Table 2: Summary of Common Analytical Functions Used in Engineering; Examples Attached

Ex.	Function name	Formula	parameters
1	Linear	$y = mx + b$	m, b
2a	Quadratic Upward opening	$y = ax^2 + bx + c$	$a > 0, b, c$
2b	Quadratic Upward opening Centered at x_0	$y = a(x - x_0)^2 + b'(x - x_0) + c'$	$a > 0, b', c', x_0$
3a	Quadratic Downward opening	$y = (-a)x^2 + bx + c$	$a > 0, b, c$
3b	Quadratic Downward opening Centered at x_0	$y = (-a)(x - x_0)^2 + b''(x - x_0) + c''$	$a > 0, b'', c'', x_0$
4	Exponential Growth $(x > 0)$	$\frac{y}{y_0} = e^{x/\lambda}$	$\lambda > 0, y_0$
5	Exponential Decay $(x > 0)$	$\frac{y}{y_0} = e^{-x/\lambda}$	$\lambda > 0, y_0$
6	First-Order Response $(x > 0)$	$\frac{y}{y_\infty} = 1 - e^{-x/\lambda}$	$\lambda > 0, y_\infty$
7	Logarithmic Positive $(x > 0)$	$\frac{y}{\alpha} = \ln\left(\frac{x}{\lambda}\right)$	$\lambda > 0, \alpha > 0$
8	Logarithmic Negative $(x > 0)$	$\frac{y}{\alpha} = -\ln\left(\frac{x}{\lambda}\right)$	$\lambda > 0, \alpha > 0$
9	Power Law Positive $(x, y > 0)$	$\frac{y}{\alpha} = \left(\frac{x}{\lambda}\right)^n$	$\lambda > 0, \alpha > 0, n > 0$
10	Power Law Negative $(x, y > 0)$	$\frac{y}{\alpha} = \left(\frac{x}{\lambda}\right)^{-n}$	$\lambda > 0, \alpha > 0, n > 0$

2 December 2014

The above Table 2 is from a CM3215 Laboratory Exercise written by Prof. Faith Morrison. The appropriate citation is:

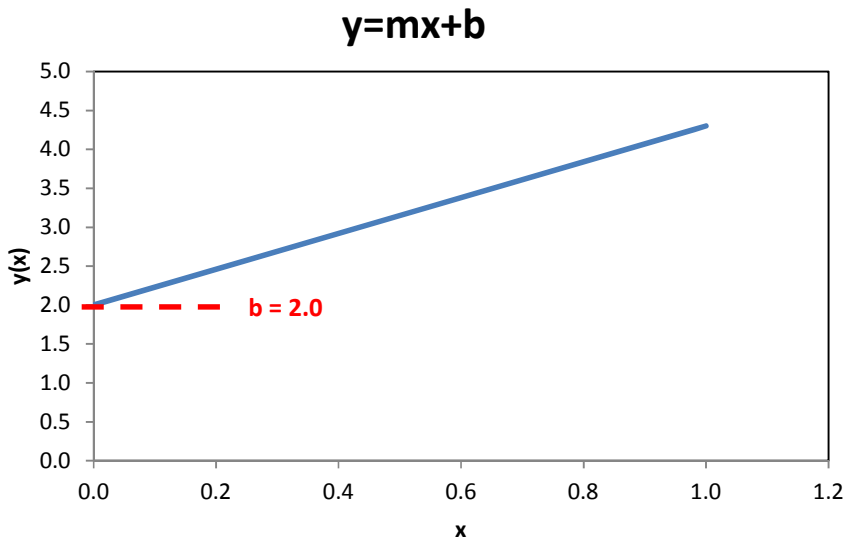
Morrison, Faith A., "Model Identification for Laboratory Data, Appendix, Table 2: Summary of Common Analytical Functions Used in Engineering," CM3215 Fundamentals of Chemical Engineering, Michigan Technological University, Department of Chemical Engineering, Houghton, MI, 2014. Available on the web at (give web address), accessed (give date).

