To understand and more complex heat transfer units, we turn now to…

CM3120 Transport/Unit Operations 2

Unsteady State Heat Transfer

Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

Dimensional Analysis

CM3110: Momentum and Heat Xfer

Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis (momentum):

- Flow in pipes at all flow rates (laminar and turbulent)
  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\mathrm{Re}, L/D)$
- Rough pipes — ***Solution:*** add additional length scale; then nondimensionalize
- Non-circular conduits — ***Solution:*** Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes
  Solution: Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\mathrm{Re})$
- Boundary layers — ***Solution:*** Two components of velocity need independent lengthscales

CM3120 Transport/Unit Operations 2

Dimensional Analysis

Towards Understanding Unsteady State Heat Transfer (and more)

Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

Includes review

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

Heat Transfer: Steady vs. Unsteady

What is our usual strategy for complex phenomena?

Answer: Dimensional Analysis

CM3110: Momentum and Heat Xfer

Complex Heat Transfer – Dimensional Analysis

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- Flow in pipes at all flow rates (laminar and turbulent) — ***Solution:*** Navier-Stokes, Re, Fr, L/D, dimensionless wall force = f; $f = f(\text{Re}, L/D)$
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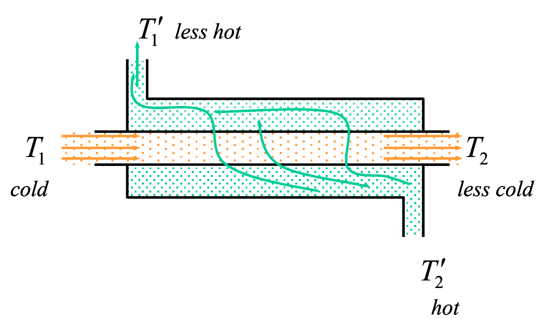
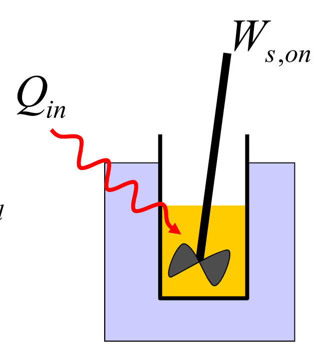
Let's review

Complex Heat Transfer (CM3110)

CM3110 REVIEW

How do we handle complex geometries, complex flows, complex machinery?



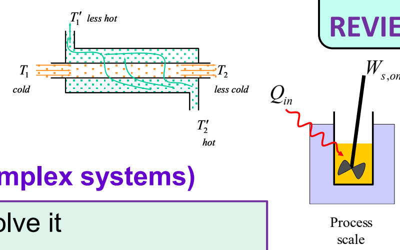


Process scale

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

(***Answer***: *Use the same techniques we have been using in fluid mechanics)*



Engineering Modeling (complex systems)

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Experience with Dimensional Analysis (momentum):

