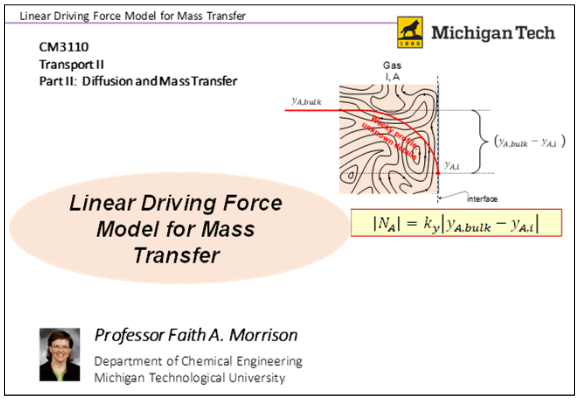
Contining work with the linear driving force for mass transfer, i.e. mass transfer coefficients, k_c



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Mass Transport "Laws"

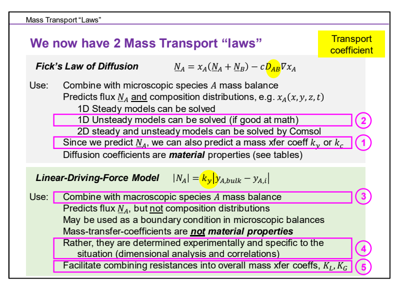
We have 2 Mass Transport "laws"

Remaining Topics to round out our understanding of mass transport:

Fick's law of diffusion

D_{AB}

1. Since we predict N_A with Fick's law, we can also predict a mass transfer coefficients k_y or k_c *Relate k_c and D_{AB}*
2. 1D Unsteady models can be solved (if good at math) *Solutions are analogous to heat transfer*



Mass transfer coefficients

k_c

3. Combine with macroscopic species A mass balance
4. Are not material properties; rather, they are determined experimentally and specific to the situation (dimensional analysis and correlations)
5. Facilitate combining resistances into overall mass transfer coefficients, K_L, K_G, to be used in modeling unit operations

Model macroscopic processes, design units

2

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Dimensional Analysis in Mass Transfer

Michigan Tech

CM3110
Transport II
Part II: Diffusion and Mass Transfer

momentum
energy
mass

Dimensionless Numbers

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material*** *properties*).

f – Friction Factor $= \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

Nu – Nusselt $= \frac{hD}{k}$

Sh – Sherwood $= \frac{k_c D}{D_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario*** *properties*).

$St_h = Nu/Pe_h$, $St_m = Sh/Pe_m$ – Stanton

Dimensional Analysis in Mass Transfer

"D.A."

4

Professor Faith A. Morrison

Department of Chemical Engineering
Michigan Technological University

3

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What do we do to understand complex ~~flows?~~ ~~heat transfer?~~ mass transfer?

Same strategy as:

flows
- *Turbulent tube flow*
- *Noncircular conduits*
- *Drag on obstacles*
- *Boundary Layers*

heat transfer
- *Forced-convection heat transfer coefficients*
- *Natural-convection heat transfer coefficients*
- *Problems with multiple kinds of physics*

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

Solve Real Problems.

Powerful.

4

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Solve Real Problems. Powerful.

What do we do to understand complex ~~flows?~~ ~~heat transfer?~~ mass transfer?

Same strategy as:

flows

- *Turbulent tube flow*
- *Noncircular conduits*
- *Drag on obstacles*
- *Boundary Layers*

heat transfer

- *Forced-convection heat transfer coefficients*
- *Natural-convection heat transfer coefficients*
- *Problems with multiple kinds of physics*

Mass transfer

- *From fluid to plate*
- *To a falling film*
- *In pipes and ducts*
- *Past submerged objects*
- *To/from bubbles, drops*
- *In agitated systems*
- *In fixed and fluidized beds*
- *In packed 2-phase contactors (absorption, distillation, cooling towers)*

5

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Dimensional Analysis in Mass Transfer

Let's review ***our review*** of dimensional analysis…

CM3120, Lecture 5

heat transfer

Heat Transfer: Steady vs. Unsteady

What is our usual strategy for complex phenomena?

Answer: Dimensional Analysis

CM3110: Momentum and Heat Xfer

Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis (momentum):

- Flow in pipes at all flow rates (laminar and turbulent) — ***Solution:*** Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\mathrm{Re}, L/D)$
- Rough pipes — ***Solution:*** add additional length scale; then nondimensionalize
- Non-circular conduits — ***Solution:*** Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes — ***Solution:*** Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\mathrm{Re})$
- Boundary layers — ***Solution:*** Two components of velocity need independent lengthscales

Let's review

6

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

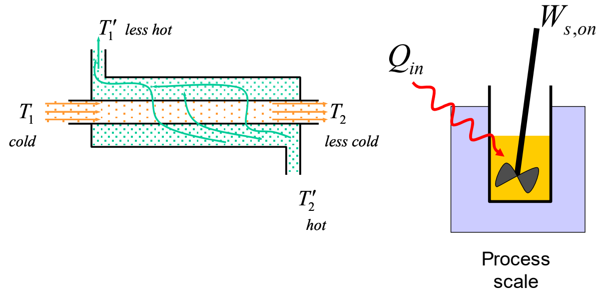
CM3120 Lecture 5

momentum transfer

Complex Heat Transfer (CM3110)

CM3110 REVIEW

How do we handle complex geometries, complex flows, complex machinery?