Currently this is available at this link:
www.chem.mtu.edu/~fmorriso/cm3215/AnalyticalFunctionsTable2.pdf

Example 1: Linear

m=	2.3
b=	2.0

x	y
0.00	2.00
0.02	2.05
0.04	2.09
0.06	2.14
0.08	2.18
0.10	2.23
0.12	2.28
0.14	2.32
0.16	2.37
0.18	2.41
0.20	2.46
0.22	2.51
0.24	2.55
0.26	2.60
0.28	2.64
0.30	2.69
0.32	2.74
0.34	2.78
0.36	2.83
0.38	2.87
0.40	2.92
0.42	2.97
0.44	3.01
0.46	3.06
0.48	3.10
0.50	3.15
0.52	3.20
0.54	3.24
0.56	3.29
0.58	3.33
0.60	3.38
0.62	3.43
0.64	3.47
0.66	3.52
0.68	3.56
0.70	3.61
0.72	3.66
0.74	3.70
0.76	3.75
0.78	3.79
0.80	3.84

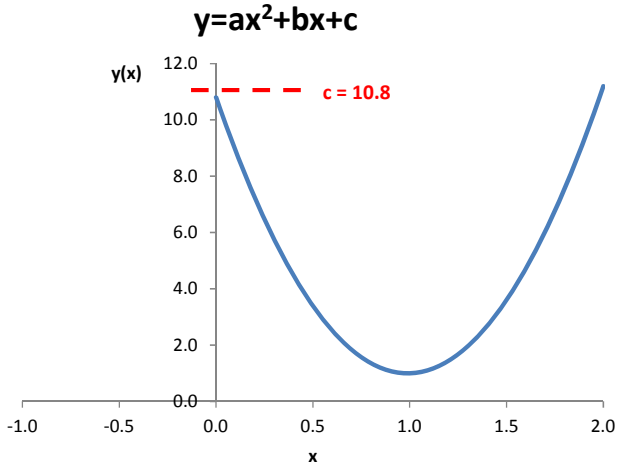


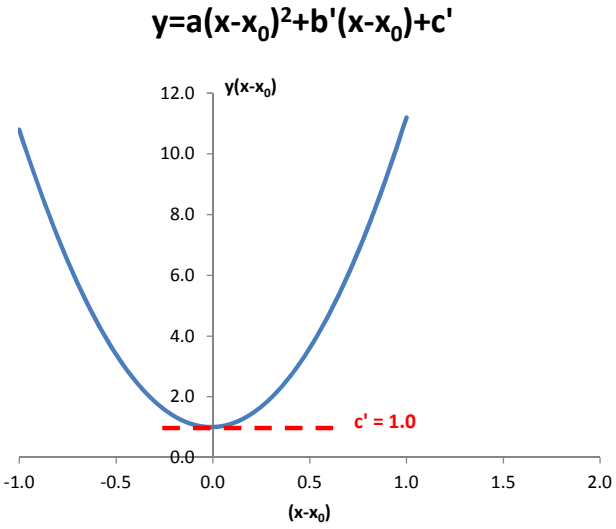
Example 2: Quadratic, Upward Opening

a=	10
b'=	0.2
c'=	1
x_0=	1

a=	10
b=	-19.8
c=	10.8

$x-x_0$	x	y
-1.00	0.00	10.80
-0.96	0.04	10.02
-0.92	0.08	9.28
-0.88	0.12	8.57
-0.84	0.16	7.89
-0.80	0.20	7.24
-0.76	0.24	6.62
-0.72	0.28	6.04
-0.68	0.32	5.49
-0.64	0.36	4.97
-0.60	0.40	4.48
-0.56	0.44	4.02
-0.52	0.48	3.60
-0.48	0.52	3.21
-0.44	0.56	2.85
-0.40	0.60	2.52
-0.36	0.64	2.22
-0.32	0.68	1.96
-0.28	0.72	1.73
-0.24	0.76	1.53
-0.20	0.80	1.36
-0.16	0.84	1.22
-0.12	0.88	1.12
-0.08	0.92	1.05
-0.04	0.96	1.01
0.00	1.00	1.00
0.04	1.04	1.02
0.08	1.08	1.08
0.12	1.12	1.17
0.16	1.16	1.29
0.20	1.20	1.44
0.24	1.24	1.62
0.28	1.28	1.84
0.32	1.32	2.09
0.36	1.36	2.37
0.40	1.40	2.68
0.44	1.44	3.02
0.48	1.48	3.40
0.52	1.52	3.81
0.56	1.56	4.25