- Flow in pipes at all flow rates (laminar and turbulent)
  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless drag $= f$; $f = f(\text{Re}, L/D)$
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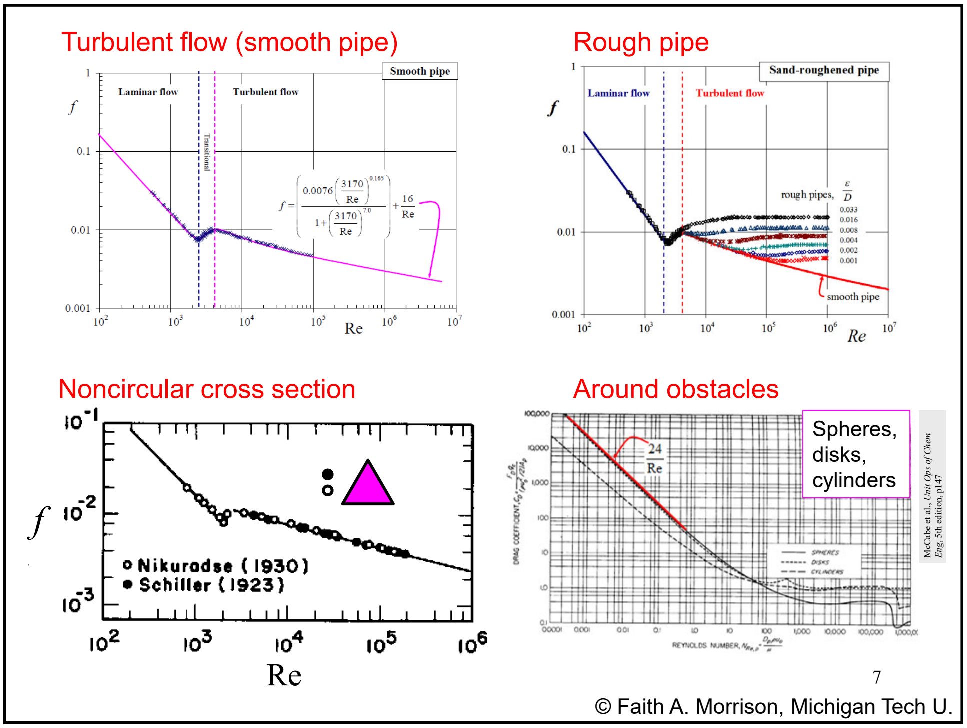
Turbulent flow (smooth pipe)
Smooth pipe
Laminar flow
Turbulent flow
Transitional
f
Re
Rough pipe
Sand-roughened pipe
Laminar flow
Turbulent flow
rough pipes, ε/D
0.033
0.016
0.008
0.004
0.002
0.001
smooth pipe
f
Re
Noncircular cross section
Nikuradse (1930)
Schiller (1923)
f
Re
Around obstacles
Spheres, disks, cylinders
24/Re
REYNOLDS NUMBER
McCabe et al., Unit Ops of Chem Eng, 5th edition, p147
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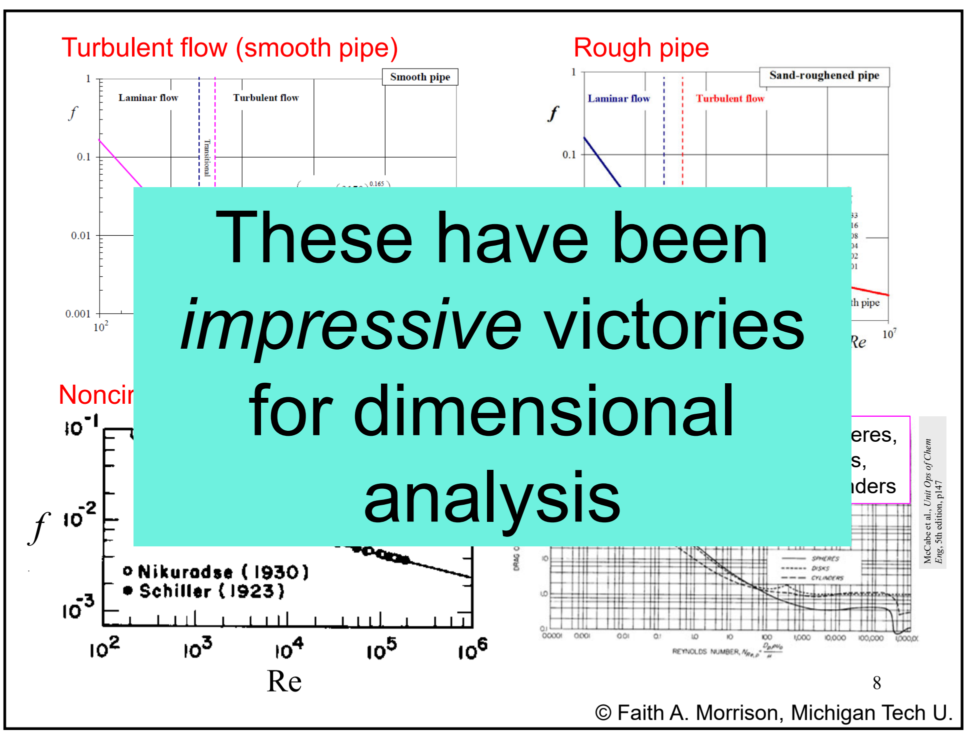
Turbulent flow (smooth pipe)
Rough pipe
These have been *impressive* victories for dimensional analysis
Nikuradse (1930)
Schiller (1923)
Re
McCabe et al., Unit Ops of Chem Eng, 5th edition, p147
8
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Heat Transfer: Steady vs. Unsteady

CM3110 REVIEW

How did Dimensional Analysis work for ***steady heat transfer***?