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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

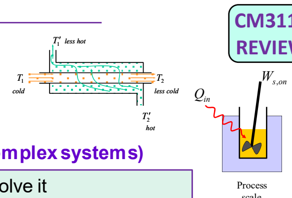
CM3120 Lecture 5

momentum transfer

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

(***Answer***: *Use the same techniques we have been using in fluid mechanics)*



Engineering Modeling (complex systems)

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 5

momentum transfer

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Experience with Dimensional Analysis (momentum):

- Flow in pipes at all flow rates (laminar and turbulent)
  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless drag = f; $f = f(\text{Re}, L/D)$
- Rough pipes
  Solution: add additional length scale; then nondimensionalize
- Non-circular conduits
  Solution: Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes
  Solution: Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\text{Re})$
- Boundary layers
  Solution: Two components of velocity need independent lengthscales

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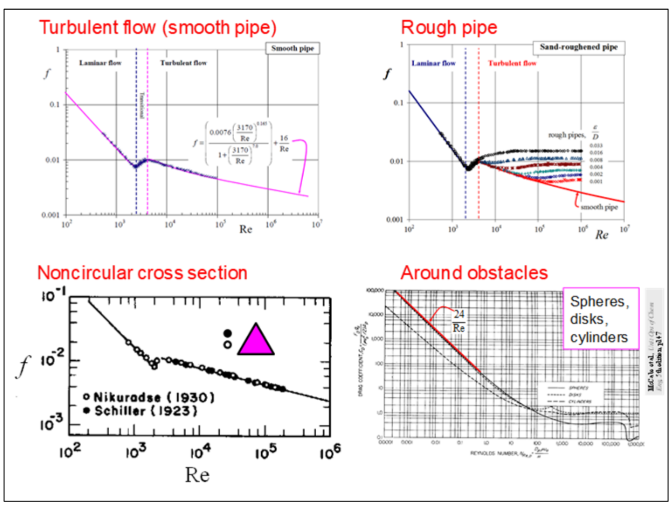
Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 5

Correlations compared with data

momentum transfer



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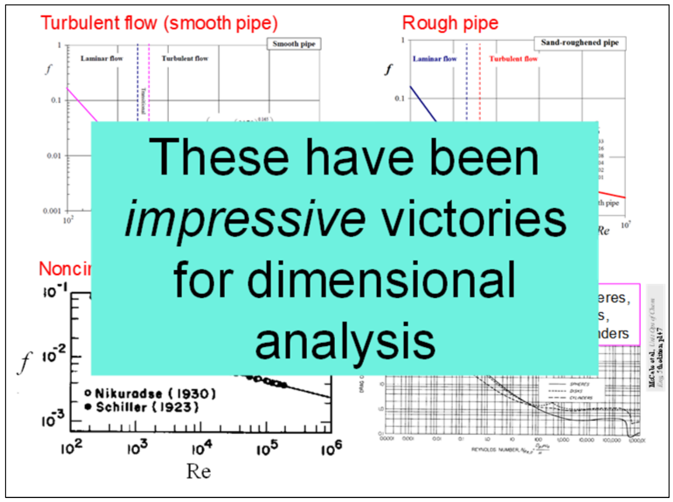
Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 5

Correlations compared with data

momentum transfer



These have been *impressive* victories for dimensional analysis

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3110 Lecture 5

steady heat transfer

Heat Transfer: Steady vs. Unsteady

CM3110 REVIEW

How did Dimensional Analysis work for *steady heat transfer*?

Answer: Here's the method:

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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Dimensional Analysis in Mass Transfer

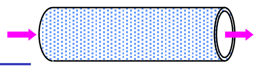
Review of dimensional analysis

CM3120 Lecture 5

steady heat transfer

Choose "typical" or "characteristic" values; can only know if they are the right choices if the D.A. works.

Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

Choose "characteristic" values

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Dimensional Analysis in Mass Transfer

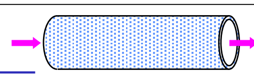
Review of dimensional analysis

CM3120 Lecture 5

steady heat transfer

Choose "typical" or "characteristic" values; can only know if they are the right choices if the D.A. works.

Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

Choose "characteristic" values

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Dimensional Analysis in Mass Transfer

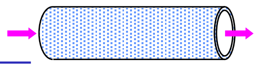
Review of dimensional analysis

CM3120 Lecture 5

Oops, re-used the "*" notation; *here* it is dimensionless variable, not molar average velocity

steady heat transfer

Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

non-dimensional variables:

time: $t^* \equiv \frac{tV}{D}$

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

velocity: $v_z^* \equiv \frac{v_z}{V}$, $v_r^* \equiv \frac{v_r}{V}$, $v_\theta^* \equiv \frac{v_\theta}{V}$

driving force: $P^* \equiv \frac{P}{\rho V^2}$, $g_z^* \equiv \frac{g_z}{g}$

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

Choose "characteristic" values

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Dimensional Analysis in Mass Transfer

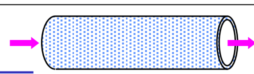
Review of dimensional analysis

CM3120 Lecture 5

Choose "typical" or "characteristic" values; can only know if they are the right choices if the D.A. works.

steady heat transfer

Forced Convection Heat Transfer



CM3110 REVIEW

Energy

Microscopic energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

non-dimensional variables:

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

temperature: $T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$

source: $S^* \equiv \frac{S}{S_0}$

Choose:
T – use a characteristic interval (since distance from T = 0*K* is not part of this physics)
S – use a reference source, S_0

$S_0 \equiv \frac{(T_1 - T_0)V\rho\hat{C}_p}{D} [=] \frac{W}{m^2}$

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 5

steady heat transfer

Micro E-Balance produces $\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re}$

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \frac{1}{\mathrm{Pe}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re} = \frac{\hat{C}_p\mu}{k}\frac{\rho VD}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr}g^*$$

$$\mathrm{Pr} \equiv \frac{\hat{C}_p\mu}{k}$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

$$\frac{Dv_z}{Dt} \equiv \frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 5

The D.A. goes with a *particular* problem (a *particular* physics)

Here, the "engineering property of interest" is the heat transferred across the boundary, Q.

steady heat transfer

Forced Convection Heat Transfer

Linear driving force model $\left|\frac{q_x}{A}\right| = h|T_1 - T_0|$

Apply at the interface:

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S \left[\hat{e}_r \cdot \tilde{q}\right]_{surface} dS$$

$$(2\pi RL)(h)(T_1 - T_0) = Q = \int_0^{2\pi}\int_0^L -k\left.\frac{\partial T}{\partial r}\right|_{r=R} Rdzd\theta$$

Now, non-dimensionalize this expression as well.

