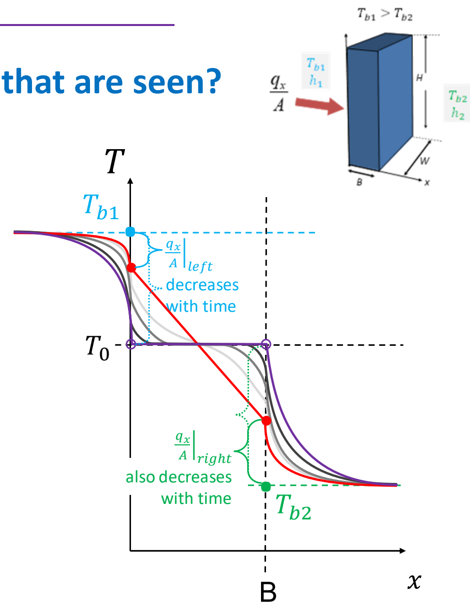
Heat Transfer: Steady vs. Unsteady

What are the various cases that are seen?

- If h_i is large, the wall temp is just the bulk temperature (fast convection)
- If k is large, the temp profile is always straight (quasi-steady state in the slab) and the convection works to keep up (heat transfer is limited by h_i; fast conduction in slab)
- If neither mechanism dominates, it's complicated!
- If the boundary conditions vary with $t, \underline{x}$, it's complicated!



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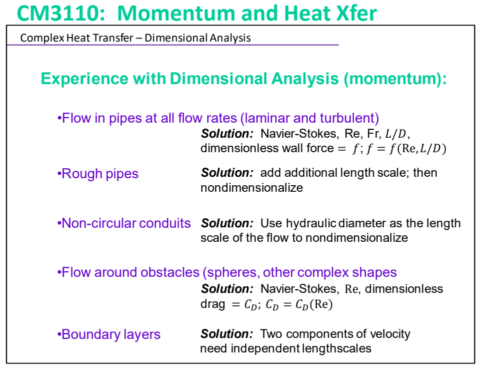
Heat Transfer: Steady vs. Unsteady

What is our usual strategy for complex phenomena?

Answer: Dimensional Analysis

CM3110: Momentum and Heat Xfer



Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis (momentum):

- Flow in pipes at all flow rates (laminar and turbulent) — ***Solution:*** Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\text{Re}, L/D)$
- Rough pipes — ***Solution:*** add additional length scale; then nondimensionalize
- Non-circular conduits — ***Solution:*** Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes — ***Solution:*** Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\text{Re})$
- Boundary layers — ***Solution:*** Two components of velocity need independent lengthscales

Let's review

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