Answer: Here's the method:

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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Complex Heat Transfer – Dimensional Analysis

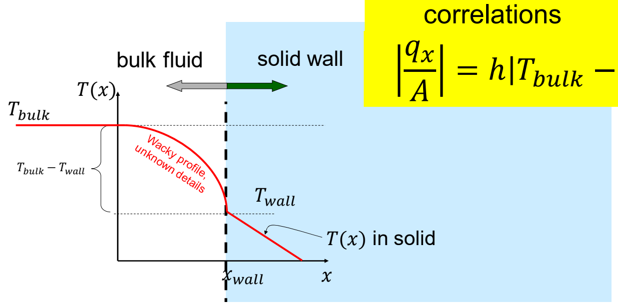
CM3110 REVIEW

Complex Heat transfer Problems to Solve:

- Forced convection heat transfer from fluid to wall
  Solution: ?
- Natural convection heat transfer from fluid to wall
  Solution: ?
- Radiation heat transfer from solid to fluid
  Solution: ?

Nondimensionalize h and produce data correlations

$$\left|\frac{q_x}{A}\right| = h|T_{bulk} - T_{wall}|$$



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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

Let's look at forced convection in a pipe. There are three pieces to the physics:

Pipe flow

Energy

Boundary conditions

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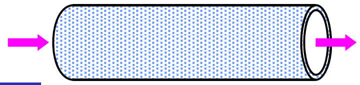
Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

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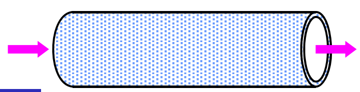
Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

non-dimensional variables:

time:	position:	velocity:	driving force:
$t^* \equiv \frac{tV}{D}$	$r^* \equiv \frac{r}{D}$	$v_z^* \equiv \frac{v_z}{V}$	$P^* \equiv \frac{P}{\rho V^2}$
	$z^* \equiv \frac{z}{D}$	$v_r^* \equiv \frac{v_r}{V}$	$g_z^* \equiv \frac{g_z}{g}$
		$v_\theta^* \equiv \frac{v_\theta}{V}$	

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

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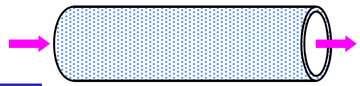
Forced Convection Heat Transfer



CM3110 REVIEW

Energy

Microscopic energy balance:

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

non-dimensional variables:

position:

$$r^*\equiv\frac{r}{D}$$

$$z^*\equiv\frac{z}{D}$$

temperature:

$$T^*\equiv\frac{T-T_o}{(T_1-T_o)}$$

source:

$$S^*\equiv\frac{S}{S_0}$$

Choose:
T – use a characteristic interval (since distance from T = $0K$ is not part of this physics)
S – use a reference source, S_0

$$S_0\equiv\frac{(T_1-T_0)V\rho\hat{C}_p}{D}\;[=]\;\frac{W}{m^2}$$

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*}+v_r^*\frac{\partial T^*}{\partial r^*}+\frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta}+v_z^*\frac{\partial T^*}{\partial z^*}\right)=\frac{1}{\mathrm{Pe}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial T^*}{\partial r^*}\right)+\frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2}+\frac{\partial^2 T^*}{\partial z^2}\right)+S^*$$

$$\mathrm{Pe}=\mathrm{Pr}\,\mathrm{Re}=\frac{\hat{C}_p\mu}{k}\frac{\rho VD}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*}=-\frac{\partial P^*}{\partial z^*}+\frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^*+\frac{1}{Fr}g^*$$

$$\mathrm{Pr}\equiv\frac{\hat{C}_p\mu}{k}$$

Non-dimensional Continuity Equation

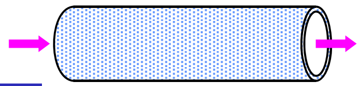
$$\frac{\partial v_x^*}{\partial x^*}+\frac{\partial v_y^*}{\partial y^*}+\frac{\partial v_z^*}{\partial z^*}+=0$$

Quantity of interest

$$Nu=\frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D}-\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2}dz^*d\theta$$

$$\frac{Dv_z}{Dt}\equiv\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}$$

Forced Convection Heat Transfer



CM3110 REVIEW

Boundary conditions

$$\left|\frac{q_x}{A}\right| = h|T_1 - T_0|$$

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S \left[\hat{e}_r \cdot \underline{\tilde{q}}\right]_{surface} dS$$

$$(2\pi RL)(h)(T_1 - T_0) = Q = \int_0^{2\pi}\int_0^{L} -k\left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta$$

Now, non-dimensionalize this expression as well.