Yields correlations for nondimensional heat transfer coefficient, h

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 5

steady heat transfer

The heat transferred from the fluid (LHS) equals the heat transferred into the wall (RHS).

LHS RHS

$$h(\pi DL)(T_1 - T_o) = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{(T_1 - T_o)}{D}\frac{D^2}{2} dz^* d\theta$$

This is a function of Re and Pr through fluid $\underline{v}$ distribution and energy balance

$$2\pi\left(\frac{hD}{k}\right)\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu (dimensionless heat-transfer coefficient)

$$Nu = Nu\left(T^*, \frac{L}{D}\right)$$

one additional dimensionless group

The engineering quantity of interest produces Nu

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 5

steady heat transfer

The D.A. produces:

$$\mathrm{Nu} = \mathrm{Nu}\left(\mathrm{Re}, \mathrm{Pr}, \frac{L}{D}\right)$$

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ three dimensionless groups:

no free surfaces

$$Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)$$

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Now, do the experiments.

Can only know if the D.A. is right, if the D.A. works.

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

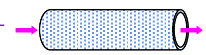
CM3120 Lecture 5

The experiments produce (turbulent flow):

$$\mathrm{Nu} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

steady heat transfer

Complex Heat Transfer – Dimensional Analysis



Now, do the experiments.

Forced Convection Heat Transfer

- Build apparatus (*several* actually, with different D, L)
- Run fluid through the inside (at different $\underline{v}$; for different fluids $\rho, \mu, \hat{C}_p, k$)
- Measure T_{bulk} on inside; T_{wall} on inside
- Measure rate of heat transfer, Q
- Calculate h: $|Q| = hA|T_{bulk} - T_{wall}|$
- Report h values in terms of dimensionless correlation:

$$\mathrm{Nu} = \frac{hD}{k} = f\left(\mathrm{Re}, \mathrm{Pr}, \frac{L}{D}\right)$$

AND IT WORKS!

It should only be a function of these dimensionless numbers (***if*** our Dimensional Analysis is correct…..)

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Dimensional Analysis in Mass Transfer

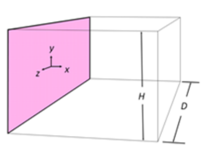
Review of dimensional analysis

CM3120 Lecture 6

We also applied D.A. to *unsteady* heat transfer:

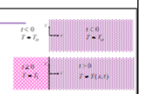
***Unsteady* heat transfer**

Let's nondimensionalize the governing equations and BCs.
Let's sort out the various cases.



1D Heat Transfer: Unsteady State

Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{\partial T}{\partial t} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

Initial condition: $t = 0 \quad T = T_0 \quad \forall\ x$

Boundary conditions:

$$x = 0 \quad \frac{q_x}{A} = -k\frac{dT}{dx} = h(T_1 - T) \quad t > 0$$

$$x = \infty \quad T = T_0 \quad \forall\ t$$

(Review:
How did we do this before?)

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 6

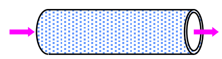
We also applied D.A. to *unsteady* heat transfer:

Unsteady heat transfer

Had to adjust the characteristic time

We'll modify our solution for

Convective Heat Transfer



Pipe flow

Dimensional Analysis

non-dimensional variables:

time: $t^* \equiv \frac{tV}{D}$

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

velocity: $v_z^* \equiv \frac{v_z}{V}$, $v_r^* \equiv \frac{v_r}{V}$, $v_\theta^* \equiv \frac{v_\theta}{V}$

driving force: $P^* \equiv \frac{P}{\rho V^2}$, $g_z^* \equiv \frac{g_z}{g}$

Energy

non-dimensional variables:

position: $r^* \equiv \frac{r}{D}$, $z^* \equiv \frac{z}{D}$

temperature: $T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$

source: $S^* \equiv \frac{S}{S_0}$

Slight problem: We need to nondimensionalize t for the unsteady case also, but there is no characteristic velocity in thermal conduction in a solid.

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 6

We also applied D.A. to *unsteady* heat transfer:

Unsteady heat transfer

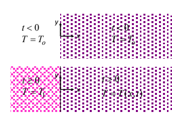
Had to adjust the characteristic time

Dimensional Analysis, Unsteady State Convection

Non-dimensionalize (eqns, BCs)

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} \right)$$

$$q_x = -k \frac{\partial T}{\partial x} = hA(T_1 - T)$$



non-dimensional variables:

position: $x^* \equiv \frac{x}{D}$

temperature: $Y \equiv \frac{(T_1 - T)}{(T_1 - T_0)}$

time: $t^* \equiv \frac{\alpha t}{D^2}$

This dimensionless time is called Fourier number Fo.

$$\text{Fo} - \text{Fourier Number} = \frac{\alpha t}{D^2}$$

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Dimensional Analysis in Mass Transfer

Review of dimensional analysis

CM3120 Lecture 6

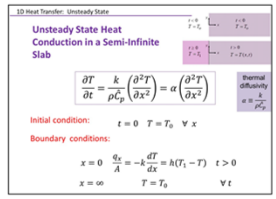
We also applied D.A. to *unsteady* heat transfer:

$$\frac{(T_1 - T)}{(T_1 - T_0)} = f\left(\frac{x}{D}, \text{Fo}, \text{Bi}\right)$$

Unsteady heat transfer

In dimensionless form, we see that this problem reduces to

$$Y = Y\left(\frac{x}{D}, \text{Fo}, \text{Bi}\right)$$



(Heissler charts)

Dimensionless quantities:

$Y = \frac{(T_1 - T)}{(T_1 - T_0)}$ — **Y** (dimensionless temperature interval)

$t^* = \text{Fo} = \frac{\alpha t}{D^2}$ — **Fourier number** (dimensionless time)

$x^* = \frac{x}{D}$

$\text{Bi} = \frac{hD}{k}$ — **Biot number** (pronounced BEE-OH) Ratio of heat transfer resistance at the boundary to resistance in the solid. This is a transport issue.