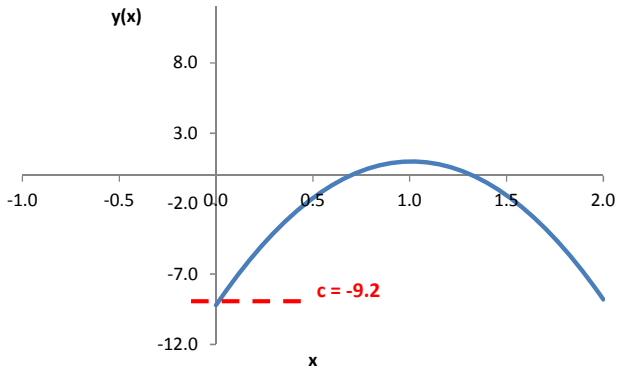
Example 3: Quadratic, Downward Opening

a=	10
b''=	0.2
c''=	1
x_0=	1

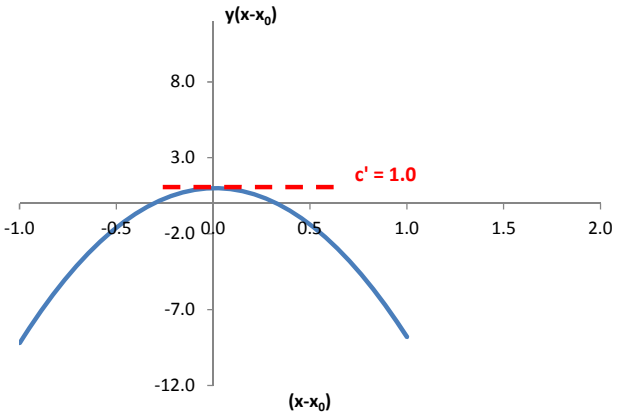
a=	-10
b=	20.2
c=	-9.2

$x-x_0$	x	y
-1.00	0.00	-9.20
-0.96	0.04	-8.41
-0.92	0.08	-7.65
-0.88	0.12	-6.92
-0.84	0.16	-6.22
-0.80	0.20	-5.56
-0.76	0.24	-4.93
-0.72	0.28	-4.33
-0.68	0.32	-3.76
-0.64	0.36	-3.22
-0.60	0.40	-2.72
-0.56	0.44	-2.25
-0.52	0.48	-1.81
-0.48	0.52	-1.40
-0.44	0.56	-1.02
-0.40	0.60	-0.68
-0.36	0.64	-0.37
-0.32	0.68	-0.09
-0.28	0.72	0.16
-0.24	0.76	0.38
-0.20	0.80	0.56
-0.16	0.84	0.71
-0.12	0.88	0.83
-0.08	0.92	0.92
-0.04	0.96	0.98
0.00	1.00	1.00
0.04	1.04	0.99
0.08	1.08	0.95
0.12	1.12	0.88
0.16	1.16	0.78
0.20	1.20	0.64
0.24	1.24	0.47
0.28	1.28	0.27
0.32	1.32	0.04
0.36	1.36	-0.22
0.40	1.40	-0.52
0.44	1.44	-0.85
0.48	1.48	-1.21
0.52	1.52	-1.60
0.56	1.56	-2.02

