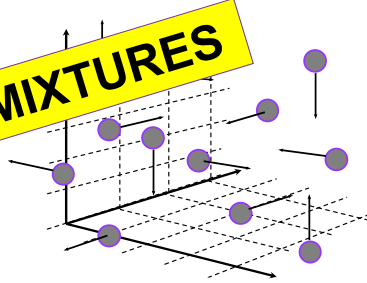


CM3120 Transport/Unit Operations 2

Diffusion and Mass Transfer

in MIXTURES



Professor Faith A. Morrison

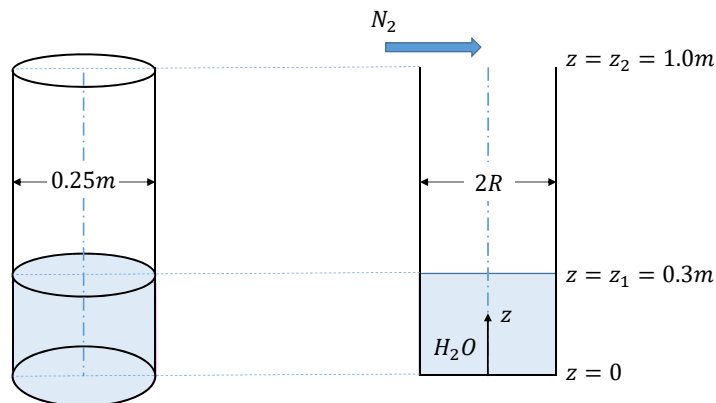
Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

© Faith A. Morrison, Michigan¹ Tech U.

QUICK START

Example: Water (40°C , 1.0 atm) slowly and steadily evaporates into nitrogen (40°C , 1.0 atm) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. **What is the rate of water evaporation?**



BSL2, p547

© Faith A. Morrison, Michigan² Tech U.

Interrogating the problem:

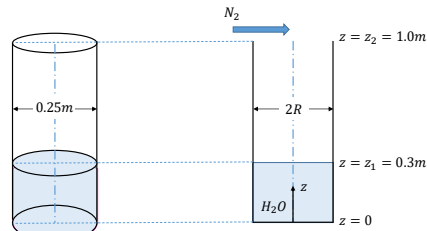
Why does the water evaporate?

What limits the rate of evaporation?

What could be done to accelerate the evaporation?

What could be done to slow down the evaporation?

QUICK START
Example: Water (40°C , 1.0 atm) slowly and steadily evaporates into nitrogen (40°C , 1.0 atm) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. **What is the rate of water evaporation?**



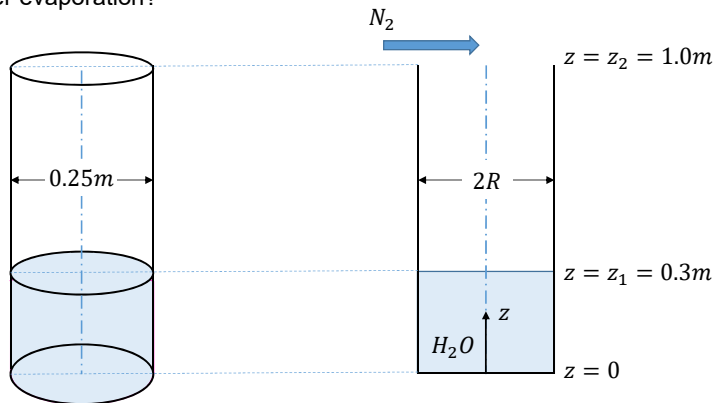
What is the driving physics?

BSL2, p547

© Faith A. Morrison, Michigan Tech U. ³

QUICK START

Example: Water (40°C , 1.0 atm) slowly and steadily evaporates into nitrogen (40°C , 1.0 atm) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. The geometry is as shown in the figure. **What is water mole fraction as a function of vertical position? You may assume ideal gas properties. What is the rate of water evaporation?**



BSL2, p547

© Faith A. Morrison, Michigan Tech U. ⁴

Example: Water (40°C, 1.0 atm) slowly and steadily evaporates into nitrogen (40°C, 1.0 atm) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. **What is the rate of water evaporation?**

Equation of Species Mass Balance in Terms of Concentration

EXAMPLE 1: Steady Diffusion from a Cylindrical Tank

micro species mass bal (steady) in z-dir

$$\frac{dN_A}{dz} = 0$$

integrate: $N_A = C_1$

Flux out (steady) = $N_A = x_A N_{A0} - c D_{AB} \frac{dx_A}{dz}$

Flux in = $x_A N_{A0} = -c D_{AB} \frac{dx_A}{dz}$

To solve:

- substitute
- obtain 2 equations, 2 unknowns
- solve for C_1, C_2

$$C_1 = \frac{c D_{AB}}{(z_1 - z_2)} \ln \left(\frac{1 - x_{A1}}{1 - x_{A2}} \right)$$

$$C_2 = \ln(1 - x_{A1}) - \frac{z_1}{(z_1 - z_2)} \ln \left(\frac{1 - x_{A1}}{1 - x_{A2}} \right)$$

- substitute back. Final answer:

$$\left(\frac{1 - x_{A1}}{1 - x_{A2}} \right)^{\frac{z_1}{z_1 - z_2}} = \left(\frac{1 - x_{A2}}{1 - x_{A1}} \right)^{\frac{z_2}{z_1 - z_2}}$$

What is the rate of evaporation?