AND IT WORKS!

END REVIEW

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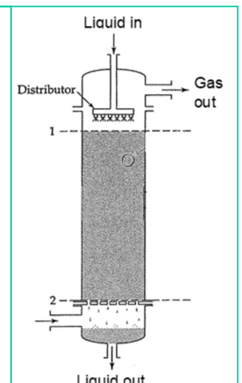
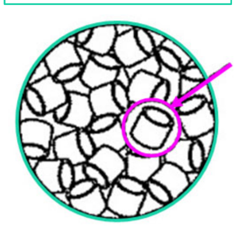
Dimensional Analysis in Mass Transfer

Returning to our question:

What do we do to understand complex mass transfer?

1. Find a simple problem that allows us to identify the physics
2. Non-dimensionalize:
   a. Choose characteristic values
   b. Produce a non-dimensional governing equation
   c. Produce a non-dimensional engineering quantity of interest
3. Explore that problem
4. Take data and correlate (confirm D.A. for chosen problem)
5. Solve real problems with the correlation





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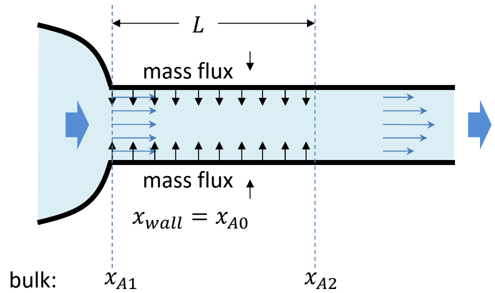
Dimensional Analysis in Mass Transfer

Example: What is the mass transfer through the walls of a permeable tube (laminar or turbulent flow)?



Assumptions:
1. Isothermal
2. Steady flow
3. Uniform inlet composition x_{A1}
4. Constant interfacial liquid composition of x_{A0}
5. ρ, μ, c, D_{AB} all constant
6. Radial mass flux (negative)

$$\text{Total mass in} = \int_0^L \int_0^{2\pi} +cD_{AB} \left.\frac{\partial x_A}{\partial r}\right|_{r=R} R\,d\theta\,dz$$

$$= k_x (2\pi RL)(x_{A0} - x_{A1})$$

BSL2 p679

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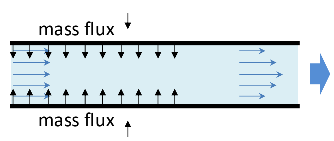
Dimensional Analysis in Mass Transfer

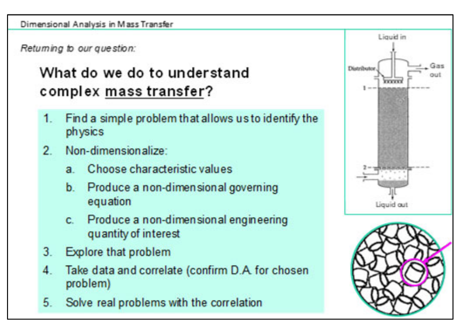
Forced Convection Mass Transfer

Pipe flow



$$k_x (2\pi RL)(x_{A0} - x_{A1}) = \int_0^L \int_0^{2\pi} +cD_{AB} \left.\frac{\partial x_A}{\partial r}\right|_{r=R} R\,d\theta\,dz$$

Next?



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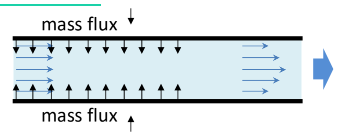
Dimensional Analysis in Mass Transfer

Forced Convection Mass Transfer



Pipe flow

non-dimensional variables:

time:	position:	velocity:	driving force:
$t^* \equiv \frac{tV}{D}$	$r^* \equiv \frac{r}{D}$	$v_z^* \equiv \frac{v_z}{V}$	$P^* \equiv \frac{P}{\rho V^2}$
	$z^* \equiv \frac{z}{D}$	$v_r^* \equiv \frac{v_r}{V}$	$g_z^* \equiv \frac{g_z}{g}$
		$v_\theta^* \equiv \frac{v_\theta}{V}$	

- Choose “typical” values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

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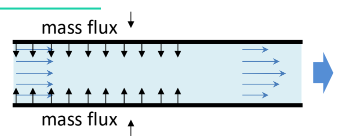
Dimensional Analysis in Mass Transfer

Forced Convection Mass Transfer



Species A Mass

Microscopic species A mass balance (no reaction):

$$c\left(\frac{\partial x_A}{\partial t} + v_r \frac{\partial x_A}{\partial r} + \frac{v_\theta}{r}\frac{\partial x_A}{\partial \theta} + v_z \frac{\partial x_A}{\partial z}\right) = cD_{AB}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial x_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 x_A}{\partial \theta^2} + \frac{\partial^2 x_A}{\partial z^2}\right)$$

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

composition

$$x_A^* = \frac{(x_A - x_{A0})}{(x_{A1} - x_{A0})}$$

Choose:
x_A –use a characteristic interval

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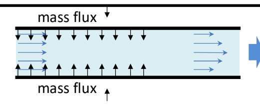
Dimensional Analysis in Mass Transfer—Forced Convection



Non-dimensional Species A Mass Equation

$$\left(\frac{\partial x_A^*}{\partial t^*}+v_r^*\frac{\partial x_A^*}{\partial r^*}+\frac{v_\theta^*}{r^*}\frac{\partial x_A^*}{\partial \theta}+v_z^*\frac{\partial x_A^*}{\partial z^*}\right)=\frac{1}{\mathrm{Pe}_m}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r\frac{\partial x_A^*}{\partial r^*}\right)+\frac{1}{r^{*2}}\frac{\partial^2 x_A^*}{\partial \theta^2}+\frac{\partial^2 x_A^*}{\partial z^2}\right)$$

$$\mathrm{Pe}_m=\mathrm{ReSc}=\frac{VD}{D_{AB}}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*}=-\frac{\partial P^*}{\partial z^*}+\frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^*+\frac{1}{Fr}g^*$$

$$\mathrm{Sc}=\frac{\mu}{\rho D_{AB}}$$

Schmidt number

(a material property)