$y=(-a)x^2+bx+c$



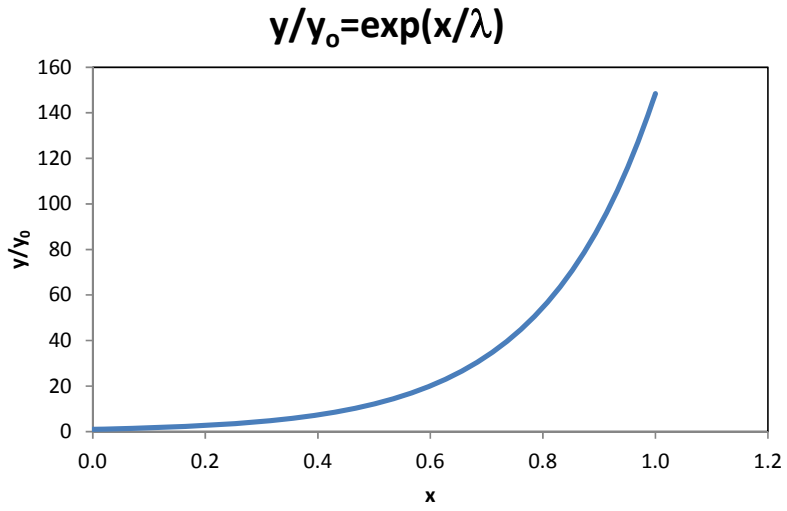
$y=(-a)(x-x_0)^2+b''(x-x_0)+c''$

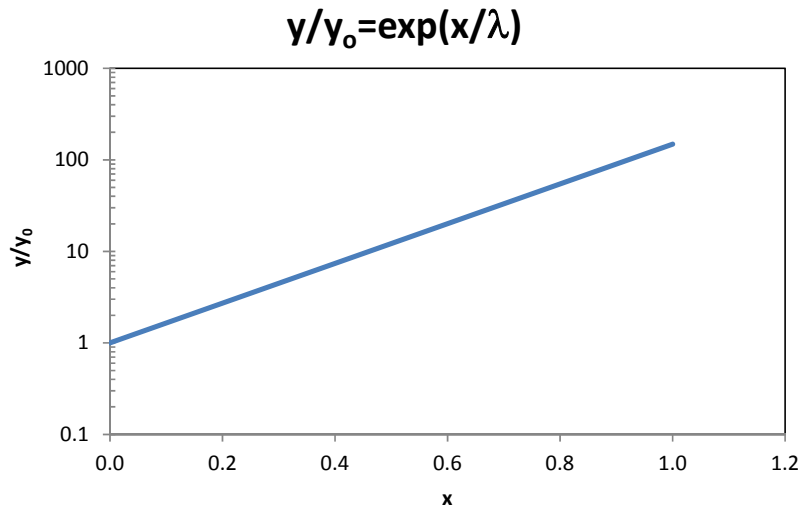


Example 4: Exponential, positive

λ=	0.2

x	y/y_0
0.00	1.00
0.02	1.11
0.04	1.22
0.06	1.35
0.08	1.49
0.10	1.65
0.12	1.82
0.14	2.01
0.16	2.23
0.18	2.46
0.20	2.72
0.22	3.00
0.24	3.32
0.26	3.67
0.28	4.06
0.30	4.48
0.32	4.95
0.34	5.47
0.36	6.05
0.38	6.69
0.40	7.39
0.42	8.17
0.44	9.03
0.46	9.97
0.48	11.02
0.50	12.18
0.52	13.46
0.54	14.88
0.56	16.44
0.58	18.17
0.60	20.09
0.62	22.20
0.64	24.53
0.66	27.11
0.68	29.96
0.70	33.12
0.72	36.60
0.74	40.45
0.76	44.70
0.78	49.40