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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

$$h(\pi \cancel{D} L)\cancel{(T_1 - T_o)} = \int_0^{2\pi}\int_0^{L/D} -k\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{\cancel{(T_1 - T_o)}}{\cancel{D}}\frac{\cancel{D^2}}{2}\,dz^*\,d\theta$$

$$2\pi\left(\frac{hD}{k}\right)\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^*\,d\theta$$

Nusselt number, Nu (dimensionless heat-transfer coefficient)

$$Nu = Nu\left(T^*, \frac{L}{D}\right)$$

one additional dimensionless group

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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

$$h(\pi D L)(T_1 - T_o) = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{(T_1 - T_o)}{D}\frac{D^2}{2} dz^* d\theta$$

This is a function of Re and Pr through fluid $\underline{v}$ distribution and energy balance

$$2\pi\left(\frac{hD}{k}\right)\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu (dimensionless heat-transfer coefficient)

$$Nu = Nu\left(T^*, \frac{L}{D}\right)$$

one additional dimensionless group

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Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ three dimensionless groups:

no free surfaces

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

$$Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)$$

Now, do the experiments.

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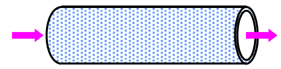
Complex Heat Transfer – Dimensional Analysis



Now, do the experiments.

Forced Convection Heat Transfer

- Build apparatus (*several* actually, with different D, L)
- Run fluid through the inside (at different $\underline{v}$; for different fluids $\rho, \mu, \hat{C}_p, k$)
- Measure T_{bulk} on inside; T_{wall} on inside
- Measure rate of heat transfer, Q
- Calculate h: $|Q| = hA|T_{bulk} - T_{wall}|$
- Report h values in terms of dimensionless correlation:

$$\mathrm{Nu} = \frac{hD}{k} = f\left(\mathrm{Re}, \mathrm{Pr}, \frac{L}{D}\right)$$

It should only be a function of these dimensionless numbers (***if*** our Dimensional Analysis is correct…..)

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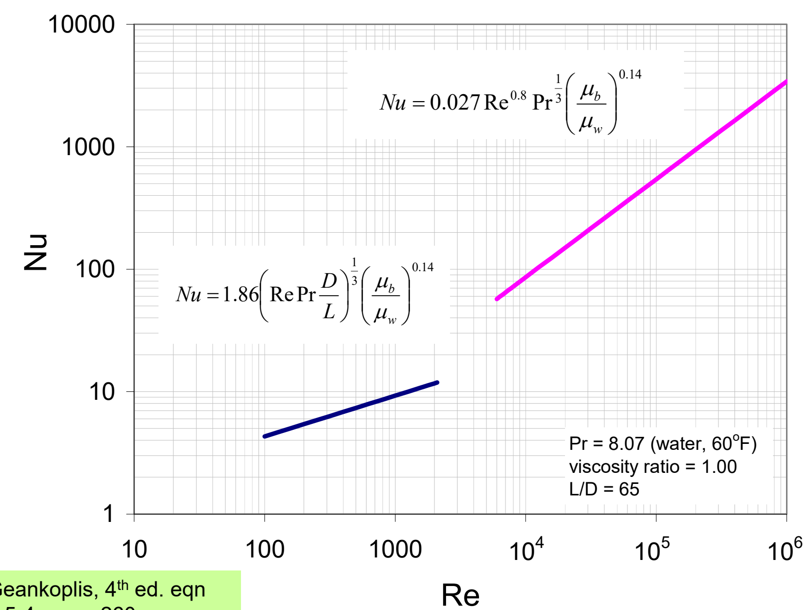
Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients