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*}+\frac{\partial v_y^*}{\partial y^*}+\frac{\partial v_z^*}{\partial z^*}+=0$$

$$\frac{Dv_z}{Dt}\equiv\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}$$

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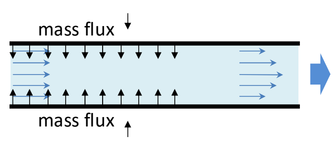
Dimensional Analysis in Mass Transfer

Forced Convection Mass Transfer

Pipe flow

Now, non-dimensionalize this expression as well.



$$k_x(2\pi RL)(x_{A0}-x_{A1})=\int_0^L\int_0^{2\pi}+cD_{AB}\left.\frac{\partial x_A}{\partial r}\right|_{r=R}Rd\theta dz$$

$$\mathrm{Sh}=\frac{k_xD}{cD_{AB}}=\frac{1}{2\pi\left(\frac{L}{D}\right)}\int_0^L\int_0^{2\pi}\left(-\left.\frac{\partial x_A^*}{\partial r}\right|_{r^*=\frac{1}{2}}\right)d\theta dz^*$$

This is a function of Re and Sc through fluid $\underline{v}$ distribution and species A mass balance

Sherwood number, Sh
(dimensionless mass-transfer coefficient)

$$\mathrm{Sh}=\mathrm{Sh}\left(x_A^*,\frac{L}{D}\right)$$

one additional dimensionless group

BSL2 p680

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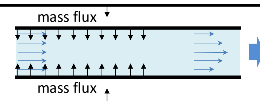
Dimensional Analysis in Mass Transfer



According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of <u>three</u> dimensionless groups:

Peclet number

$$\mathrm{Pe}_m = \mathrm{ReSc} = \frac{VD}{D_{AB}}$$

Schmidt number

$$\mathrm{Sc} = \frac{\mu}{\rho D_{AB}}$$

$$\mathrm{Sh} = \mathrm{Sh}\left(\mathrm{Re}, \mathrm{Sc}, \frac{L}{D}\right)$$

Now, do the experiments.

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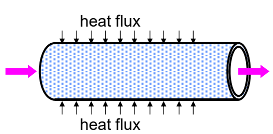
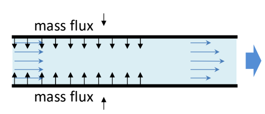
Dimensional Analysis in Mass Transfer

<u>Note</u> this development has been exactly the same as a related heat transfer development:





Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ three dimensionless groups:

no free surfaces

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k}\frac{\rho V D}{\mu}$$

Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

$$Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)$$

Now, do the experiments.

Dimensional Analysis in Mass Transfer

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of <u>three</u> dimensionless groups:

Peclet number

$$\mathrm{Pe}_m = \mathrm{ReSc} = \frac{VD}{D_{AB}}$$

Schmidt number

$$\mathrm{Sc} = \frac{\mu}{\rho D_{AB}}$$

$$\mathrm{Sh} = \mathrm{Sh}\left(\mathrm{Re}, \mathrm{Sc}, \frac{L}{D}\right)$$

Now, do the experiments.

In many cases, heat and mass transfer are **analogous**

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Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum — Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}(\nabla^{*2} v_z^*) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy — Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}(\nabla^{*2} T^*) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass — Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}(\nabla^{*2} x_A^*)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

ref: BSL1, p581, 644

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Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede in the governing equations.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum — Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}(\nabla^{*2} v_z^*) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy — Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}(\nabla^{*2} T^*) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass — Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}(\nabla^{*2} x_A^*)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

Oops! This is dimensionless $\underline{v}$, NOT molar average velocity; sorry!

ref: BSL1, p581, 644

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Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

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Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

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Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

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Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

Transport coefficients

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Dimensional Analysis

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario property*).

Dimensionless numbers from the
Engineering Quantities of Interest

momentum

Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\mathrm{Re}}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(\frac{\partial v_z^*}{\partial r^*}\right)\bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

(Fanning)
f – Friction Factor
$\frac{L}{D}$ – Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy

Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\bigg|_{r^*=1/2} dz^* d\theta$$

Nu – Nusselt
$\frac{L}{D}$ – Aspect Ratio

$$\mathrm{St}_h = \frac{h}{\rho V \hat{C}_p} = \frac{\mathrm{Nu}}{\mathrm{RePr}}$$

$$\mathrm{Nu} = \frac{hD}{k}$$

mass xfer

Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(-\frac{\partial x_A^*}{\partial r^*}\right)\bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

Sh – Sherwood
$\frac{L}{D}$ – Aspect Ratio

$$\mathrm{St}_m = \frac{k_c}{V} = \frac{\mathrm{Sh}}{\mathrm{ReSc}}$$

$$\mathrm{Sh} = \frac{k_c D}{D_{AB}}$$

St – Stanton

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momentum
energy
mass

Dimensionless Numbers

Re – Reynolds $= \frac{\rho VD}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho VD}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario** properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material** properties*).

f – Friction Factor $= \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

Nu – Nusselt $= \frac{hD}{k}$

Sh – Sherwood $= \frac{k_c D}{D_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario** properties*).

$\mathrm{St}_h = \mathrm{Nu}/\mathrm{Pe}_h, \mathrm{St}_m = \mathrm{Sh}/\mathrm{Pe}_m$ – Stanton

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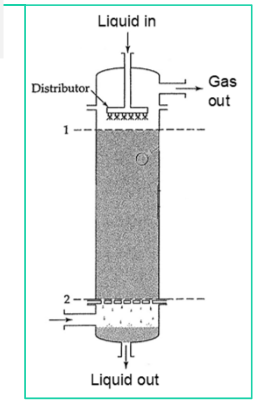
Dimensional Analysis in Mass Transfer

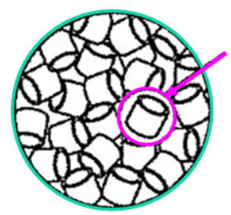
Steps to produce correlations

Returning to our question:

What do we do to understand complex <u>mass transfer</u>?