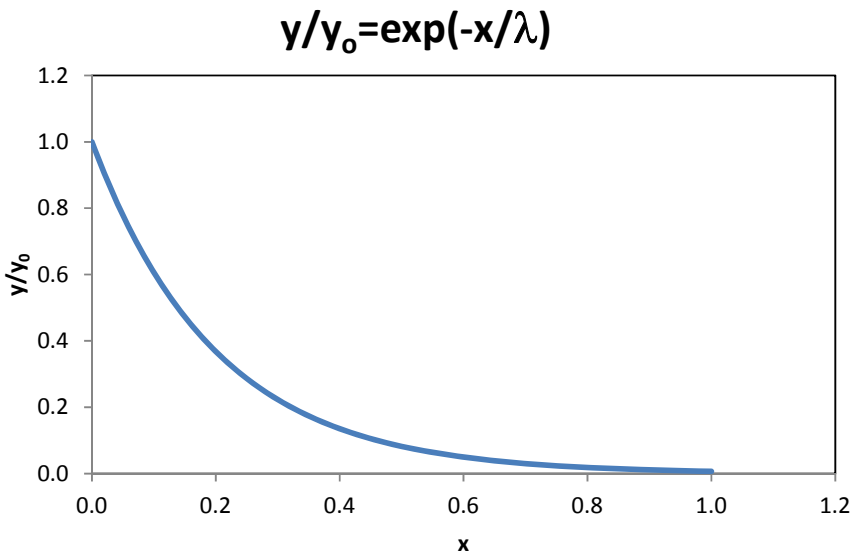


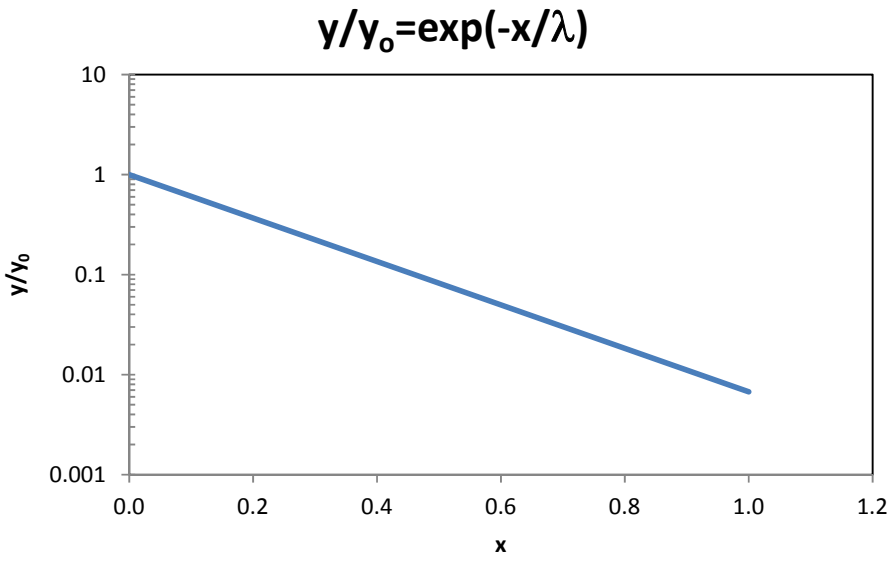


Example 5: Exponential, negative

λ=	0.2

x	y/y_0
0.00	1.00
0.02	0.90
0.04	0.82
0.06	0.74
0.08	0.67
0.10	0.61
0.12	0.55
0.14	0.50
0.16	0.45
0.18	0.41
0.20	0.37
0.22	0.33
0.24	0.30
0.26	0.27
0.28	0.25
0.30	0.22
0.32	0.20
0.34	0.18
0.36	0.17
0.38	0.15
0.40	0.14
0.42	0.12
0.44	0.11
0.46	0.10
0.48	0.09
0.50	0.08
0.52	0.07
0.54	0.07
0.56	0.06
0.58	0.06
0.60	0.05
0.62	0.05
0.64	0.04
0.66	0.04
0.68	0.03
0.70	0.03
0.72	0.03
0.74	0.02
0.76	0.02
0.78	0.02
0.80	0.02