Answer: N_{A0}

$\Rightarrow C_1$

$x_{A1} = \frac{p_A}{P} = 0.028374$ (see table for $P^*(A_0, T)$)

$x_{A2} = 0.02$ (given)

$c = \frac{P}{RT} = 3.91367 \times 10^{-5} \frac{\text{mol}}{\text{cm}^3}$

$P D_{AB} = 2.074 \frac{\text{m}^2 \text{Pa}}{\text{s}}$ (see J)

... Answer: $N_{A0} = 0.026 \text{ mol/s}$ //

BSL2, p547

5

© Faith A. Morrison, Michigan Tech U.

Writing to Learn: Free Writing

Topic: What is the role/importance of diffusion or mass transfer in the unit you are studying as part of your Friday Project?

Instructions

Write for **five minutes** on the announced topic.

- Keep your hand moving until the time is up..
- Pay no attention to grammar, spelling, punctuation, neatness, or style.
- If you get off the topic or run out of ideas, keep writing anyway.
- If you feel bored or uncomfortable as you're writing, write about that.
- Afterwards, look over what you've written and mark passages that contain ideas or phrases that might be worth keeping.

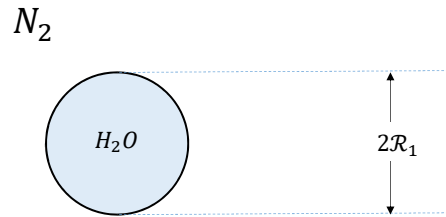
Put your free writing into your Friday project binder.

https://en.wikipedia.org/wiki/Free_writing, cited as Natalie Goldberg's and Jack Kerouac

© Faith A. Morrison, Michigan Tech U.

QUICK START

Example: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). What is the water mole fraction as a function distance from the droplet?



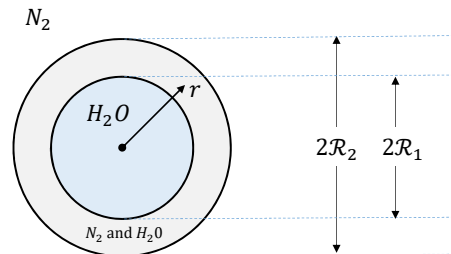
Let's Interrogate the problem.

BSL2, p550

© Faith A. Morrison, Michigan Tech U. ⁷

QUICK START

Example: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. What is the water mole fraction in the film as a function of radial position? You may assume ideal gas properties for air.



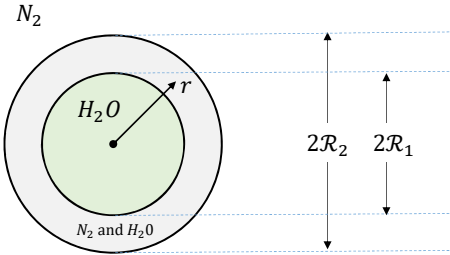
BSL2, p550

© Faith A. Morrison, Michigan Tech U. ⁸

QUICK START

Example: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. **The temperature in the film is not constant but varies as $T(r)/T(\mathcal{R}_1) = (r/\mathcal{R}_1)^n$.** What is the water mole fraction in the film as a function of radial position?

Note: *not* isothermal



Let's Interrogate the problem.

BSL2, p550

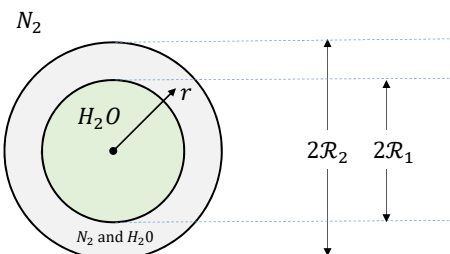
© Faith A. Morrison, Michigan Tech U.

QUICK START

Example: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. We can model the diffusion process as shown in the figure. **The temperature in the film is not constant but varies as $T(r)/T(\mathcal{R}_1) = (r/\mathcal{R}_1)^n$.** What is the water mole fraction in the film as a function of radial position? You may assume ideal gas properties for air; **you may assume that the diffusivity varies with temperature as follows:**

$$D_{AB}(T)/D_{AB,1} = (T/T_1)^{3/2}$$

Note: *not* isothermal



HW3,
problem 12
(stretch)

BSL2, p550

© Faith A. Morrison, Michigan Tech U.

QUICK START

Example: Heterogeneous catalysis

Let's Interrogate the problem.

An irreversible, instantaneous chemical reaction ($2A \rightarrow B$) takes place at a catalyst surface in a reactor as shown. How might mass transfer affect the observed rate of reaction?

BSL2, p552 11
© Faith A. Morrison, Michigan Tech U.

QUICK START

Example: Heterogeneous catalysis

An irreversible, instantaneous chemical reaction ($2A \rightarrow B$) takes place at a catalyst surface, as shown. The reaction is "diffusion-limited," however, because the rate of completion of the reaction is determined by the rate of diffusion through the "film" near the catalyst surface. Calculate the steady state composition distribution in the film ($x_A(z)$).

BSL2, p552 12
© Faith A. Morrison, Michigan Tech U.

Recurring Modeling Assumptions in Diffusion

- Near a liquid-gas interface, the region in the gas near the liquid is a film where diffusion takes place
- The vapor near the liquid-gas interface is often saturated (Raoult's law, $x_A = p_A^*/p$)
- If component A has no sink, $\underline{N}_A = 0$.
- If A diffuses through stagnant B , $\underline{N}_B = 0$.
- If a binary mixture of A and B are undergoing steady equimolar counter diffusion, $\underline{N}_A = -\underline{N}_B$.
- If, for example, two moles of A diffuse to a surface at which a rapid, irreversible reaction converts it to one mole of B , then at steady state $-0.5\underline{N}_A = \underline{N}_B$.
- Because diffusion is slow, we can make a quasi-steady-state assumption
- Homogeneous reactions appear in the mass balance; heterogeneous reactions appear in the boundary conditions
-
-