1. Find a simple problem that allows us to identify the physics
2. Non-dimensionalize:
   a. Choose characteristic values
   b. Produce a non-dimensional governing equation
   c. Produce a non-dimensional engineering quantity of interest
3. Explore that problem
4. Take data and correlate (confirm D.A. for chosen problem)
5. Solve real problems with the correlation





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Dimensional Analysis in Mass Transfer

Steps to use correlations

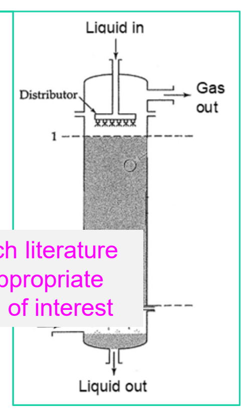
Returning to our question:

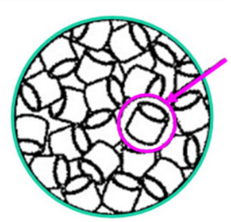
What do we do to understand complex mass transfer?

1. Find a simple problem that allows us to identify the physics
~~2. Non-dimensionalize:~~
   ~~a. Choose characteristic values~~
   ~~b. Produce a non-dimensional governing equation~~
   ~~c. Produce a non-dimensional engineering quantity of interest~~
~~3. Explore that problem~~
~~4. Take data and correlate (confirm D.A. for chosen problem)~~

2. Determine which literature correlation is appropriate for the problem of interest

3. ~~5.~~ Solve real problems with the correlation (be sure to validate choice)





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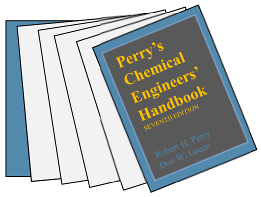
Dimensional Analysis in Mass Transfer



Perry's Chemical Engineers' Handbook

7th edition *(1997)*
Robert H. Perry
Don W. Green

See also: (Green and Southard, 9th edition, 2019)

Section 5: Heat and Mass Transfer

Authors of Mass Transfer:
Phillip C. Wankat
Kent S. Knaebel

Table 5-21: *Correlations* for Mass Transfer: *(pp 5-59 thru 5-77)*

- *From fluid to plate*
- *To a falling film*
- *In pipes and ducts*
- *Past submerged objects*
- *To/from bubbles, drops*
- *In agitated systems*
- *In fixed and fluidized beds*
- *In packed two-phase contactors (absorption, distillation, cooling towers)*

(T)-theoretical
(S)-semi-empirical
(E)-empirical

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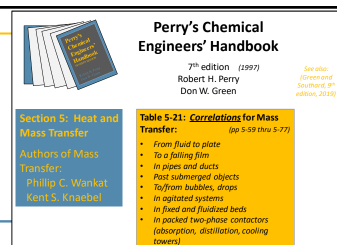
Dimensional Analysis in Mass Transfer



Advice from Wankat and Knaebel

$$Sh = Sh(Re, Sc)$$

1. Because of its importance, there are many studies of mass transfer in the literature
2. For simple geometries, theoretical results are obtainable (T)
3. For very complex systems, only empirical (E) forms can be found
4. Theoretical correlations can be "improved" by fitting to data, resulting in a semi-empirical correlation (S)
5. The major limits and constraints are listed in Perry's Table 5-21; many details are not included, however
6. Readers are *strongly encouraged* to check the references before using the correlations; look for comparisons to actual data
7. Even authoritative sources have typos

(Perry's, 7th ed, p 5-58)

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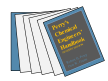
Dimensional Analysis in Mass Transfer

MORE Advice from Wankat and Knaebel



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- *In packed two-phase contactors (absorption, distillation, cooling towers)*

$\mathrm{Sh} = \mathrm{Sh}(\mathrm{Re}, \mathrm{Sc})$

When there are several correlations that are applicable (which often happens), how do we choose?

1. Determine which correlations are closest to the situation under study (similarity of geometries, checking the range of dimensionless numbers and other parameters)
2. Check to see if correlations under consideration have been compared in the literature, both to each other, and to data
3. Check for "rules of thumb" shared by experts

(Perry's, 7th ed, p 5-58 through 5-60)

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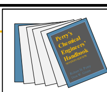
Dimensional Analysis in Mass Transfer

MORE Advice from Wankat and Knaebel



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- *In agitated systems*
- *In fixed and fluidized beds*
- *In packed two-phase contactors (absorption, distillation, cooling towers)*

$\mathrm{Sh} = \mathrm{Sh}(\mathrm{Re}, \mathrm{Sc})$

Rules of Thumb

1. If arithmetic concentration difference was used to determine k for the correlation , that should only be used in such an expression
2. Semi-empirical correlations are often preferred to empirical (do *not* extrapolate empirical) or purely theoretical (can be far off; assumptions)
3. Correlations with a broader data base are preferred
4. Heat/mass transfer analogy is pretty good within its bounds; good heat transfer data (without radiation) can often be used to predict mass-transfer coefficients
5. Recent data is preferred over older data
6. With complex geometries, $k_y a$ (or HTU) correlations are more accurate than k_y correlations
7. **If a mass-transfer correlation looks too good to be true, it probably is.**

(Perry's, 7th ed, p 5-60)

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Dimensional Analysis in Mass Transfer

Routes to Mass Transfer Correlations

Theoretical

- Based on a model of the situation; can be solved for flux, and thus for k_x
- All models have assumptions; how good the assumptions are determine how good the correlations are
- The heat-transfer analogy counts as a theoretical pathway