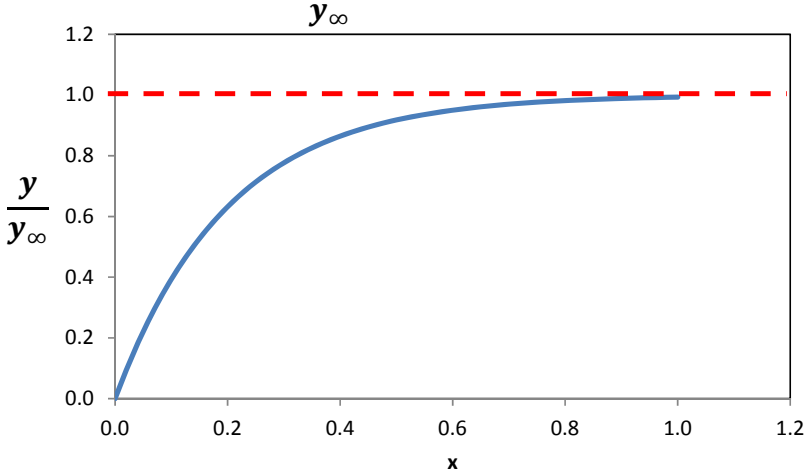


Example 6: First order response

λ=	0.2

$$\frac{y}{y_\infty} = 1 - e^{-x/\lambda}$$

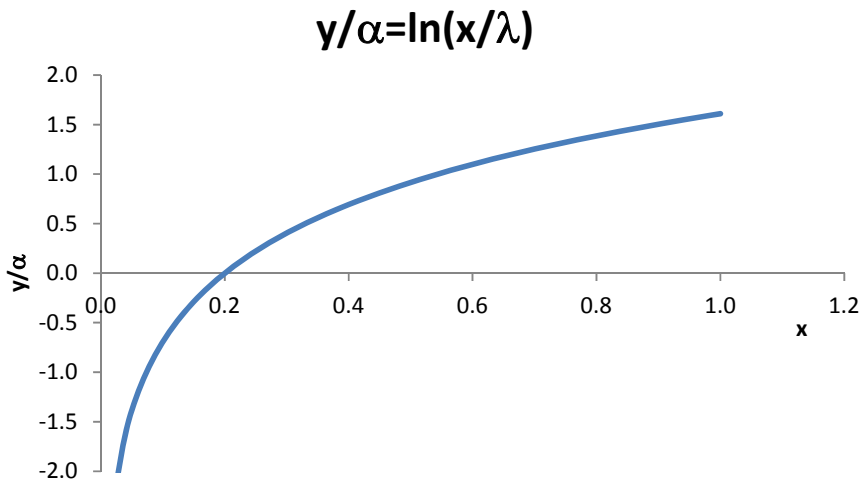
x	y/y_{infty}
0.00	0.00
0.02	0.10
0.04	0.18
0.06	0.26
0.08	0.33
0.10	0.39
0.12	0.45
0.14	0.50
0.16	0.55
0.18	0.59
0.20	0.63
0.22	0.67
0.24	0.70
0.26	0.73
0.28	0.75
0.30	0.78
0.32	0.80
0.34	0.82
0.36	0.83
0.38	0.85
0.40	0.86
0.42	0.88
0.44	0.89
0.46	0.90
0.48	0.91
0.50	0.92
0.52	0.93
0.54	0.93
0.56	0.94
0.58	0.94
0.60	0.95
0.62	0.95
0.64	0.96
0.66	0.96
0.68	0.97
0.70	0.97
0.72	0.97
0.74	0.98
0.76	0.98
0.78	0.98
0.80	0.98

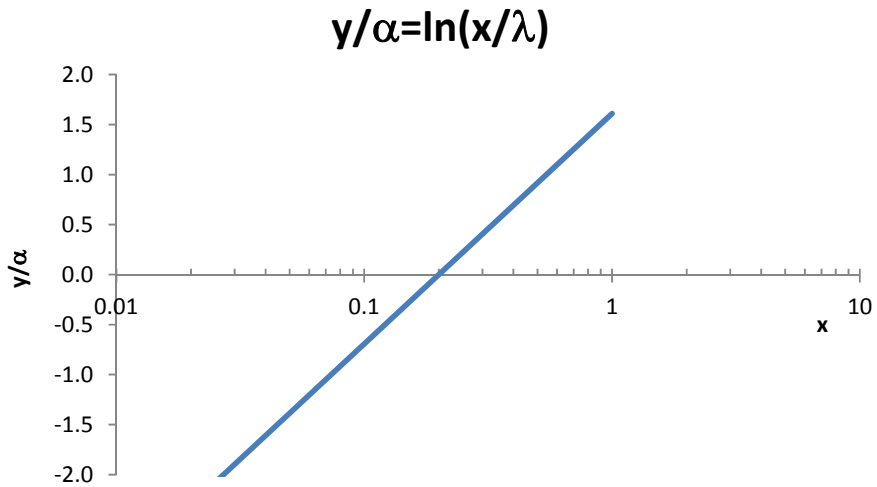


Example 7: Logarithmic, positive

λ=	0.2

x	y/α
0.00	
0.02	-2.30
0.04	-1.61
0.06	-1.20
0.08	-0.92
0.10	-0.69
0.12	-0.51
0.14	-0.36
0.16	-0.22
0.18	-0.11
0.20	0.00
0.22	0.10
0.24	0.18
0.26	0.26
0.28	0.34
0.30	0.41
0.32	0.47
0.34	0.53
0.36	0.59
0.38	0.64
0.40	0.69
0.42	0.74
0.44	0.79
0.46	0.83
0.48	0.88
0.50	0.92
0.52	0.96
0.54	0.99
0.56	1.03
0.58	1.06
0.60	1.10
0.62	1.13
0.64	1.16
0.66	1.19
0.68	1.22
0.70	1.25
0.72	1.28
0.74	1.31
0.76	1.34
0.78	1.36
0.80	1.39
0.82	1.41