Geankoplis, 4th ed. eqn 4.5-4, page 260

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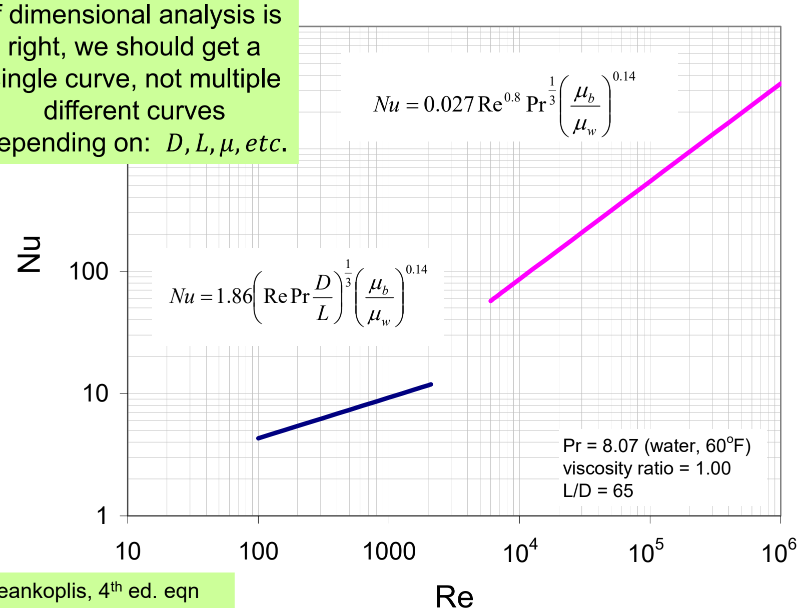
Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients

If dimensional analysis is right, we should get a single curve, not multiple different curves depending on: $D, L, \mu, etc.$



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Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients

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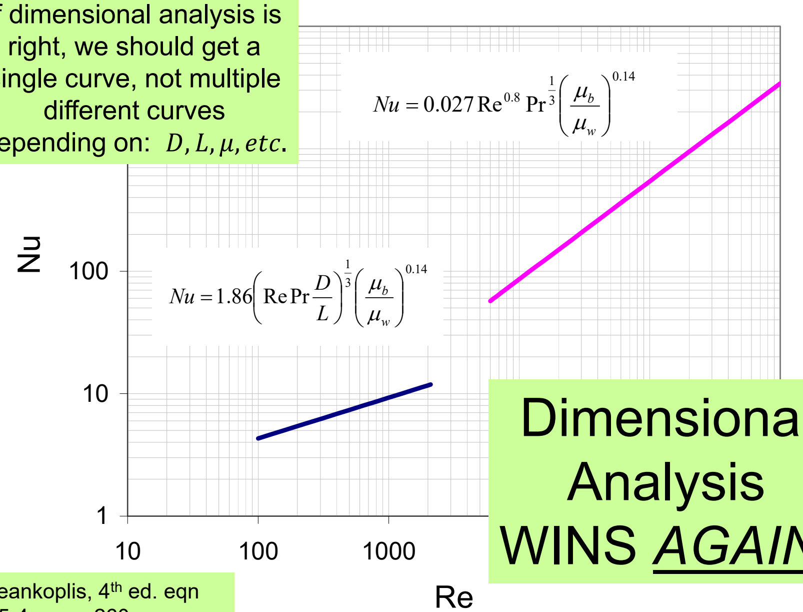

Dimensional Analysis WINS ***AGAIN***!

Geankoplis, 4th ed. eqn 4.5-4, page 260

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Example of *partial* summary of correlations from the literature

CM3110 REVIEW

Laminar flow in pipes	$Nu_a = \frac{h_a D}{k} = 1.86\left(\text{Re}\,\text{Pr}\,\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re<2100, (RePrD/L)>100, horizontal pipes, eqn 4.5-4, page 238; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.
Turbulent flow in smooth pipes	$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027\,\text{Re}^{0.8}\,\text{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re>6000, 0.7 <Pr <16,000, L/D>60 , eqn 4.5-8, page 239; all properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the wall … **(studied in CM3110)**
Air at 1atm in turbulent flow in pipes	$h_{lm} = \frac{3.52V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = \frac{0.5V(ft/s)^{0.8}}{D(ft)^{0.2}}$	
Water in turbulent flow in pipes	$h_{lm} = 1429\left(1+0.0146T(^{o}C)\right)\frac{V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = 150\left(1+0.011T(^{o}F)\right)\frac{V(ft/s)^{0.8}}{D(ft)^{0.2}}$	4 < T(°C)<105, equation 4.5-10, page 239