Semi-empirical

- Shortcomings in theoretical models may be "fixed" by adjusting a theoretically derived coefficient or exponent to achieve a better fit to the data
- The theoretical underpinnings give some hope that extrapolation outside the range of the data fit may be valid

Empirical

- A pure fit to the data is a convenient way to use data directly in downstream calculations
- With no theoretical underpinnings it is extremely unwise to use empirical correlations outside the range of the data fit

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Dimensional Analysis in Mass Transfer

Routes to Mass Transfer Correlations

Theoretical

- Based on a model of the situation; can be solved for flux, and thus for k_x
- All models have assumptions; how good the assumptions are determine how good the correlations are
- The heat-transfer analogy counts as a theoretical pathway

Semi-empirical

- Shortcomings in theoretical models may be "fixed" by adjusting a theoretically derived coefficient or exponent to achieve a better fit to the data
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The Heat/Mass Transfer Analogy

Theoretical Pathway to Mass Transfer Coefficients:

The Heat/Mass Transfer Analogy

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mass

Example: A spherical medical pill dissolves slowly in a stagnant fluid (water). What is the mass transfer coefficient k_c? What is the Sherwood number for this situation?

heat

Example: A spherical pellet of reacting solid slowly emits heats steadily into a stagnant fluid such that the surface temperature is constant. What is the heat transfer coefficient h? What is the Nusselt number for this situation?

BSL2 p321, problem 10B.1, p678

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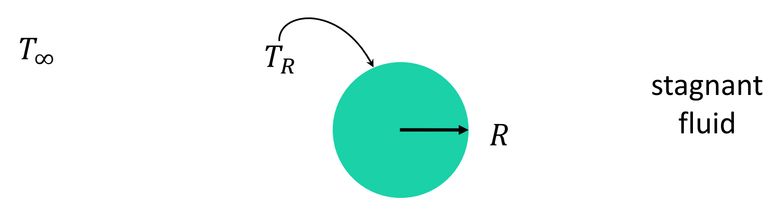

Solve.

BSL2 p321, problem 10B.1

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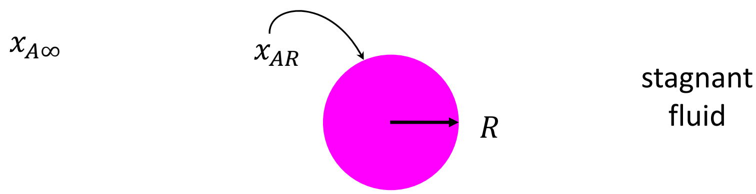

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BSL2 p321, problem 10B.1

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Theoretical Pathway to Mass Transfer Coefficients:

The Heat/Mass Transfer Analogy

Results for transfer from sphere to stagnant fluid:

- $\mathrm{Sh} = \mathrm{Nu} = 2$
- **Limited to low mass transfer rates ($\underline{v}^* \approx 0$)**
- At low mass transfer rates and stagnant fluid, $\underline{J}_A^* = \underline{N}_A$ and $\underline{j}_A = \underline{n}_A$; this makes it easy to convert units moles to mass

Assumptions of the analogy between heat and mass

1. Constant physical properties
2. **Small net mass transfer rates**
3. No chemical reactions
4. No viscous dissipation heating
5. No absorption or emission of radiant energy
6. No pressure diffusion, thermal diffusion, or forced diffusion

BSL2 p681

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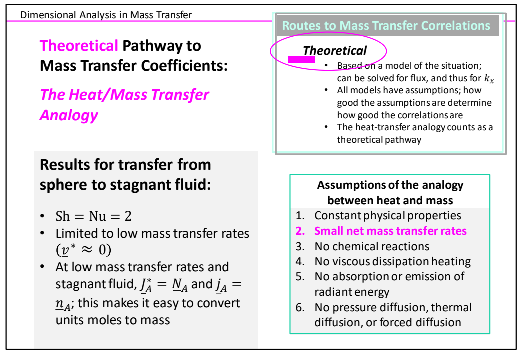
The Heat/Mass Transfer Analogy



Comment from the experts:

> "It would be very misleading to leave the impression that all mass transfer coefficients can be obtained from the analogous heat transfer coefficient correlations. For mass transfer we encounter a much wider variety of boundary conditions and other ranges of relevant variables."

BSL2 p682

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Dimensional Analysis in Mass Transfer

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Dimensional Analysis in Mass Transfer

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Semi-Empirical Pathway to Mass Transfer Coefficients

Inspired by theoretical results and a model (a picture of how the mass transfer may be explained), correlations may be created that are then fine-tuned to match the data

For example, **Colburn's extension of the Reynolds analogy**

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Dimensional Analysis in Mass Transfer

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Reynolds Analogy, Colburn's, Prandtl's extensions

- Reynolds noted the similarities in mechanism between energy and momentum transfer
- He derived, for restrictive conditions ($\mathrm{Pr}=1$, no form drag), the following equation:

$$\frac{h}{\rho V_\infty \hat{C}_p} = \mathrm{St}_h = \frac{f}{2}$$

(Stanton number for heat transfer)

- Coleburn modified the Reynolds result to work at more values of Pr and proposed the following:

$$\mathrm{St}_h \mathrm{Pr}^{2/3} = \frac{f}{2}$$

- This improved relationship does a better job of predicting heat transfer coefficients and
- Separating the turbulent core from the laminar sublayer in boundary layer flow allows it to be extended to mass transfer (Prandtl), resulting in a refined empirical correlation (WRF eqn 28-54)

BSL2 p681; WRF p580

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Dimensional Analysis in Mass Transfer

Empirical Pathway to Mass Transfer Coefficients

Routes to Mass Transfer Correlations

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Inspired by looking at data from a variety of systems, correlations may be created that are fine-tuned to match the data.

These may be based purely on dimensional analysis or there may be a model that the researchers have in mind.

Empirical models are judged by how accurately they represent the data.