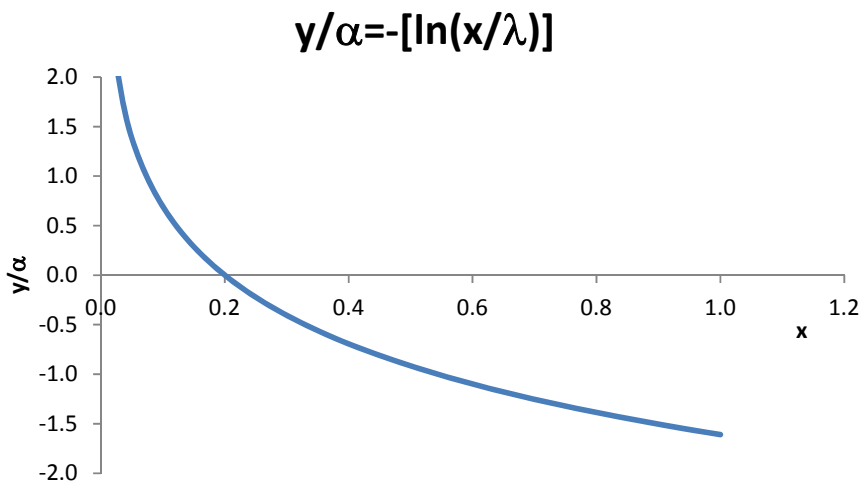


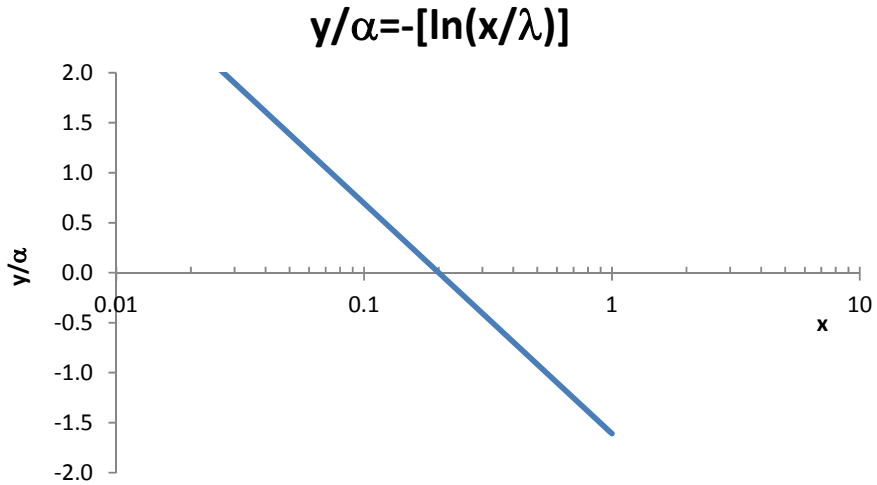


Example 8: Logarithmic, negative

λ=	0.2

x	y/α
0.00	
0.02	2.30
0.04	1.61
0.06	1.20
0.08	0.92
0.10	0.69
0.12	0.51
0.14	0.36
0.16	0.22
0.18	0.11
0.20	0.00
0.22	-0.10
0.24	-0.18
0.26	-0.26
0.28	-0.34
0.30	-0.41
0.32	-0.47
0.34	-0.53
0.36	-0.59
0.38	-0.64
0.40	-0.69
0.42	-0.74
0.44	-0.79
0.46	-0.83
0.48	-0.88
0.50	-0.92
0.52	-0.96
0.54	-0.99
0.56	-1.03
0.58	-1.06
0.60	-1.10
0.62	-1.13
0.64	-1.16
0.66	-1.19
0.68	-1.22
0.70	-1.25
0.72	-1.28
0.74	-1.31
0.76	-1.34
0.78	-1.36
0.80	-1.39
0.82	-1.41