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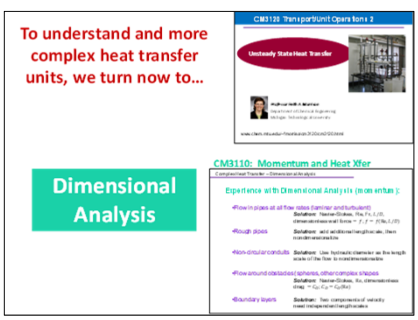

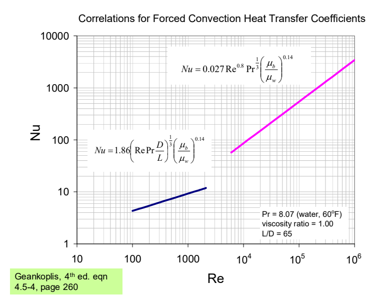

$$\text{Nu} = \text{Nu}\left(\text{Re}, \text{Pr}, \frac{L}{D}\right)$$

Summary

- Dimensional analysis works as well in heat transfer as in momentum transfer
- We should use it (and probably also in mass transfer, but…)
- These dimensionless numbers are stacking up (and…)
- What do they really mean?

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Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum

Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^{*2} v_z^*\right) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}\left(\nabla^{*2} T^*\right) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass

Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}\left(\nabla^{*2} x_A^*\right)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

ref: BSL1, p581, 644

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Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

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Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$

$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

$\text{Pe} - \text{Péclet}_m = \text{ReSc} = \frac{VD}{D_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

$\text{Pr} - \text{Prandtl} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

$\text{Sc} - \text{Schmidt} = \text{LePr} = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

$\text{Le} - \text{Lewis} = \frac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

Transport coefficients

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Dimensional Analysis

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario property*).

Dimensionless numbers from the
Engineering Quantities of Interest

momentum

Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\text{Re}}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(\frac{\partial v_z^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

(Fanning)
f — Friction Factor
$\frac{L}{D}$ — Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy

Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\Bigg|_{r^*=1/2} dz^* d\theta$$

Nu — Nusselt
$\frac{L}{D}$ — Aspect Ratio

$$\text{Nu} = \frac{hD}{k}$$

mass xfer

Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(-\frac{\partial x_A^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

Sh — Sherwood
$\frac{L}{D}$ — Aspect Ratio

$$\text{Sh} = \frac{k_m D}{D_{AB}}$$

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momentum
energy
mass

Dimensionless Numbers

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\text{Fr} - \text{Froude} = \frac{V^2}{gD}$

$\text{Pe} - \text{Péclet}_h = \text{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

$\text{Pe} - \text{Péclet}_m = \text{ReSc} = \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

$\text{Pr} - \text{Prandtl} = \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

$\text{Sc} - \text{Schmidt} = \text{LePr} = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

$\text{Le} - \text{Lewis} = \frac{\alpha}{D_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material*** *properties*).

$f - \text{Friction Factor} = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

$\text{Nu} - \text{Nusselt} = \frac{hD}{k}$

$\text{Sh} - \text{Sherwood} = \frac{k_m D}{D_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario*** *properties*).

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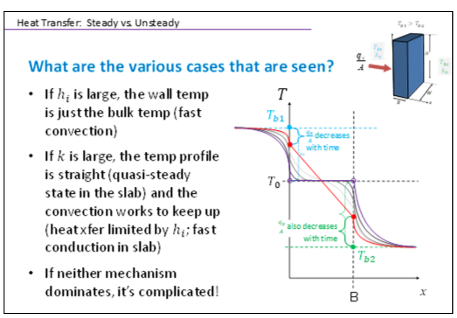
Unsteady State Heat Transfer: Dimensional Analysis

NEW STUFF!

***Question*: What now?**

Answer: Let's apply Dimensional Analysis to something new, unsteady state heat transfer, to sort out the various effects.



Engineering Modeling (complex systems)

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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