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Dimensional Analysis in Mass Transfer

Chilton-Colburn Analogy

Inspired by semi-empirical analogies such as the Reynolds Analogy, define the "j factors":

$$j_H \equiv \frac{\text{Nu}}{\text{RePr}^{\frac{1}{3}}} = \frac{h}{\rho \hat{C}_p V_\infty}\left(\frac{\hat{C}_p \mu}{k}\right)^{2/3}$$

$$j_M \equiv \frac{\text{Sh}}{\text{ReSc}^{1/3}} = \frac{k_x}{c V_\infty}\left(\frac{\mu}{\rho D_{AB}}\right)^{2/3}$$

Compare to data.

Routes to Mass Transfer Correlations

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Chilton-Colburn Analogy

$$j_H = j_M = \frac{f}{2}$$

(f is the Fanning friction factor)

WRF p581

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Dimensional Analysis in Mass Transfer

Chilton-Colburn Analogy

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Conditions:

- Exact for flat plates
- Satisfactory in other geometries as long as form drag is not present
- Relates convective heat and mass transfer
- Permits evaluation of one transfer coefficient through information obtained on another
- Experimentally validated for gases and liquids within the ranges $0.60 \leq \text{Sc} \leq 2500, 0.6 \leq \text{Pr} \leq 100$
- Constant physical properties data

WRF pp581-2

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Dimensional Analysis in Mass Transfer

Final thoughts on Literature Mass-Transfer Coefficient Correlations

- Choose correlation carefully
- Check the original reference (how k_y defined, what are the assumptions, how well does it represent the data
- With complex geometries, $k_y a$ (or HTU) correlations are more accurate than k_y correlations

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Mass Transport "Laws"

We have 2 Mass Transport "laws"

Remaining Topics to round out our understanding of mass transport:

Fick's law of diffusion

D_{AB}

1. ✓ Since we predict $\underline{N}_A$ with Fick's law, we can also predict a mass transfer coefficients k_y or k_c *Relate k_c and D_{AB}*
2. ✓ 1D Unsteady models can be solved (if good at math) *Solutions are analogous to heat transfer*

Mass transfer coefficients

k_c

3. ✓ Combine with macroscopic species A mass balance
4. ✓ Are not material properties; rather, they are determined experimentally and specific to the situation (dimensional analysis and correlations)
5. Facilitate combining resistances into overall mass transfer coefficients, K_L, K_G, to be used in modeling unit operations

Model macroscopic processes, design units

$\mathrm{Sh} = f(\mathrm{ReSc})$

Mass Transport "Laws"

We now have 2 Mass Transport "laws" — Transport coefficient

Fick's Law of Diffusion $\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB}\nabla x_A$

Use: Combine with microscopic species A mass balance
Predicts flux $\underline{N}_A$ and composition distributions, e.g. $x_A(x, y, z, t)$
1D Steady models can be solved
1D Unsteady models can be solved (if good at math) (2)
2D steady and unsteady models can be solved by Comsol
Since we predict $\underline{N}_A$, we can also predict a mass xfer coeff k_y or k_c (1)
Diffusion coefficients are ***material*** properties (see tables)

Linear-Driving-Force Model $|N_A| = k_y|y_{A,bulk} - y_{A,i}|$

Use: Combine with macroscopic species A mass balance (3)
Predicts flux $\underline{N}_A$, but not composition distributions
May be used as a boundary condition in microscopic balances
Mass-transfer-coefficients are ***not material properties***
Rather, they are determined experimentally and specific to the situation (dimensional analysis and correlations) (4)
Facilitate combining resistances into overall mass xfer coeffs, K_L, K_G (5)

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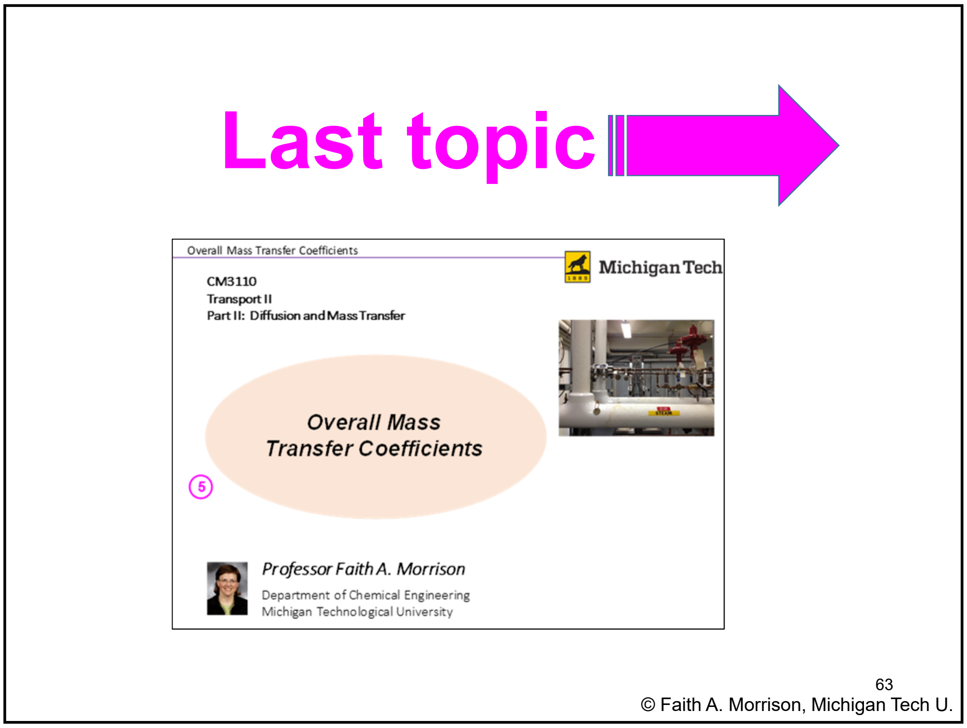
Last topic
Overall Mass Transfer Coefficients
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Transport II
Part II: Diffusion and Mass Transfer
Overall Mass
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Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University
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