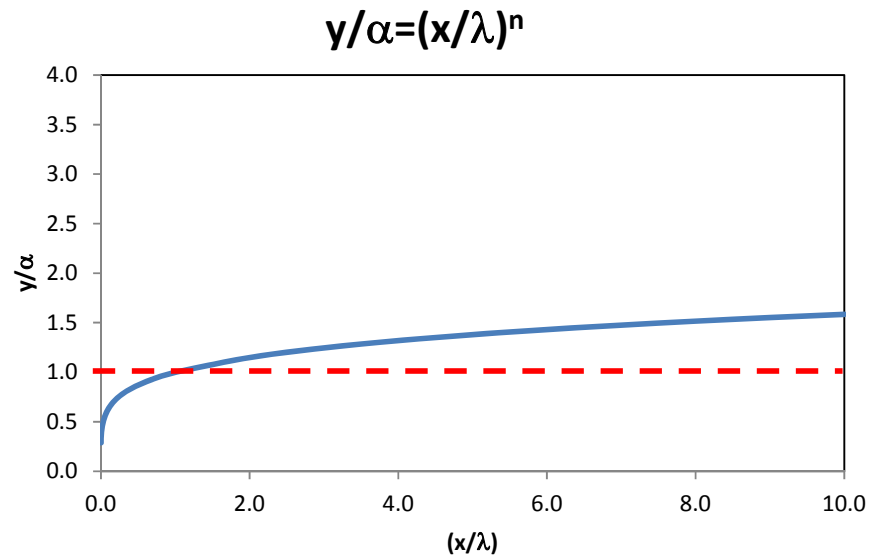


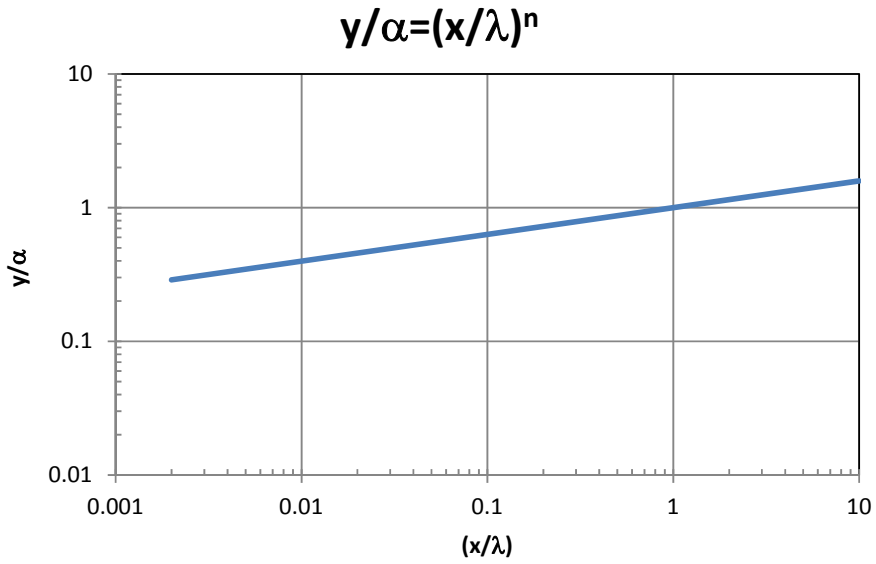


Example 9: Power law, positive

n=	0.2

x/λ	y/α
0.00	
0.002	0.29
0.005	0.35
0.01	0.40
0.02	0.46
0.04	0.53
0.06	0.57
0.08	0.60
0.10	0.63
0.12	0.65
0.14	0.67
0.16	0.69
0.18	0.71
0.20	0.72
0.22	0.74
0.24	0.75
0.26	0.76
0.28	0.78
0.30	0.79
0.32	0.80
0.34	0.81
0.36	0.82
0.38	0.82
0.40	0.83
0.42	0.84
0.44	0.85
0.46	0.86
0.48	0.86
0.50	0.87
0.52	0.88
0.54	0.88
0.56	0.89
0.58	0.90
0.60	0.90
0.62	0.91
0.64	0.91
0.66	0.92
0.68	0.93
0.70	0.93
0.80	0.96
0.90	0.98
1.00	1.00





Example 10: Power law, negative

n=	0.2

x/λ	y/α
0.00	
0.002	3.47
0.005	2.89
0.01	2.51
0.02	2.19
0.04	1.90
0.06	1.76
0.08	1.66
0.10	1.58
0.12	1.53
0.14	1.48
0.16	1.44
0.18	1.41
0.20	1.38
0.22	1.35
0.24	1.33
0.26	1.31
0.28	1.29
0.30	1.27
0.32	1.26
0.34	1.24
0.36	1.23
0.38	1.21
0.40	1.20
0.42	1.19
0.44	1.18
0.46	1.17
0.48	1.16
0.50	1.15
0.52	1.14
0.54	1.13
0.56	1.12
0.58	1.12
0.60	1.11
0.62	1.10
0.64	1.09
0.66	1.09
0.68	1.08
0.70	1.07
0.80	1.05
0.90	1.02
1.00	1.00